New Procedures for the Adjustment of Photogrammetric Blocks

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1 Introduction

The aim of photogrammetry is to obtain a metric description of objects from images.

The output of photogrammetric processing consists in a set of features of the objects depicted in the images: position, orientation, size and shape (Kraus, 2007). The position and the attitude of the sensor that generated the images enter into the problem as nuisance parameters to be determined together with the main unknowns or in a preliminary operation. Different approaches have been developed to solve this classic problem (Kraus, 2007).

A single image can be oriented using the collinearity equations and some known object points (*space resection*), but this procedure is seldomly used. In fact most of the photogrammetric procedures rely on stereopsis to obtain 3D positions from images, therefore two or more images must be used. It is generally convenient to perform the orientation of the images in a combined manner.

The orientation is sometime performed in steps: images are oriented relatively to each other forming the *models*. Each model is essentially a portion of the object space equipped with an arbitrary reference system. Finally, the models are oriented in the object reference system using a 7 parameters transformation.

The photogrammetric block adjustment can be performed with block-units that are models (model block) or images (bundles block).

In modern photogrammetric procedures the bundles block adjustment is generally preferred (Forlani et al., 2015) especially in automatic aerial triangulation (Schenk, 1997), (Heipke, 1997) and in the feature based matching (Schenk, 2004) developed

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in computer vision.

However, model orientation is gaining new interest because it is currently investigated and deployed for the co-registration of laser scanner point clouds using similar equations and procedures (Markiewicz, 2016). The orientation of lidar-generated point clouds can require efficient algorithms, given the huge amount of data involved (Zhang et al., 2016). Moreover, model orientation is also used for the combined orientation of data from multiple sources, such as laser scanner point clouds, images and digital surface and bathymetric models (Colomo-Jiménez et al., 2016), (Fabris et al., 2010).

The adjustment of a photogrammetric block is always based on non-linear observation equations. It is therefore necessary to incorporate in the procedure the iterative solution of the system of equations and some algorithm for the computation of an approximate solution used as starting point.

2 Summary

In the present work we only treat the case of model block exploiting a quite simple idea. The orientation parameters of the models with respect to the object and the orientation parameters between the models are linked together by means of proper equations. The equations can be used in a non redundant system for the direct (non iterative) computation of all the parameters or they can be used in linearized form for the iterative least squares solution of a redundant system.

Here is an outline of this work. Sections (3,4,5) present some well-known mathematical models of photogrammetry and some elements of the theory of rotations. They are not a primer of the subject. They are just a review of classical topics that introduce the following sections. The main ideas of this work are presented in sections (6,7,8,9). Section (10) contains the description of the computer program and some examples.

3 Equations of the Helmert transformation

The relation between the Cartesian coordinates of the same point in two different coordinate systems is expressed by the well-known relation:

$$\underline{y} = \lambda R \underline{x} + \underline{t} \tag{1}$$

where $\underline{x}$ and $\underline{y}$ are the (vectors of) coordinates of the same point in the two coordinate systems, λ is a scalar that accounts for the different length-unit, R is a rotation matrix that accounts for the different axis orientation and $\underline{t}$ is a vector that accounts for the different position of the origin.

We generally use lower-case letters (Latin or Greek) for scalars, underlined lower-case letters for vectors and upper case for matrices.

The equation (1) has many applications in geodesy, photogrammetry and other disciplines. In photogrammetry the most frequent use is to express the relation between model coordinates and object coordinates. The same equation can be used to transform the coordinates when the parameters of the transformation are already known and to estimate the parameters when some points have known coordinates in both the coordinate systems.

The latter computation is generally performed with redundant data and with the application of the least squares estimation principle. This is a quite peculiar estimation problem that can be solved directly (i.e. without iterations) in spite of the non-linearity of the equations. More than one solution exist, see e.g. the works by Sansò (1973) and by Teunissen (1985).

4 Model block adjustment

We have a set of models that must be oriented. Some points are known in more than one model. Some points are known in the object coordinate system. It is convenient to use the following notation:

- n is the number of points,
- m is the number of models,
- j is the index used for points,
- i and k are indices used for models,
- $\underline{x}_j$ are the initially unknown coordinates of point i in the object coordinate system (with j=1 ... n),
- R_i is the unknown rotation matrix that gives the orientation of the model i with respect to the object,
- λ_i is the unknown scale factor between the model i and the object,
- $\underline{t}_i$ are the unknown coordinates of the origin of model i in the object coordinate system,
- $\underline{y}_{i,j}$ are the given coordinates of point j in the coordinate system of model i ((i,j) only ranges over a proper subset of (1 ... n)×(1 ... m)),
- $\underline{y}_{0,j}$ are the given coordinates of point j in the object coordinate system (j only ranges over a proper subset of (1 ... n)),

With the stated notation the following equation arises:

$$\underline{y}_{i,j} = \lambda_i R_i (\underline{x}_j - \underline{t}_i) \tag{2}$$

which is an adapted version of (1) and:

$$\underline{y}_{0,j} = \underline{x}_j \tag{3}$$

that includes the contribution of the control points.

5 Rotations and quaternions

It is obviously necessary to choose a proper set of parameters to represent the rotations involved in the orientation of the elements of the photogrammetric block. Quaternion algebra gives a representation of rotations that exhibits some nice property. It is therefore convenient to briefly introduce the essential elements of quaternion algebra. We define the quaternions as hyper-complex numbers with a ordinary real number as real part and a three-component vector as imaginary part. There are obviously four scalar components altogether. The notation that is used in this paper is the following:

- $\mathbf{q}$: quaternions are written in bold,
- (q_0, q_1, q_2, q_3) : a subscript denotes the components,
- $\mathbf{q}=q_0+i\underline{q}$: the scalar real component and the vector imaginary component are evidenced in this "sum".

Two quaternions are equal if and only if all their components are equal. The sum and the difference of quaternions are operated on the individual components. The product of quaternions is defined by means of the dot product (represented by $\underline{q}\cdot\underline{p}$) and the vector product (represented by $\underline{q}\wedge\underline{p}$). The expression is:

$$\mathbf{qp} = (q_0p_0 - \underline{q}\cdot\underline{p}) + i(q_0\underline{p} + p_0\underline{q} + \underline{q}\wedge\underline{p}). \tag{4}$$

It is also useful to define the conjugate of a quaternion: $\overline{\mathbf{q}} = q_0 - i\underline{q}$ and the norm of a quaternion: $|\mathbf{q}|=(\mathbf{q}\,\overline{\mathbf{q}})^{1/2}$. The definition of the reciprocal is possible (quaternions are therefore a division algebra) but it is not necessary here. A short but more complete primer on quaternions can be found in (Sansò, 1973). A more complete treatment is in the book by Altmann (1986).

The use of quaternions for the representation of rotation is based on the expression:

$$\mathbf{y} = \mathbf{q}\mathbf{x}\overline{\mathbf{q}} \tag{5}$$

with $|\mathbf{q}|=1$ and $\mathrm{Re}(\mathbf{x})=0$. The analysis of the property of equation (5) shows that it represents a rotation in the ordinary three dimensional space identified with the (sub-)space of imaginary quaternions. The representation of rotations by means of quaternions has some good properties that are exploited in the present work. The composition of rotations is represented by the product of the corresponding quaternions because the quaternionic product is associative. Sansò (1973) proposed a direct (i.e. non iterative) solution of the Helmert problem based on quaternion algebra. Furthermore the quaternion that represents a rotation has a nice geometric meaning that is useful in other applications but is not necessary here.

6 Equations relating global and local rotations of the elements of a block of models

The coordinates of one point in two models are obviously related by a transformation that is again a version of (1) namely:

$$\underline{y}_{k,j} = \lambda_{k,i} R_{k,i} \underline{y}_{i,j} + \underline{t}_{k,i} \tag{6}$$

and it is then easy to obtain the relation between the scale factors:

$$\lambda_{k,i} = \lambda_k \lambda_i^{-1} \tag{7}$$

and between the rotations:

$$R_{k,i} = R_k R_i^{-1}. \tag{8}$$

The relation between the rotations can be conveniently expressed using quaternions:

$$\mathbf{q}_{k,i} = \mathbf{q}_k \overline{\mathbf{q}}_i \,. \tag{9}$$

Other relations can be written for the additive term of the transformation.

It is also necessary to derive a linearized version of equations (7) and (9).

In order to linearize equation (7) we express an unknown factor λ as the product of a known factor $\tilde{\lambda}$ (the approximate solution) and a factor near to the unity: $\lambda = \tilde{\lambda}\ (1+\mu)$ with $|\mu| << 1$. The linearized form of equation (7) is therefore:

$$\lambda_{k,i} = \tilde{\lambda}_k \tilde{\lambda}_i^{-1} (1 + \mu_k - \mu_i). \tag{10}$$

This equation can be manipulated to obtain:

$$\lambda_{k,i} \tilde{\lambda}_k^{-1} \tilde{\lambda}_i - 1 = \mu_k - \mu_i. \tag{11}$$

In order to linearize equation (9) we express an unknown quaternion as the product of a known quaternion (the approximate solution) and an unknown quaternion that must represent a small rotation. The most convenient expression is: $\mathbf{q}_k = \tilde{\mathbf{q}}_k \mathbf{v}_k$ that gives:

$$\mathbf{q}_{k,i} = \tilde{\mathbf{q}}_k \mathbf{v}_k \overline{\mathbf{v}}_i \overline{\tilde{\mathbf{q}}}_i \tag{12}$$

and then:

$$\bar{\tilde{\mathbf{q}}}_k \, \mathbf{q}_{k,i} \, \tilde{\mathbf{q}}_i = \mathbf{v}_k \bar{\mathbf{v}}_i \tag{13}$$

and finally:

$$\bar{\tilde{\mathbf{q}}}_k \, \mathbf{q}_{k,i} \, \tilde{\mathbf{q}}_i = 1 + i(\underline{v}_k - \underline{v}_i) \tag{14}$$

where the second order terms have been dropped.

The imaginary part of (14) gives the vectorial equation:

$$\mathrm{Im}(\bar{\tilde{\mathbf{q}}}_k \, \mathbf{q}_{k,i} \, \tilde{\mathbf{q}}_i) = \underline{v}_k - \underline{v}_i \, . \tag{15}$$

An alternative procedure to obtain the same equation is the following. An unknown quaternion is expressed as the sum of a known quaternion (the approximate solution) and an unknown quaternion that it is assumed to be small in modulus. The expression is:

$$\mathbf{q}_\mathrm{k} = \tilde{\mathbf{q}}_k + \mathbf{r}_k \tag{16}$$

and the constraint $|\mathbf{q}_k|=1$ becomes:

$$\mathrm{Re}(\bar{\tilde{\mathbf{q}}} \, \mathbf{r}_k) = 0 \tag{17}$$

which is valid in first order approximation. Equation (9) becomes:

$$\mathbf{q}_{k,i} = (\tilde{\mathbf{q}}_k + \mathbf{r}_k)\,(\bar{\tilde{\mathbf{q}}}_i + \bar{\mathbf{r}}_i) \tag{18}$$

and then:

$$\mathbf{q}_{k,i} = \tilde{\mathbf{q}}_k \bar{\tilde{\mathbf{q}}}_i + \mathbf{r}_k \bar{\tilde{\mathbf{q}}}_i + \tilde{\mathbf{q}}_k \bar{\mathbf{r}}_i \tag{19}$$

where the second order terms have been dropped. If we remember the constraint (17) we can state: $\bar{\tilde{\mathbf{q}}}_k \mathbf{r}_\mathrm{k} = i\underline{v}_k$. It is therefore easy to derive (14) and (15) from (19).

7 Outline of two new procedures for the computation of a block of models

The main idea of this work is to use the Sansò's direct solution of the Helmert problem to link models in pairs and to use equation (7) and (9) to obtain a global solution. The procedures described in the next sections can produce both a direct approximate solution and a sub-optimal iterative efficient adjusted solution of the model block orientation problem. The following sections contain the description of two procedures. The first procedure is based on a minimal set of equations of type (7) and (9) for the direct computation of the orientation parameters of all the models. The second procedure is based on a redundant system of equations of type (7) and (9). The least squares solution of the system must be performed iteratively using the linearized equations presented in the previous section. The parameters obtained by the first procedure can serve as Taylor-point for the linearization.

8 A procedure for the direct computation of the orientation parameters of a block of models

The procedure for the computation of approximate orientation parameters of all the models can be better described as a sequence of steps.

1. Creation of a list of pairs of models that contains a proper number of common tie points. This list constitutes a graph. Each model is a node of the graph. Each possible connection is a edge of the graph. Each edge has an associated quality index that will be defined later on.
2. Extraction of an optimal spanning tree from the graph.
3. Sorting the edges of the spanning tree, so that starting from a "root" the sequential scan of the edges always connects a not-yet-visited node to an already visited node.
4. Computing the orientation between models for all the edges of the tree. The result of this computation is the quaternion $\mathbf{q}_{k,i}$ for all the k,i pairs of models that are edges of the tree. The scale factor and the additive parameters are also computed.
5. Assignment of an arbitrary orientation to the first node of the first edge of the tree.
6. Computation of the orientation of all the models by means of equations (7) and (9). Other equations are used for the computation of the additive terms, if they are needed. The computations are performed traveling all the edges of the tree according to the order generated in step 3. At the end of the current step the models have an arbitrary but coherent orientation.
7. The set of models are oriented with respect to the object using the object control points.

Some remarks are necessary to explain the details of the algorithm.

Step 1 is performed by a complete search among all the pairs of models. A pair is accepted if the number of tie points is above a certain threshold. The quality index is simply the number of tie points. A more refined acceptance test and a more refined quality index can be defined taking into account the number of tie points, their spatial distribution and the $\hat{\sigma}_0^2$ that result from the orientation. (This would require to perform the computation described in step 4 for all the edges and to perform it in advance.) We assume that the resulting graph is connected.

Step 2 is performed with the Kruscal algorithm described e.g. in the book by Chartrand and Zhang (2012). (Note that we use a quality index for the edge of the graph instead of a cost.)

The direct solution of the Helmert problem due to Sansò (1973) is applied in steps 4 and 7. (The use of a procedure based on a symmetric stochastic model would be more appropriate in step 4.)

The requirement that the step 1 generates a connected graph is obviously necessary for the success of the procedure. It is anyway possible to define a multi-step procedure that “aggregate” models in a more general manner.

9 A procedure for the sub optimal adjustment of a block of models

The procedure for the computation of a sub-optimal adjusted solution of the model block orientation problem is mainly based on the solution of a redundant system of equations of type (7), a redundant system of equations of type (9) and a redundant system of linear equations for the additive terms. Redundant systems are solved in the least squares sense. Equations (11) and (15), i.e. the linearized versions of (7) and (9), generate a normal system that splits into four separate systems with the same normal matrix. The normal matrix is generated like the normal matrix of a leveling network described by the graph generated in the step 1 of the other procedure.

It is important to remember that the solution is not optimal for several reasons. The main reasons are:

- the unknowns are treated in groups (not globally),
- the stochastic model is not handled consistently,
- the control points are only used in the final global orientation.

10 Implementation and test of the procedures

We have realized an experimental computer program that realizes the two procedures described in Sections 8 and 9. We have performed some tests based on both simulated data and real data. The tests based on simulated data include the simulation of measuring errors. The orientation parameters of the models are recovered with no apparent bias. The object coordinates of the points are estimated with inter-model discrepancies that are compatible with the errors generated in the simulation process. The results obtained with simulations are therefore quite satisfactory although they are clearly only preliminary results. The scheme of the graph of a simulated block of 9 models is in Figure 1.

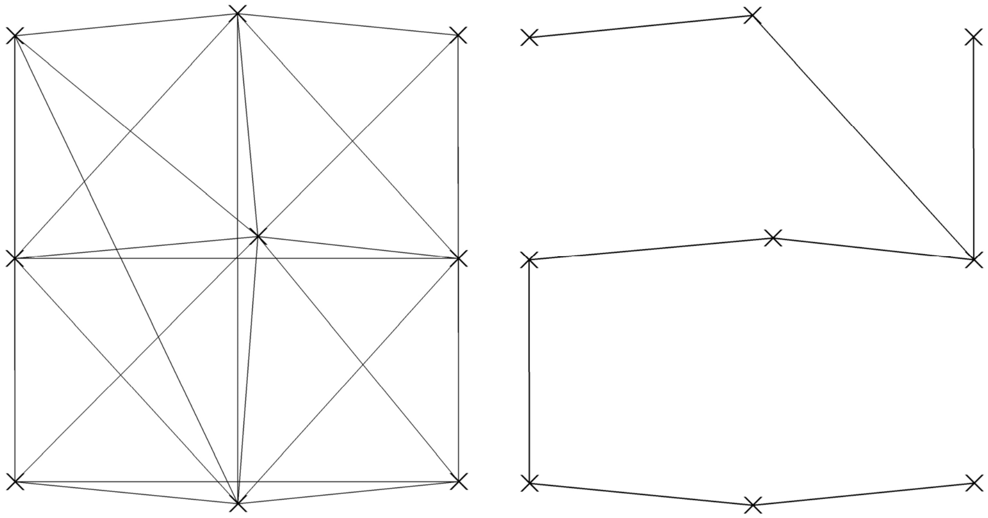

Figure 1: *Graph of a small block used in the first test. Complete graph on the left, optimal tree on the right.*

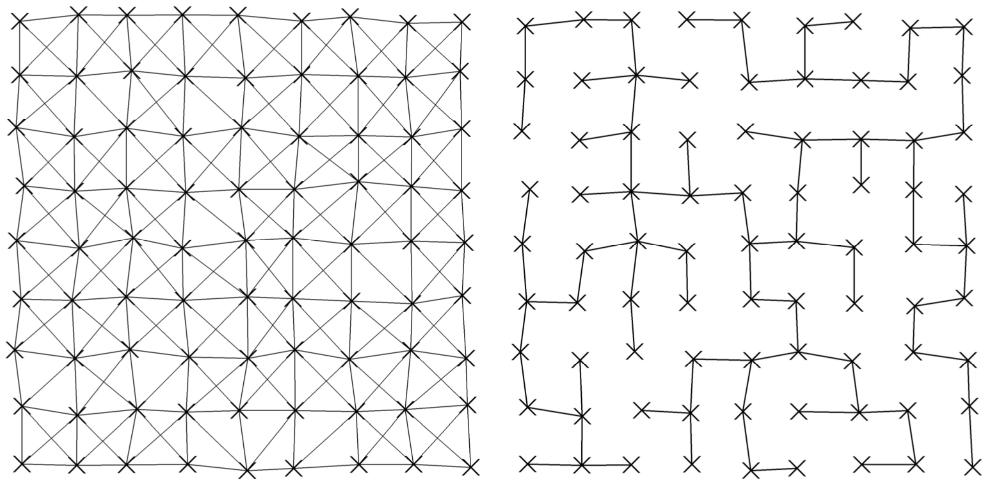

Figure 2: *Graph of a medium-size block used in the second test. Complete graph on the left, optimal tree on the right.*

The same block has 128 distinct points, 374 measured model points and 7 object points. Each node is plotted according with the planimetric position of the origin of the model coordinates. Figure 1 also shows the corresponding tree. Figure 2 refers to a larger block of 81 models (4060 distinct points, 6703 measured model points and 263 object points).

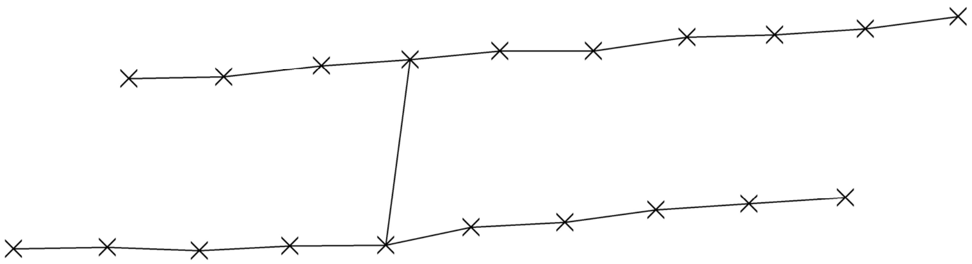

Figure 3: *The graph of a real photogrammetric block.*

We also realized a small test with a real photogrammetric block of 20 models (94 distinct points, 185 measured model points and 6 object points). The graph of the block is represented in Figure 3. The graph has no cycles, i.e. no redundancy. Therefore we can only apply the procedure described in Section 8. The object coordinates of the points are estimated with inter-model discrepancies generally < 10 cm and always < 60 cm. Ground control points are fitted with discrepancies of 30−100 cm.

11 Conclusions, comments and perspectives

The algorithm described in Section 8 is a valuable procedure to obtain the approximate values of all the parameters of a block of models. The set of approximate values can be used as starting point of a rigorous or simplified adjustment.

The algorithm described in Section 9 is a simplified adjustment procedure with a quite low computational burden. The quality of the attainable results must be accessed by means of further tests that must include the comparison with respect to the results of a rigorous adjustment.

The generalization of the described models and of the described procedures to the treatment of bundles blocks is the most important perspective for the development of the present work.

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