Geometric correction factor for elliptic holes in perforated plates and its role in sound absorption coefficient

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ABSTRACT

A comprehensive analysis was carried out to describe the influence on the sound absorption performance of elliptical holes in perforated plates (PP) and micro-perforated plates (MPP). The analysis was performed numerically using two end correction expressions (by Mechel, 1995, and Laurens et al., 2014) for elliptical holes and two impedance models (by Bauer, 1977, and Ingard, 1994). From formulas mentioned above, two geometric correction factors (GCF) were derived for the end correction of elliptic holes, in terms of a symmetry factor related to eccentricity. Impedance models were used to numerically compare how the GCFs influence sound absorption characteristics, using a typical configuration of a thin perforated panel backed by an air gap and a solid wall. The two GCFs show similar behavior for the high and low limit of eccentricity values but differ for medium values. Using the Mechel's GCF, low eccentricity ellipses can be assumed as circular perforations whereas high eccentricity perforations show a GCF<1; on the other hand, Laurens' GCF predicts a value>1 in a wide range of eccentricities. In PP applications, the eccentricity affects the frequency response of the system using both GCFs. In conclusion, both GFCs models predict that the performance of the resonant absorber may be influenced by the elliptical geometry of the holes, which significantly affects the end correction; however, experimental measures should be performed to define the validity range of each expression.

1. INTRODUCTION

Perforated plates (PP) and microperforated plates (MPP) are widely used in sound absorption applications [1]. The main difference between PP and MPP is the diameter of the perforation, which is less than 1 mm for MPP. The total vibrating mass of each perforation has an effective length $l' = l + 2\Delta l$, where l is the thickness of the plate and $2\Delta l$ represents the added length of the two perforation sides and is commonly known as end correction [2].

For a plate with multiple holes the effective conductivity is affected by the interaction between the holes and the modified end correction can be obtained by $\Delta l' = M(\varepsilon)\Delta l$, where $M(\varepsilon)$ represents the interaction between holes with a value always lower than 1. Early works of Fok [3] and Nesterov [4] describe the expressions for $M(\varepsilon)$. More recently, Jaouen and Chevillotte [5] showed that for symmetric holes all correction formulas are almost equivalent, and they proposed the approximation shown in the Eqn. (1), also valid for non-symmetric (rectangular) holes: the proposed formula has a very low deviation if the form factor is near to 1:

$$M(\varepsilon) = 1 - 1.33\varepsilon - 0.07\varepsilon^2 + 0.40\varepsilon^3 \qquad (1)$$

For elliptical holes, Laurens et al. (2014) [6] analytically found that an upper limit (equivalent to a piston) and a lower limit (equivalent to a circular hole) to the end correction exist, according to Eqn. (2):

$$\frac{\pi}{4}\tau(\beta)\,(2b) < 2\Delta l < \frac{8}{3\pi}\tau(\beta)\,(2b) \qquad (2)$$

In Eqn. (2) $\tau(\beta)$ is the Geometric Correction Factor (GCF) that represents the deviation from the circular aperture for ellipses, and an *elliptic integral of the first order* is used to directly estimate the correction. A second option to calculate the end correction for elliptical apertures is found in Mechel [7, 8], where the derivation is slightly different but also uses elliptic integrals. The proposed formulas are resumed in Eqn. (3) and Eqn. (4):

$$\frac{\Delta l}{b} = \frac{16}{3\pi^2}K(1-\beta^2) \qquad (3)$$

$$K(1-\beta^2) = \begin{cases} \frac{4+\beta^2}{8}\ln\frac{16}{\beta^2} - \frac{\beta^2}{4}, & \beta = b/a \le 0.641 \\ \frac{\pi}{2}\frac{11+5\beta^2}{7+9\beta^2}, & \beta = b/a > 0.641 \end{cases} \qquad (4)$$

The objective of this work is to develop a numerical comparison between the two models in order to underline the differences in the end correction calculation for an elliptical hole and their influence on the sound absorption coefficient for a typical configuration.

2. METHODOLOGY

A directly comparison of the geometric correction factors (GCF) of both formulations using two impedance models was used to numerically show how the GCFs influence sound absorption characteristics. A typical configuration of a thin perforated panel backed by an air gap and a solid wall was tested. The sound absorption curves are calculated from the impedance models of Bauer 1977 [9] and Ingard 1994 [10, 11] and the hole interaction formula reported in Eqn. (1).

3. RESULTS

The end correction normalized to semi-major axis as a function of β is showed in Fig.1, where is also represented the end correction for a rectangular perforation according to [7, 8]. The general trends show different information if a normalization for equivalent area holes is used (not showed here).

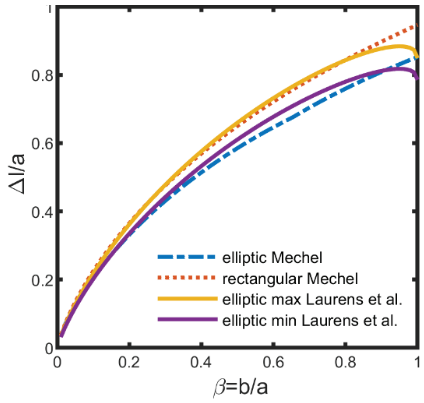

Figure 1. End correction for a single hole as a function of form factor β.

The sound absorption of a typical configuration of a PP resonator is presented in Fig.2, characterized by a thin PP with low porosity backed by an air gap and a rigid wall. (porosity 5%, plate thickness 2 mm, perforation minor axis 1 mm, air layer thickness 0.1 m). The discontinuity in the formulation of Mechel for $\beta = 0.641$ is evident.

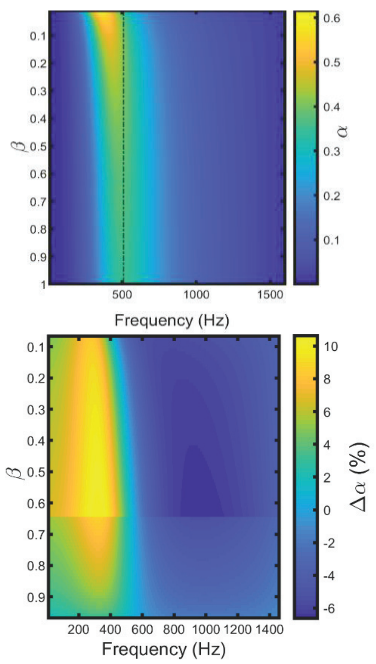

Figure 2. Absorption coefficient response to GCF geometric correction factor, Mechel/Ingard (upper image). Laurens et al. upper limit vs. Mechel GCF/Bauer (lower image).

4. DISCUSSION AND CONCLUSIONS

A complete expression for the end correction of non-circular holes must include not only the holes interaction factor $M(\varepsilon)$ but also for geometrical correction factors such as $\tau(\beta)$. The two models analyzed for the GCF show similar behavior. The elliptic perforation affects the performance of resonant absorbers and can significantly change the reactive part of the characteristic impedance adding (with same semi-minor axis perforations) or subtracting (with equivalent area perforations) vibrating mass. Modifying existing circular holes can be an effective technique to perform fine-tuning of a resonant absorber. Our results suggest that the use of the two semiempirical models for the impedance of PP shows similar behaviour for the evaluated parameters. It is possible to perform in future studies an extensive parametric comparison between the two impedance models. Experimental measurements should also be performed in order to validate the numerical results showed here.

5. REFERENCES

[1] T. Cox and P. D'Antonio, *Acoustic Absorbers and Diffusers: Theory, design and application*, Second. Abigdon, Oxon: Taylor & Francis Group, 2009.

[2] S. N. Rschevkin, *A Course of Lectures on The Theory of Sound*. England: Pergamon Press, 1963.

[3] V. A. Fok, "Theoretical study of the conductance of a circular hole, in a partition across a tube.," *Proceedings of the USSR Academy of Sciences*, vol. 31, no. 9, pp. 875–878, 1941.

[4] V. S. Nesterov, "Experimental study of the conductance of a circular hole, in a partition across a tube," *Proceedings of the USSR Academy of Sciences*, vol. 31, no. 9, pp. 879–882, 1941.

[5] L. Jaouen and F. Chevillotte, "Length Correction of 2D Discontinuities or Perforations at Large Wavelengths and for Linear Acoustics," *Acta Acustica united with Acustica*, vol. 104, no. 2, pp. 243–250, Mar. 2018.

[6] S. Laurens, E. Piot, A. Bendali, M. Fares, and S. Tordeux, "Effective conditions for the reflection of an acoustic wave by low-porosity perforated plates," *J. Fluid Mech.*, vol. 743, pp. 448–480, Mar. 2014.

[7] F. P. Mechel, *Schallabsorber*, vol. II. Stuttgart: S. Hirzel Verlag, 1995.

[8] F. P. Mechel, *Formulas of Acoustics*, 1st ed. Berlin, Heidelberg: Springer - Verlag, 2002.

[9] A. B. Bauer, "Impedance Theory and Measurements on Porous Acoustic Liners," *Journal of Aircraft*, vol. 14, no. 8, pp. 720–728, Aug. 1977.

[10] U. Ingard, *Notes On Sound Absorption Technology*, vol. 3, 94-02 vols. Kittery Point, USA: Noise Control Foundation, 1994.

[11] U. Ingard, *Noise Reduction Analysis*. Sudbury, Massachusetts, USA: Jones and Bartlett Publishers, 2010.