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Incompatibility and entailment in the logic of norms

In this paper I examine critically some theses on the incompatibility and the implication between norms that appear in a well-known article by Bulygin on the conceptions of the logic of norms elaborated, respectively, by Weinberger and Kelsen. I also analyze Bulygin's thesis according to which the expressive conception of norms, defended by Kelsen in the last part of his career, is perfectly capable of explaining the relationship of justification that mediates between general norms and particular norms (especially, between legislated norms and judicial decisions) and, being capable of this, cannot be accused of "irrationalism", as Weinberger argues instead. This work is concluded by formulating a dilemma that Bulygin should face: either the notion of the satisfaction of norms is accepted and the "rationalism" of the expressive conception is saved, or this notion is rejected and with it also the possibility of "expressive" rationality in legal reasoning. | A prior version of this text was presented on 28 July 2015, at the Special Workshop "Bulygin's Philosophy of Law", XXVII IVR Congress, Washington, DC (USA).

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1 FOREWORD

In this essay, I aim at critically examining some of the theses concerning the incompatibility and the entailment between and among norms, which are defended by Eugenio Bulygin in a well-known article,[1] bearing upon the conceptions of the logic of norms articulated by Weinberger and Kelsen (section 2).

I also intend to analyse Bulygin's interesting thesis that given a charitable reading of the theses defended by the "last" Kelsen, the expressive conception of norms (that is to say, that theory that conceives the norms as the results of illocutionary acts of prescribing, carried out on propositional contents) is perfectly capable of explaining the relation of justification that mediates general norms and particular norms (especially, between legislated norms and judicial decisions). Being capable of this, such a conception is immune to accusations of "irrationalism", as argued by Weinberger (section 3).

This work is concluded by formulating a theoretical problem that Bulygin (and with him a large number of of legal theorists and deontic logicians) should

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1 Bulygin 2015.

face: either the notion of satisfiability of norms is accepted and the "rationality" of the expressive conception is saved, or this notion is rejected and with it also the possibility of an "expressive rationality" in legal reasoning (section 4).

2 SATISFIABILITY AND INCOMPATIBILITY

The starting point of Bulygin's analysis is constituted by the well-known theses of the last Kelsen on law and logic.

Kelsen (1991) argues that logic cannot be applied to law because (1) norms lack truth-values, and being logicaly truth-functional, genuinely logical inferences with norms cannot be carried out, (2) norms are linked to the performance of linguistic acts, and logic does not mediate between acts, but between propositional contents.

While admitting that norms lack truth-values, Weinberger (1981) argues that logic can be applied to norms. However, as Bulygin brilliantly shows, some fundamental logical relations remain very obscure in his approach.

For example, Weinberger argues that the norm "~Op" is not the negation of the norm "Op" – as one might expect in analogy with propositional logic – but constitutes its repeal (or, better put, the name of the illocutionary act by means of which such an operation is carried out). This being so, it is impossible to construct the logical connectives in the traditional way, that is, by using the negation for the purposes of inter-defining disjunction, conjunction, and conditional (which, by the way, makes it impossible to apply De Morgan's laws to norms and the definition of the conditional in terms of disjunction or conjunction). With this alone, we would already face a much-weakened form of logic of norms.

In addition, this has important repercussions on the correlated notions of incompatibility and implication. Following Bulygin, let us start with incompatibility. The intuitive way to draw an analogy between propositional contradiction ("p & ~p") and incompatibility between norms is to identify the complex sentence "Op & ~Op" as the paradigmatic case of contradiction between norms. However, we have seen that Weinberger regards "~Op" as the representation of an act of repeal and therefore the notion of contradiction in the normative realm cannot be analogous to the propositional notion.

The other possibility for Weinberger's theory (1981: 70) consists in identifying a normative incompatibility in the statement "Op & O~p", that is between the mandatory and the prohibited.

On this point, Bulygin (2015: 212) observes:

> In what sense can these two norms be said to be inconsistent? Clearly not in the same sense as "p" and "~p" are inconsistent, for norms are neither true nor false. Nor, for reasons of logic, would it do to say that these two norms cannot both be obeyed or

> satisfied (at the same time). This is certainly correct, but the impossibility of satisfaction is due to the fact the propositions "p" and "~p" (that is, the contents of the two norms) cannot both be true, so we are faced here with an inconsistency of norm-content and not of norms. This inconsistency of norm-content entails the possibility of satisfying both norms, but it does not follow then that the norms "Op" and "O~p" are inconsistent as well. Therefore, if the alleged contradiction between "Op" and "O~p" were only to mean that the two norms cannot be satisfied for reasons of logic (that is, independently of all experience), this would be just another way of saying that the propositions "p" and "~p" are contradictory, that is, there would be nothing other than an inconsistency between (descriptive) propositions.

Beyond the philological problems in Weinberger's work, which occupy the remaining pages of the section to which this quotation belongs and on which I will not elaborate here, it is important to discuss, even briefly, the idea apparently suggested by Bulygin that the concept of satisfiability (that is, the fulfilment of what a norm prescribes[2]) must be dismissed as a conceptual tool for examining the notion of incompatibility between norms.

By rejecting the notion of satisfiability, Bulygin (2015: 214) comes to the general conclusion that "the coexistence of 'Op' and 'O~p' would certainly be undesirable and impractical", precisely because they cannot both be fulfilled at the same time, but "this alone does not justify our calling them logically inconsistent".

At its core, Bulygin argument is that there is *no univocal way* to determine when two norms are incompatible, once the possibility of constructing, or assuming, an analogy with what occurs in the propositional realm with respect to negation, is discarded.

However, this thesis seems liable to at least two different readings.

1) The first reading, which we might call *cautious*, is limited to arguing that the notion of incompatibility between norms is an intuitive notion, susceptible of being assumed rather than explained. This seems to be the understanding of the typical incompatibility elaborated by the authors that embraces some form of hyletic conception of norms,[3] that is to say, the position that conceives norms like quasi-propositional entities. The main argumentative move these authors make consists in drawing some kind of analogy with the

2 It is often argued that the concept of satisfiability would not apply to permissions. This thesis is not entirely correct, since one only needs to introduce the temporal dimension in order to distinguish the conditions of satisfiability of the permissions and the other normative modalities (or situations). Cf. Moreso & Vilajosana 2004: 79-80. However, it is true that, from a strictly synchronic perspective, permissions and obligations seem to have the same conditions of satisfiability.

3 Alchourrón & Bulygin 1971 seem to defend this view.

alethic modalities and/or the quantifiers of predicate logic,[4] and then developing some asserted peculiarities of the logic of deontic operators.

2) By contrast, the second reading, which we can call *radical*, holds that it is impossible to delineate a genuine notion of logical incompatibility between norms, so that – as Bulygin says – the simultaneous presence of "Op" and "O~p" in the same set of norms would be undesirable, but it would not allow us to conclude without further ado that the set at hand is logically incoherent. This reading, taken to its extremes, not only irremediably undermines the concept of incompatibility between norms, but also seems to question such basic principles for the concept of implication in standard deontic logic as "Ought implies may". This is due to the well-known fact that, in the standard system of deontic logic, the equivalence "Op⊃Pp≡~(Op & O~p)" is valid, that is, the derivation of a permission from an obligation is equivalent (by definition of the conditional) to the simultaneous non-admissibility (or, according to different interpretations, obligation or satisfiability) of the corresponding obligation and prohibition.

The latter would also have obvious repercussions on the notion of implication between norms. By rejecting the notion of incompatibility between obligation and prohibition, we would also reject one of the axioms of the logic of norms and therefore eliminate many of the inferences that can be made with them, unless we elaborate a new logic of norms with quite different rules of inference. This is precisely what the expressive conception of norms intends to do. Such a conception, in its most radical form, does not admit the derivability of permissions from obligations, since the mere existence of an act of allowing cannot be inferred from the existence of an act of prescribing. Although, as we shall see shortly, this conception admits some "mediated" forms of derivability between norms based on the derivation between normative contents.

I will not, however, elaborate here on the attempts made by Bulygin and others,[5] to build a genuine expressive logic of norms. Instead, in the next section I shall underline the main aspects of the explanation of the justification of judicial decisions offered by the expressive conception.

3 EXPRESSIVE CONCEPTION AND LOGICAL DERIVATION

In the last part of his article, Bulygin offers a criterion of justification for judicial decisions from the perspective of the expressive conception of norms.

4 A remarkable exception is found in Navarro & Rodríguez 2014, who develop a complex possible worlds logic as the foundation of the logic of norms.

5 See, for instance, the seminal Alchourrón & Bulygin 1984 and, in more recent literature, the excellent Kristan 2014.

This criterion consists in the logical derivability not between norms, which for this conception are the results of acts of prescribing and therefore logically inert facts, but between the propositional contents of such acts.

On this point, Bulygin (2015: 218) writes:

> If norms depend on acts of prescribing, then there clearly is no logical entailment between a general and an individual norm (for example, between the act of the lawmaker and the act of the judge). There may, however, be a logical relation of deducibility between the contents of these two acts […] I shall illustrate this with an example. Suppose the legislative authority issues a general norm to the effect that all landowners should pay a special tax. The proposition commanded by the lawmaker (that is, the content of this norm) is that all landowners are to pay a special tax, and so it is true that all landowners have an obligation to pay the tax (or, as we might also say, they ought to pay it). Now, from "all landowners are to pay the tax", it follows that landowner A is to pay the tax, so the proposition "A is to pay the tax" belongs to the commanded set, and therefore it is true that A has an obligation to pay the tax.

Obviously, connecting the logical derivation between two norms with different degrees of generality with the truth of the normative contents ordered by them, is the same as saying that one norm entails the other when their conditions of satisfaction (or equivalently, the truth-conditions of its normative contents) are logically linked: it is not possible for the sentence "All landowners ought to pay the tax X" to be true and the sentence "Landowner A pays the tax X" be false. Put another way, it is not possible that the general rule "All landowners ought to pay the tax X" is satisfied (that is, that it is fulfilled by all its recipients) while the individual norm "The landowner A ought to pay the tax X" is not satisfied (that is, the recipient A does not comply).

This seems to lead Bulygin towards a conceptual tension in his expressivist approach. Indeed, the notion of satisfiability of norms, which, as we have seen, had been rejected previously, suddenly reappears here, since a norm can be regarded as satisfied precisely when the propositional content it prescribes corresponds to reality.

Beyond this, it should be noted that the conditions for satisfying a norm do not say anything about reality (that is, whether certain norms are *actually fulfilled* or not), in the same way that the truth conditions of a proposition say nothing about whether a certain proposition is actually true or not. These well-known circumstances lead Bulygin (2015: 218) to affirm that "Being a landowner, A has an obligation to pay the tax, but it may very well be the case that he does not pay the tax in the allotted time and so does not fulfil his obligation". This is correct: from the fact that the effective satisfaction of a general norm necessarily entails the satisfaction of an individual norm, it does not follow that both norms are actually fulfilled. Moreover, the lack of satisfaction of the individual norm is a reason to conclude that the general norm by which it is entailed can no longer be satisfied (and this is nothing more than *modus tollens* applied

to the abstract proposition that the satisfaction of the general norm entails that of the individual norm). All this would seem to offer a logical criterion to determine, from an expressivist perspective, whether the judicial conclusion follows from the premises (in particular, from the normative ones).

As Bulygin (2015: 218) observes, however, events in law do not usually stop here. If A does not fulfil his obligation, then

> he can be brought before a court. Now the norm that regulates the activity of the judge does not prescribe that all landowners who fail to pay their taxes be sentenced, that is, all those of whom it is true that they ought to pay their taxes and did not pay in the allotted time. Rather, the norm prescribes that all of those of whom it has been *proven in a court of law* that they ought to pay and did not pay be sentenced.

Here, Bulygin already moves away from the strictly logical problem of justification according to the expressive conception to point to a contingent problem. The problem consists in the fact that the relations between what must be done according to the law and the legal consequences that follow from not having realized what was due according to the law, are mediated, at least in contemporary legal systems, by procedural law. Such a branch of law requires that the facts, on the basis of which a certain subject has been prosecuted, be proved.

Profitting from the well-known distinction between the primary or subject's system and the secondary and judge's systems, Bulygin (2015: 218–219) affirms:

> The two systems are related in the sense that the secondary or the judge's system presupposes the existence of the primary or the subject's system; thus, they are found at different levels. This gives rise to some interesting situations that might well seem paradoxical. For instance, it may be true that A ought to pay his taxes and did not pay them but that judge nevertheless ought not to sentence him (if, for example, A's failure to pay has not been proven in court). Vice versa, it may be true that the judge ought to sentence A for not having paid his taxes, although it is not true that A failed to pay.

All this does not, however, have much to do with the expressive conception in itself, but concerns wider considerations of a general theory of law.[6] What

6 However, it should be noted in passing that from this discussion, Bulygin draws two remarkable consequences. First, what is to be proven in the trial is determined by substantive law and not by procedural rules. Second, the fact that the judge's decision, although legal, is not justified by substantive law makes it possible to say that it is based on a wrong deliberation. From this it follows, according to Bulygin (2015: 219), that "those theories that tend to interpret all legal norms as directives addressed to the courts are deeply mistaken. They lead not only to a distortion of the function of the law but also to a most inconvenient limitation on the expressive capacity of legal language". It seems to me that on this point Bulygin's reasoning ends up being a *non sequitur*. If, as Bulygin himself seems to affirm, an individual norm has as a justification condition its derivability, because of its normative content from a general norm, how can it be that an individual norm that correctly applies the relevant procedural rules brings about a wrong decision? If norm X imposes on the judge the duty to fine the subjects whose tax evasion has been proven at trial, and the tax evasion of A has not been proven at trial, how can it be held, from Bulygin's own perspective, that the decision that avoids sanctioning A is wrong? The interaction between the subject's system and the judge's system is a

must be stressed here is that there does not seem to be room for an expressive logic without the concept of satisfiability. However, to the extent that it is in the jurists' interest to explain the derivability of particular norms from general norms, the expressive conception seems completely capable of performing this task.

4 CONCLUSION

What has been said so far generates a fundamental problem for Bulygin himself, and with him, for a large part of legal theory and deontic logic.

The problem is the following: The expressive conception is undoubtedly right in underlining that norms lack truth-values, and in maintaining that the existence of any positive norm depends on an act of promulgation. On this matter, it seems to reconstruct the ideas of the jurists much better than the hyletic conception. From these premises, it is easy to conclude that there is no logic of norms and therefore no way to control the rationality of norms-based decisions.

One possible way out of this pessimistic conclusion (perhaps the only truly viable way out) is to construct a logic of norms based on the notion of satisfiability, as Alchourrón and Bulygin themselves sometimes seem to suggest by constructing a logic of normative contents.

However, we have seen that Bulygin is reluctant to define the main logical relationships on the basis of the notion of satisfiability. This is a fairly common resistance in the literature on the logic of norms,[7] for – so it is argued – to found a logic of norms on a factual notion as satisfiability cancels any normative peculiarity of such logic. Perhaps, this is the "price" to pay to harmonize all the ideas at stake. It does not, however, seem to be too high a price, since – as Bulygin correctly points out – the main logical relation that jurists try to explain concerns the derivation of individual norms from general norms: on this sco-

contingent problem, so to speak, of a systematic interpretation of law; but there is little doubt that, in contemporary legal systems, it is the judge, in the end, who has to decide about such systematic relationships. And there is nothing wrong in saying that 'A has an obligation to pay taxes according to the individual norm Y', derived from the substantive general norm Y, and at the same time, that the judge has the obligation to fine those subjects whose tax evasion has been proven and, therefore, has no obligation to sanction A, whose tax evasion has not been proven. It seems clear that the theories criticized by Bulygin, which hold that all legal norms are directives addressed to the courts, have theoretical objectives different from those of Bulygin. In particular, they want to provide jurists with ways to predict what the courts are going to decide, so that it is not surprising (and much less wrong) that they focus on norms that, beyond the content of substantive law, empower the judge to make ultimate decisions (that is, they attribute the value of *res judicata* to their determinations). On this point, see at least Schauer 2009: Ch. 7.

7 In recent literature, see Navarro & Rodríguez 2014: 54-55.

re, an expressive logic, based on the notion of satisfiability, seems to be above reproach.

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