

# How Long does a Generation Last? Assessing the Relationship Between Infinite and Finite Horizon Dynamic Models\*

## Abstract

This note aims at assessing the temporal relationship that exists between the time reference of dynamic models with infinite and finite horizon. Specifically, comparing the optimal inter-temporal plans arising from an infinite-horizon model and a 2-period overlapping generations model in their stationary equilibria, I suggest way to assess the number of time periods of the former that form a time unit of the latter. Relying on an argument grounded on consumption smoothing, I show that the theoretical length of a generation is an increasing function of the discount factor of the optimizing agent. Moreover, from an empirical point of view, I give evidence that this analysis corroborates the well-documented nexus that links demographic developments and the path of interest rate, and it offers interesting insights for the calibration of discount rates in computational models.

**Keywords:** Infinite horizon; Overlapping generations; Consumption smoothing; Discount rate; Demographic developments.

**JEL Classification:** C61, C68, E21, E30.

## 1 Introduction

According to a widely accepted view, the length of a generation, i.e., the number of periods between successive young-old relationships in human communities, is about 25 years.<sup>1</sup> As proof of this, a quarter of a century is also the interval usually acknowledged for the length of a generation by demographers and geneticists (e.g., Weiss, 1973; Thomson et al. 2000). On their side, economists

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<sup>1</sup> The young (old) agents are usually the children (parents) of the old (young) ones born in the previous period.

studied a variety of phenomena that involve the behaviour of human beings in different stages of their life (cf. Samuelson, 1958; Diamond, 1965; Galor and Weil, 1996). To the best of my knowledge, however, these scholars took the generation length as given without any attempt to provide a criterion to measure the duration of young-old relationships that are typical of finite-time models. In this note, I aim at filling this gap by evaluating the length of a generation from an exquisitely economic point of view.

The starting point of my investigation is the assessment of the relationship that holds between the time reference of commonly used models with infinite and finite horizon. Specifically, comparing the consumption plans arising from an infinite horizon (IH) model and from a companion 2-period overlapping generations (OLG) model in their respective stationary equilibria, I provide a way to determine the number of time periods of the former that form a time unit of the latter. In other words, analysing the behaviour of households endowed with logarithmic preferences that puts forward a plan aimed at financing their consumption expenditure by means of their wealth, I show that the hypothetical length of a generation depends on how heavily households themselves discount their future utility streams. From an empirical point of view, this analysis validates the link between demographic developments and the path of real interest rates pointed out by several contributions and provides some insights for the calibration of discount rates in real-business-cycle (RBC) models.

This note is arranged as follows. Section 2 describes the building blocks. Section 3 develops the IH model. Section 4 sets out the OLG model. Section 5 makes a comparison between the consumption plans arising from the two models. Section 6 explores the empirical implications of the theoretical analysis. Section 7 concludes.

## 2 The general framework

Taking time as a discrete phenomenon, I consider an IH and a OLG model in which a representative household endowed with logarithmic instantaneous preferences puts forward an optimal plan aimed at financing its consumption expenditure by means of its wealth with no regard for bequests. In addition, I make the hypothesis that the household discounts its future utility streams with a constant discount rate that is assumed to coincide with the yield recognized by the capital market.<sup>2</sup>

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<sup>2</sup> In this case, a stationary consumption plan implies the equality between the marginal rate of substitution between two consecutive levels of consumption and their relative price.

Positing that the household is called in to choose among  $n \geq 1$  goods, the maximandum of its problem can be written as

$$\sum_{t=s}^T \left(\frac{1}{1+r}\right)^{t-s} \sum_{i=1}^n g_i \log(c_{i,t}) \quad i = 1, \dots, n \quad t = s, \dots, T \quad (1)$$

where  $s$  ( $T$ ) is the starting (final) period,  $r > 0$  is the real interest rate,  $c_{i,t}$  is the consumption of the  $i$ -th good at time  $t$  and  $g_i$  is the weight assigned to the  $i$ -th good.

For sake of simplicity, I assume that the instantaneous utility is a log-linearization of a homogenous function of degree one so that

$$\sum_{i=1}^n g_i = 1 \quad (2)$$

In its starting period, the household is also endowed with a real wealth equal to  $W_s$ . This value can be thought as the sum of its human and financial wealth evaluated at the beginning of its optimization problem and it can be alternately consumed or – if saved – invested in the capital market at the prevailing interest rate.

From a human-capital perspective,  $W_s$  can be thought as the present discounted value of labour incomes earned by the household by selling period-by-period in a competitive labour market a fixed endowment of labour services (cf. Becker, 1962). By contrast, from a financial perspective,  $W_s$  can be seen as a universal basic income transferred to the household at the beginning of its life by a public authority for the mere fact of being born (cf. Ghatak and Maniquet, 2019). Furthermore, it can be considered as the value of the initial stock of capital augmented by the discounted value of the proceeds accruing from renting a fixed factor – such as land – to the productive sector (cf. Zhang, 2021). Consequently, the household's budget constraint is of the form

$$\sum_{t=s}^T \left(\frac{1}{1+r}\right)^{t-s} \sum_{i=1}^n c_{i,t} \leq W_s \quad (3)$$

Equation (3) states that the present value of the total consumption expenditure carried out from  $s$  to  $T$  cannot be higher than  $W_s$ . Moreover, since the interest rate used to discount future consumption expenditure is equal to the one used to discount future utilities, such a constraint will imply the stationarity of the consumption plans no matter the underlying time horizon (cf. Ramsey, 1928; Cass, 1965; Koopmans, 1965).

In the remainder of this note, I will use  $t$  ( $\tau$ ) to denote the time unit of the IH (OLG) model. Consequently,  $T$  will be equal to  $\infty$  for the IH model whereas in the OLG model – in which the household is initially young, then after a period it becomes old and then dies –  $T$  will be equal to  $s + 1$ . Obviously, it seems reasonable to argue that  $\tau > t$ , i.e., that the time horizon covered by a period of the OLG model is longer than the one covered by a single period of the IH model.

Consistently with the hypothesis that  $\tau$  is greater than  $t$ , I will assume also that the real interest rate plugged into the IH model – indicated by  $r_{IH} > 0$  – is strictly lower than the one plugged into the OLG model – denoted instead by  $r_{OLG} > 0$ . Everything else being equal, this hypothesis means that the young household of the OLG model will discount future consumption streams more heavily than the corresponding household of the IH model. At the same time, however, the old household of the OLG model will enjoy higher returns on its savings. Given this general framework, the main goal of the analysis developed below is to provide a way to assess the magnitude of  $\tau$  over  $t$ .

### 3 The IH model

Here I develop an IH model that draws on Farmer and Plotnikov (2012) and Farmer (2010, Chapter 6) in which optimizing households are endowed with preferences defined over the same commodity space conveyed by (1) and (2). Specifically, the problem of the infinitely lived household is assumed to be the following:

$$\max_{\{c_{i,t}\}_{i=1}^n}_{t=s}^{\infty} \sum_{t=s}^{\infty} \left(\frac{1}{1+r_{IH}}\right)^{t-s} \sum_{i=1}^n g_i \log(c_{i,t}) \quad (4)$$

s.to

$$\sum_{t=s}^{\infty} \left(\frac{1}{1+r_{IH}}\right)^{t-s} \sum_{i=1}^n c_{i,t} \leq W_s \quad (5)$$

The problem above can be solved by writing the implied Lagrangian. Hence,

$$\mathcal{L}(\cdot) \equiv \sum_{t=s}^{\infty} \left(\frac{1}{1+r_{IH}}\right)^{t-s} \sum_{i=1}^n g_i \log(c_{i,t}) - \lambda \left( \sum_{t=s}^{\infty} \left(\frac{1}{1+r_{IH}}\right)^{t-s} \sum_{i=1}^n c_{i,t} - W_s \right) \quad (6)$$

where  $\lambda$  is the Lagrange multiplier.<sup>3</sup>

The first-order conditions (FOCs) for (6) are given by the following sequences:

$$g_i - \lambda c_{i,t} = 0 \quad i = 1, \dots, n \quad t = s, \dots, \infty \quad (7)$$

Recalling the result in (2) and aggregating over the  $n$  consumption goods reveals that the expressions in (7) can be written as

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<sup>3</sup> Whenever a dynamic maximization/minimization problem can be written by omitting the evolution of the underlying state variables, standard optimization techniques can be used so that there are no sequences for the Lagrange multipliers but only a single expression. See, for example, the text-book treatment of the Ramsey model offered by Romer (2019, Chapter 2).

$$C_t = \frac{1}{\lambda} \quad t = s, \dots, \infty \quad (8)$$

where  $C_t \equiv \sum_{i=1}^n c_{i,t}$  is the total consumption expenditure at time  $t$ .

Plugging the result in (8) into (5), allows us to write down the Lagrange multiplier as a function of  $r_{IH}$  and  $W_s$ , that is

$$\lambda = \frac{1}{W_s} \frac{1+r_{IH}}{r_{IH}} \quad (9)$$

Substituting (9) into (8) leads to

$$C_t = \frac{r_{IH}}{1+r_{IH}} W_s \quad (10)$$

Equation (10) reveals that the optimal plan of the infinitely lived household is to consume in each period a fraction of its wealth equal to  $r_{IH}/(1+r_{IH})$  and save the remaining part by investing the corresponding amount in the capital market.<sup>4</sup> Consequently, the higher the value of the discount rate, i.e., the more impatient the household, the higher the wealth share allocated to current consumption.

## 4 The OLG model

Here I develop a 2-period OLG model that draws on Guerrazzi (2007, 2010) who – among the others – explores the behaviour of optimizing households endowed with logarithmic preferences whose choices are bound by constraints like the one in (3). Specifically, the household that lives for two periods is assumed to solve the following problem:

$$\max_{\{\{c_{i,t}\}_{i=1}^n\}_{\tau=s}^{s+1}} \sum_{\tau=s}^{s+1} \left( \frac{1}{1+r_{OLG}} \right)^{\tau-s} \sum_{i=1}^n g_i \log(c_{i,\tau}) \quad (11)$$

s.to

$$\sum_{i=1}^n c_{i,s} + \frac{1}{1+r_{OLG}} \sum_{i=1}^n c_{i,s+1} \leq W_s \quad (12)$$

As before, the problem above can be solved by writing the implied Lagrangian. Hence,

$$\mathcal{L}(\cdot) \equiv \sum_{\tau=s}^{s+1} \left( \frac{1}{1+r_{OLG}} \right)^{\tau-s} \sum_{i=1}^n g_i \log(c_{i,\tau}) - \lambda \left( \sum_{i=1}^n c_{i,s} + \frac{1}{1+r_{OLG}} \sum_{i=1}^n c_{i,s+1} - W_s \right) \quad (13)$$

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<sup>4</sup> If the infinitely lived household discounts utility at a rate  $\rho_{IH} \neq r_{IH}$ , then straightforward algebra reveals that the ratio between total consumption in two consecutive periods – say  $t$  and  $t+1$  – is given by  $C_t/C_{t+1} = (1+r_{IH})/(1+\rho_{IH})$ . Such an expression implies that total consumption is growing (shrinking) over time whenever  $r_{IH}$  is higher (lower) than  $\rho_{IH}$ .

The FOCs for (13) are given by the following sequences:

$$g_i - \lambda c_{i,\tau} = 0 \quad i = 1, \dots, n \quad \tau = \{s, s + 1\} \quad (14)$$

Recalling the result in (2) and aggregating over the  $n$  consumption goods reveals that the expressions in (14) necessarily imply that

$$C_s = C_{s+1} \quad (15)$$

where  $C_s \equiv \sum_{i=1}^n c_{i,s}$  ( $C_{s+1} \equiv \sum_{i=1}^n c_{i,s+1}$ ) is the total consumption expenditure in the starting (final) period when the household is young (old).

Substituting the result in (15) into (12) leads to

$$C_\tau = \frac{1+r_{\text{OLG}}}{2+r_{\text{OLG}}} W_s \quad (16)$$

Equation (16) shows that the optimal plan of the household that lives for two periods is to consume the fraction  $(1 + r_{\text{OLG}})/(2 + r_{\text{OLG}})$  of its wealth in each period and to save the remaining part by investing the corresponding amount in the capital market.<sup>5</sup> The fraction of consumed wealth is always higher than the share of wealth consumed by the infinitely lived household conveyed by (10) no matter the actual value of  $r_{\text{OLG}}$ .<sup>6</sup>

## 5 IH versus OLG

Here I put forward a comparison between the consumption plans described above by assessing the number of time units of the IH model that form a time unit in the OLG model. A simple way to make such an assessment is to find the number of time units over which the infinitely lived household consumes the same resources consumed in a single time unit by the household the lives for two periods. Formally speaking, this means that the theoretical generation length is given by the value of  $t$  that solves the following equation:

$$t \frac{r_{\text{IH}}}{1+r_{\text{IH}}} = \frac{1+r_{\text{OLG}}}{2+r_{\text{OLG}}} \quad (17)$$

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<sup>5</sup> Again, making the hypothesis that the household that lives for two periods discounts the utility of its old age at a rate  $\rho_{\text{OLG}} \neq r_{\text{OLG}}$ , the ratio between total consumption in two consecutive periods – say  $\tau$  and  $\tau + 1$  – is given by  $C_\tau/C_{\tau+1} = (1 + r_{\text{OLG}})/(1 + \rho_{\text{OLG}})$ . Assuming the existence of a common term structure for interest and discount rates, the condition for a growing (or shrinking) consumption in the OLG model is that same that holds in the IH model.

<sup>6</sup> In general, if  $r$  is the real interest rate prevailing on the capital market that is also used to discount future utility streams, then it would be possible to show that a household that lives for  $m$  periods consumes a fraction  $(1 + (1 + r)^{-1} + \dots + (1 + r)^{1-m})^{-1}$  of its wealth. Consequently, whenever  $m \rightarrow \infty$ , (16) collapses to (10).

Equation (17) is grounded on a resource-consumption criterion, but it does not consider the fact that given the available financial investment opportunities – when interest rates are positive – the two households consume an amount of resources that exceed their wealth over their respective time horizon.<sup>7</sup> This shortcoming could be bypassed by computing in how many periods the present value of the consumption stream from the IH model equals the value of consumption in the OLG model. As I show in Appendix, however, this ‘financially-oriented’ way to proceed pins down the value of the interest rate prevailing in the OLG model by leaving the number of periods covered by the IH one to be determined merely by the term structure of interest rates.

Aiming at finding a solution to (17) that depends on one configuration only of the interest rate, it is necessary to make some assumptions about the relationship that holds between  $r_{IH}$  and  $r_{OLG}$ . In what follows, I will assume that the rate of return prevailing in the OLG model is achieved in the IH model only after the theoretical length of a generation. Considering (17), this means that the prevailing term structure of interest rates is described by

$$(1 + r_{IH})^t = 1 + r_{OLG} \quad (18)$$

Plugging (18) into (17) reveals that for each value of  $r_{IH}$ , the length of a generation is found by retrieving the value of  $t$  that solves

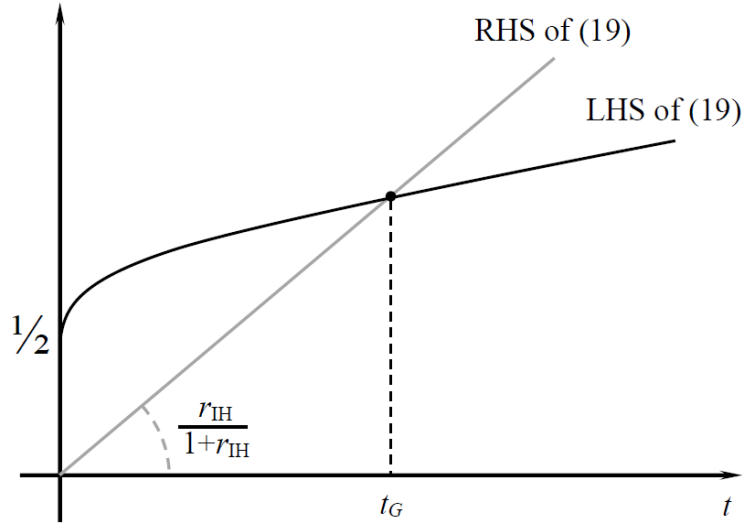
$$t \frac{r_{IH}}{1+r_{IH}} = \frac{(1+r_{IH})^t}{1+(1+r_{IH})^t} \quad (19)$$

On the one hand, the expression of the LHS of (19) is equal to zero for  $t = 0$  and thereafter it rises linearly with the time unit of the IH model. On the other hand, the expression on the RHS is equal to  $1/2$  for  $t = 0$  and thereafter it rises at decreasing rates with increases in  $t$ . Consequently, as shown in Figure 1, there will be only one meaningful solution to (19) – say  $t_G$  – and such a solution, according to the resource-consumption criterion in (17), returns the theoretical length of a generation.<sup>8</sup>

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<sup>7</sup> Moreover, it does not consider that the household that lives for two periods runs out all its wealth during its finite life, whereas the infinitely lived one cannot consume its entire wealth during *any* finite time interval.

<sup>8</sup> Obviously, since time is measured in a discrete manner, the point value of  $r_{IH}$  can always be tuned to find a value of  $t_G$  belonging to  $\mathbb{N}$ .



**Figure 1:** The length of a generation

The realized value of  $t_G$ , interestingly, is negatively related to  $r_{IH}$ ; indeed, for higher (lower) values of the real interest rate, the line and the curve depicted in Figure 1 rotate in a counter-clockwise (clockwise) direction. Given the different shapes, however, the movements of the RHS of (19) are always more pronounced than the corresponding movements of the LHS. Consequently, higher (lower) values of  $r_{IH}$  lead to lower (higher) values of  $t_G$ .

An economic rationale for the relationship described above can be given as follows. A household that does not care about its future will immediately consume all of its wealth no matter the length of its time horizon; indeed, whenever  $r_{IH} \rightarrow \infty$ , the IH as well as the OLG model deliver the same consumption plan. Therefore, in this case, the length of a generation coincides with the single period of the IH model. By contrast, when the household starts to care about its future, it will smooth its consumption expenditure throughout its lifetime. In general, whenever  $r_{IH} < \infty$ , the length of a generation increases with the implied value of the discount factor.

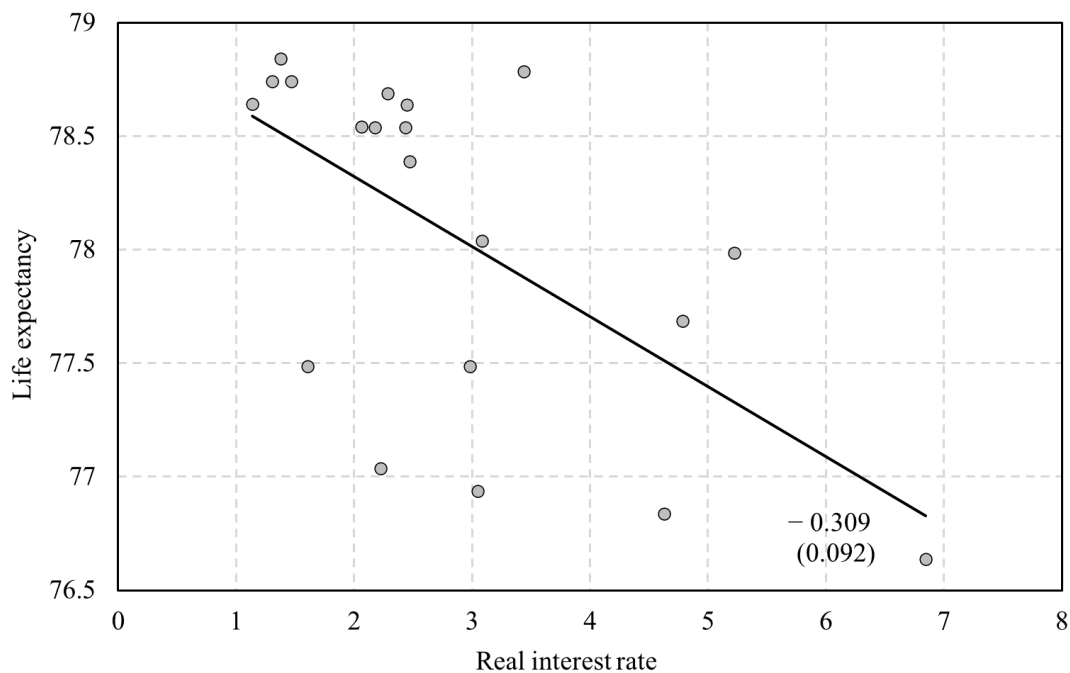
## 6 Empirical implications

From an empirical point of view, the theoretical investigation developed above has a couple of interesting implications. First, the criterion exploited to assess the length of a generation implies a close link between the demographic developments of the population and the actual path of real interest rates. In this regard, it might be worth to recalling that sometimes sociologists identified the generation length with the conventional duration of youth in human communities (cf. Berger, 1960).



According to this view, the increased longevity observed in developed societies in the last years might have delayed ageing processes by extending the ‘youthfulness’ of individuals.

Under these assumptions, the value of life expectancy at birth will be proportional to the length of a generation and it can be taken as a proxy for the realized value of  $t_G$ . Consequently, following such a sociological perspective, the analysis carried out in Section 5 involves a negative relationship between the longevity indicator and the observed values of real interest rates that can be tested with actual data. As illustrated in Figure 2, taking US values over the last 20 years, such a negative link emerges with a strong degree of significance; indeed, regressing the values of life expectancy against a constant and the corresponding figures of the real interest rate, we find a coefficient equal to -0.309 with a standard error – as shown in parenthesis – of only 0.092, and such a regression explains more than 38% of the variance of the longevity indicator.<sup>9</sup>



**Figure 2:** Real interest rates and life expectancy in the US (2000-2019)

Reversing the direction of causality, i.e., considering the effect of demographic developments on the yield recognized by the capital market, a negative relation between life expectancy and real interest rates has been highlighted in several papers. For example, Carvalho et al. (2016) calibrate a

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<sup>9</sup> Data on life expectancy and real interest rates can be downloaded from [data.worldbank.org](http://data.worldbank.org). Further details on the OLS regression illustrated in Figure 2 are available upon request.

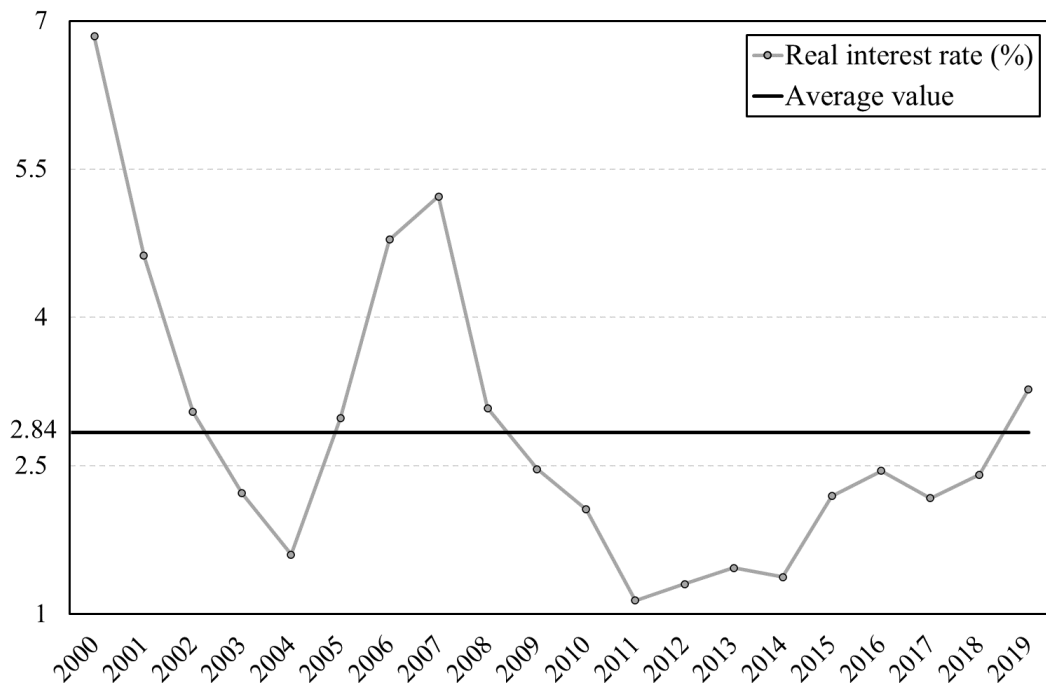
life-cycle model to match the increase in longevity observed in developed countries by finding that the corresponding increase in savings for retirement provisions leads to a significant reduction of real interest rates. Along the same line, multiperiod OLG models have been extensively used to investigate the connection between population ageing and the decline of real interest rates. For instance, Lisack et al. (2017) develop model in which households' expenditure can be directed both in consumption and housing by showing that higher life expectancy increases the share of the population in its high-wealth stages, and this pushes down interest rates. Furthermore, Papetti (2019) sets forth a framework with a perfect annuity market by showing that the downward pressure on real interest rates triggered by the higher savings of an aging population is exacerbated by the increasing scarcity of the effective labour input associated to the increased stock of retired workers.<sup>10</sup> Suggesting the possibility of a causal link that runs from interest rates to demographic developments passing through the households' consumption choices, the criterion to evaluate the length of a generation suggested in this note strengthens the soundness of the descriptive statistics surveyed in these applied works.

In addition, the analysis of Section 5 offers some insights for the calibration of discount rates in computational models. In the RBC literature, the discount rate of consumers is usually calibrated on a quarterly basis to deliver an implied value of  $r_{IH}$  around 1% (e.g., Kydland and Prescott, 1982; Long and Plosser, 1983). Implementing the procedure described above, such a value of the interest rate implies that time unit of the OLG model is about 66 periods of the IH model. Therefore, if the time reference of the IH model were a quarter, then a generation would cover about 16 years only. Consequently, if we aim at achieving the conventional reference of 25 years, then the discount factor should be set at highest values by targeting a value of  $r_{IH}$  around 0.7%. As shown by the time series in Figure 3, such a value of the quarterly real interest rate is close to the estimation of its corresponding annual mean over the horizon covered by the data analyzed in Figure 2; indeed, the average of the US real interest rate in that period amounts exactly to 2.84%.<sup>11</sup>

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<sup>10</sup> More recently, Sudo and Takizuka (2020) build a model with heterogeneous agents that derive utility from public-bond holdings by emphasizing that the effects of the demographic transition on real interest rates are both persistent and sizable.

<sup>11</sup> Since this value of the interest rate is associated to life expectancy of about 80 years, this means that a generation covers less than one-third of total expected life.



**Figure 3:** Real interest rates in the US (2000-2019)

## 7 Concluding remarks

This note aimed at assessing the relationship between the time reference of infinite and finite horizon dynamic models through the comparison of the consumption plans of the involved optimizing households. Such a theoretical exploration grounded on a resource-consumption criterion revealed three remarkable results. First, the generation length decreases with the value of the real interest rate by corroborating the nexus between demographic developments and the yield recognized by the capital market emphasized in a number of applied works. Second, conventional figures exploited to calibrate discount factors in RBC models lead to a shorter theoretical generation with respect to the reference usually acknowledged by demographers and geneticists. Furthermore, the canonical interval of 25 years can be achieved by calibrating discount rates by targeting the average value of the real interest rate observed over the last two decades.

The analysis carried out in this note could be extended in many directions. For instance, it could be interesting to see how the results summarized above change when households with different time horizons are also endowed with different preferences; indeed, whenever the households' wealth affects their marginal propensity to consume, the wealth itself would contribute to determinate the length of a generation (cf. Fiaschi and Romanelli, 2010). The implied extensions are left, however, to further developments.

## Appendix A: Equalizing the present value of the consumption stream from the IH model to the consumption of the OLG model

The present value of the consumption stream undertaken over  $t$  periods by the infinitely lived household is given by

$$\left(1 + \frac{1}{1+r_{IH}} + \left(\frac{1}{1+r_{IH}}\right)^2 + \dots + \left(\frac{1}{1+r_{IH}}\right)^{t-1}\right) C_t \quad (A1)$$

Exploiting the result in (10), (A1) reduces to

$$\left(1 - \left(\frac{1}{1+r_{IH}}\right)^t\right) W_s \quad (A2)$$

Equalizing (A2) to (16) by considering (18), leads to following expression:

$$2r_{OLG}^2 + r_{OLG} - 1 = 0 \quad (A3)$$

Ignoring the negative solution, (A3) pins down a point value for  $r_{OLG}$  equal to  $\sqrt{2} - 1$  that means a yield above 41%. Relying on the term structure of interest rate implied by (18), the value of  $r_{IH}$  consistent with a generation that lasts for 25 years amounts to 0.03%. Assuming that the time reference of the IH model is a quarter, this figure is about the half of the average value of the real interest rate observed over the last 20 years in the US.

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