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An approximate solution for the inverted four-point bending test in symmetric specimens

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Abstract

An approximate solution is derived for the interfacial energy release rate of the inverted four-point bending test. The analysis builds on a previous model developed by one of the authors for an orthotropic edge-cracked layer subject to arbitrary generalized end forces. Contact forces exerted from the upper on the lower layer of the edge-cracked portion behind the delamination tip are introduced. Their value is chosen such that the two layers undergo the same deflection. The effects of both shear deformations along the layers and friction at the point of contact are taken into account within beam theories and approximate Coulomb model with a prescribed friction coefficient. The delamination energy release rate is derived for homogeneous and symmetric specimens. A parametric analysis is performed on varying the mechanical parameters of the model to analize the influence of shear deformations and friction. The results show that both shear deformations and friction affect the value of the energy release rate for short/intermediate interfacial cracks. For very long interfacial cracks the delamination energy release rate tends to a constant limit value which corresponds to that obtained within classical Euler-Bernoulli beam theory in absence of friction.

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1. Introduction

The Four-Point Bending (4PB) test is one of the fracture mechanics tests used to characterize the delamination toughness of composite laminates. In this test a central notch through one outer layer of the specimen provides the site for interfacial crack initiation and propagation under a loading condition of four-point bending. As originally

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developed by Charalambides et al. (1989), the loads are configured to bend the central section of the specimen so that the crack opens and delamination occurs with a significant mode I component. This test is extensively used in many applications, but it is not applicable to the case of thin brittle layers, which serve as protection for a substrate, as in Thermal Barrier Coatings (TBCs). The thickness of normal coatings is usually very small, so that the energy stored is insufficient to propagate the crack.

In order to generate interfacial delamination of the coating without breaking the thin layer, Hofinger et al. (1998) proposed to modify the four-point bending specimen and introduce a stiffener layer on the top of the thin surface coating. On one end, the stiffener increases the stored energy, therefore decreasing the required load for driving the crack along the interface; on the other, fragmentation of the coating is prevented. The stiffener usually has thickness comparable with that of the substrate and can be made of the same (homogeneous specimen) or of different material (bimaterial specimen). The thickness of the coating is so small compared with the crack length and other characteristic dimensions of the specimen that its presence can be ignored in the evaluation of the energy release rate, as suggested by Suo and Hutchinson (1989). The Modified Four-Point Bending (M4PB) test was used by Théry et al. (2009) to study the degradation of two thermal barrier coating systems during thermal cycling tests. They found that the measured values of the adhesion fracture toughness in the modified bending test correspond to a mixed mode interfacial propagation that is not sufficiently far from mode I; whereas, during spontaneous growth, the interfacial delamination typically occurs under near mode II conditions.

More recently, Vaunois et al. (2017) considered another crack propagation test, the Inverted Four-Point Bending (I4PB) test, and confirmed the effectiveness of this test to evaluate the adhesion fracture toughness in TBC systems under different mode mixity loading conditions. The I4PB test, Fig. 1, was first suggested by Hutchinson and Hutchinsom (2011). The specimen geometry is the same as that of the M4PB specimen. The two tests, however, differ in the applied loading conditions. The loads (or, alternatively, the specimen positioning in the testing machine) are configured to bend the central section of the specimen so that the crack faces come into contact at the center of the specimen and near mode II loading conditions are then produced. Numerical simulations performed through the finite element method show that at least for sufficiently long delaminations the contact zone at the center of the specimen between the free end of the upper layer and the lower layer is small. Outside this small contact area the crack faces are open (Hutchinson and Hutchinson, 2011; Vaunois et al., 2017). If a rigorous calibration technique were available, the test could be used to measure the interfacial toughness under mixed-mode conditions with an important mode II component, also in other layered systems, such as in sandwiches and laminates used for structural applications (Lundsgaard-Larsen et al., 2012).

The paper deals with an approximate solution for the interfacial energy release rate in the I4PB test. The analysis builds on beam theory and employs the analytical expressions for the energy release rate for edge-cracked homogeneous and orthotropic layers subjected to arbitrary generalized end forces derived by Andrews and Massabò (2007) and recently extended to sandwich specimens in Barbieri et al. (2019). In literature, both Euler-Bernoulli and Timoshenko beam theory are at the basis of many delamination models for layered systems (Campi and Monetto, 2013; Monetto, 2015; Pelassa and Massabò, 2015; Monetto and Campi, 2017; Monetto, 2019). Here, the effects of shear deformations along the layers are taken into account through Timoshenko beam theory. Contact between the layers is assumed to occur at a point and is modelled through two opposite vertical forces applied at the point of contact and chosen such that the upper and lower layers undergo the same deflection. The related effects of friction are considered through the approximate Coulomb model with prescribed friction coefficient. The analysis is here restricted to homogeneous and symmetric specimens. In the future the analysis will be extended to multi-layered materials (Barbieri et al., 2019).

Several sets of results are shown and compared. In particular, the influence of shear deformations along the specimen and friction at the point of contact on the delamination energy release rate is analyzed and discussed. The results obtained show that both shear deformations and friction affects the values of the energy release rate for short/intermediate interfacial cracks. For very long interfacial cracks the delamination energy release rate tends to a constant limit value which corresponds to that obtained within classical Euler-Bernoulli beam theory in absence of friction and assuming the delaminated arms to be clamped-in at the crack tip (Hutchinson and Hutchinson, 2011).

Nomenclature

A	right crack tip
a	delamination semi-length
d	distance between the lines of action of the applied forces
h	specimen semi-thickness
E	material longitudinal Young's modulus
F_f	friction forces per unit width, $F_f = \mu P_c$
$\mathcal{G}$	delamination energy release rate
$\mathcal{G}_{EB}$	limit value of the energy release rate
G	material in plane shear modulus
μ	friction coefficient
M	applied moment per unit width, $M = Pd$
P	applied forces per unit width
P_c	contact forces per unit width
x	horizontal axis with the tip at the origin
y	vertical axis with the tip at the origin

2. Beam theory solution

The homogeneous and symmetric I4PB specimen shown in Fig. 1 is now considered. The pre-existing delamination of length $2a$ is positioned symmetrically with respect to the center and at mid-thickness. The material of the specimen is orthotropic and linearly elastic. Focus is then on unidirectional composites. In order to derive the collinear delamination energy release rate for this specimen, because of symmetry, reference is made to its right central portion between the center and an arbitrary section sufficiently far from the right crack tip A and the applied loads. The structure is loaded per unit width as is shown in Fig. 2a: $M = Pd$ is the applied moment transmitted along the specimen through the continuous lower layer; P_c denotes the vertical contact forces between the layers (the lower layer exerts an upward force on the upper layer at its free end to limit the deflection); F_f acts on the neutral axes of the layers and represents the horizontal friction force resisting the relative motion along the x-direction of the two layers in contact (the lower layer exerts a rightward force on the upper layer to limit the rotation of its free end); $F_f h/2$ is the compensating moment which takes into account that the friction forces are applied at the point of contact on the delamination surface.

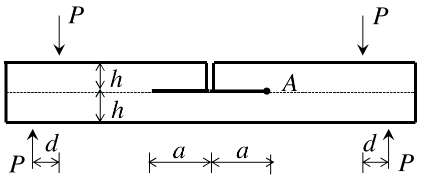

Fig. 1. Inverted four-point bending specimen: geometry and loading condition.

According to the approximate Coulomb model, the friction forces $F_f = \mu P_c$ are assumed proportional to the contact forces, being μ a prescribed friction coefficient. The contact forces are evaluated by imposing a compatibility condition between the deflection at the free end of the upper layer and that at the center of the lower layer. In order to do this, the upper and lower layers behind the crack tip A are treated as cantilever beams having the same length a and rectangular cross section with unit width and height h. According to Timoshenko beam theory, the following expression for P_c in terms of the applied moment and both geometrical and mechanical parameters is obtained

$$P_c = \frac{15aGM}{2(10a^2G + 3h^2E)} \tag{1}$$

where E and G are, respectively, the longitudinal Young's and in plane shear moduli of the material. It is worthwhile noting that friction does not affect the value of the contact force, since, because of symmetry, the related contributions in the deflection expressions cancel one each other. As a special case, the effects of shear deformations along the layers can be neglected assuming an infinite shear stiffness for the layers. In this case Eq. (1) simplifies to $P_{cEB} = 3M/(4a)$, which corresponds to the contact force according to Euler-Bernoulli beam theory (Hutchinson and Hutchinson, 2011).

When the contact force has been obtained, the stress resultants acting on the cross sections immediately preceding and following the crack tip can be determined. Such an equivalent delamination tip loading is shown in Fig. 2b.

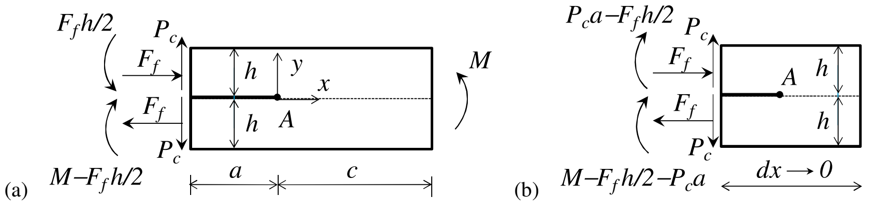

Fig. 2. (a) loads on the edge-cracked right central portion with $M = Pd$; (b) equivalent delamination tip loading.

3. Energy release rate

The energy release rate, $\mathcal{G}$, for the collinear extension of the delamination in the I4PB specimen is determined by employing a convenient expression in terms of the stress resultants acting at the crack tip, shown in Fig. 2b, which was derived by Andrews and Massabò (2007). It is straightforward to obtain

$$\mathcal{G} = \frac{3}{4h^3E}\left[4P_c^2(4a^2+\mu^2h^2)+7M^2-8MP_c(2a+\mu h)\right]+P_c^2\left(\frac{\mu^2}{hE}+\frac{6}{5hG}\right) \tag{2}$$

where the contact force is given by Eq. (1). As observed above, if the shear stiffness tends to infinite, the effects of shear deformations along the layers are neglected. $\mu = 0$ corresponds to the case of frictionless contact. As a special case, for infinite shear stiffness and $\mu = 0$ Eq. (2) reduces to the constant value $\mathcal{G}_{EB} = 3M^2/(h^3E)$ which is independent of the delamination length (Hutchinson and Hutchinson, 2011).

The effects of the shear deformations along the specimen and the presence of friction resistance at the point of contact on the delamination energy release rate are investigated by performing a parametric analysis. $\mathcal{G}$ is calculated for different values of the geometrical and mechanical parameters of the model. The special case of an isotropic material ($G = E/2/(1+\nu)$) is here considered. The results obtained varying the material Poisson's ratio $\nu = 0, 0.3, 0.5$ and friction coefficient $\mu = 0, 0.25, 0.5$ are shown in Fig. 3 and presented in terms of the dimensionless energy release rate $\mathcal{G}/\mathcal{G}_{EB}$ as a function of the dimensionless delamination length a/h.

Except for the special case where the effects of both shear deformations and friction are neglected, the delamination energy release rate increases for increasing delamination length, but the rate of increase becomes smaller as the delamination becomes longer: the percentage increase decreases from less than 1% for a/h=3.5 to less than 0.5% for a/h=4.6 and less than 0.25% for a/h=6.3.

Friction lowers the energy release rate within both Euler-Bernoulli (solid curves in Fig. 3) and Timoshenko (dashed, dotted and dashed-dotted curves in Fig. 3) beam theories. Taking into account the effects of shear deformations along the specimen further lowers the energy release rate especially for sufficiently short delaminations with a/h in the range 1−3.5. In all cases, for very long delaminations the energy release rate tends very slowly to the limit value $\mathcal{G}_{EB}$; the trend rate depends strongly on the friction coefficient and minimally on the Poisson's ratio: for $a/h = 10$ $\mathcal{G}$ is about $0.96\mathcal{G}_{EB}$ for $\mu = 0.25$ and $0.92\mathcal{G}_{EB}$ for $\mu = 0.5$; for $a/h = 20$ $\mathcal{G}$ is about $0.98\mathcal{G}_{EB}$ for $\mu = 0.25$ and $0.96\mathcal{G}_{EB}$ for $\mu = 0.5$; for $a/h = 50$ $\mathcal{G}$ is about $0.99\mathcal{G}_{EB}$ for $\mu = 0.25$ and $0.98\mathcal{G}_{EB}$ for $\mu = 0.5$.

The results in Fig. 3 are similar to those derived in Hutchinson and Hutchinson (2011) using a 2D finite element analysis. Some discrepancies are observed, which are expected to be resolved by accounting for the near tip deformations.

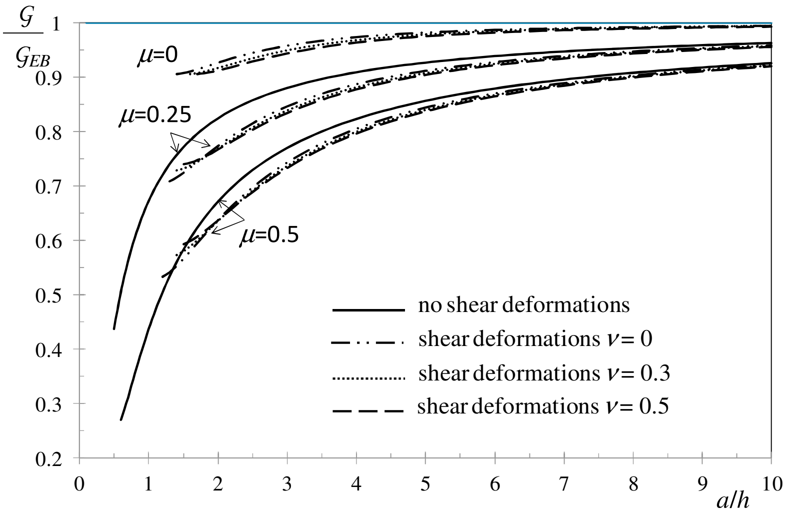

Fig. 3. Effects of friction and shear deformations on the I4PB energy release rate.

4. Conclusions

An approximate solution for the interfacial energy release rate in the I4PB test is presented. The test has been proposed to measure the interfacial fracture toughness in thermal barrier coatings, where delamination growth typically occurs under near mode II conditions; if a rigorous calibration technique were available, the test could be effectively used also for other material systems, such as sandwiches and laminates.

Beam theory is used to determine the contact forces by the imposition of a compatibility condition on deflections of the upper and lower layers at the specimen midspan. The effects of the shear deformations along the layers are taken into account by using Timoshenko beam theory. The contribution of friction at the point of contact is included through the approximate Coulomb model. The energy release rate is then calculated in terms of the stress resultants at the delamination tip.

With reference to homogeneous and symmetric specimens, which the analysis here performed is restricted to, several sets of results have been obtained. In particular, both shear deformations and friction are shown to affect the delamination energy release rate for short/intermediate interfacial cracks. Such effects become more modest for long interfacial cracks. For very long cracks the solution tends to a constant value which is obtained by neglecting both shear deformations and friction effects. Finally, the Poisson ratio appears to have a minimal influence on the adhesion toughness of I4PB specimens.

An improvement of the expression derived here is currently being studied to account for the effects of the near tip deformations. Results will be presented at the meeting. The mode mixity conditions are also currently being defined using the approach in Andrews and Massabò (2007), which provides closed form expressions for the mode I and mode II components of the stress intensity factor (see also Brandinelli and Massabò, 2006, and Massabò et al., 2003).

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