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Network performance evaluation under disruptive events through a progressive traffic assignment model

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Abstract: The purpose of this paper is to present an assignment model as a basis for evaluating the performance of a traffic network, capable of describing its evolution immediately after the occurrence of a disruptive event. First of all, a User-Equilibrium traffic assignment problem is solved in order to obtain an estimation of the system state before the disruption. Starting from the critical event, a Progressive Assignment procedure is performed in order to obtain reasonable traffic assignments on the network, taking into account the users' tolerance to increases in travel times as well as the inherent inertia of the system. Therefore a metric for the description of the network performance is proposed as well as implementation of the model on a test network.

Keywords: traffic assignment model, traffic network, performance evaluation, resilience, optimization problem

1. INTRODUCTION

Transport infrastructure systems are the basis on which the mobility of people and goods has developed over the centuries. They connect businesses, support supply chains and provide access to vital resources for everyday activities. Unfortunately, among similar and equally crucial infrastructures, transport systems are those which have suffered most from disasters. Natural and/or man-made disasters not only directly damage transport infrastructure, but, being highly interdependent with all human activities, indirectly and severely affect our economies and social systems.

The increasing awareness of these issues has led to a growing body of literature on the subject of performance evolution of transport systems during disruptive events, generally aimed at providing adequate estimations of the system operativity level in order to assess its resilience, defined as the ability of a system potentially exposed to a risk of adapting, resisting or changing, in order to achieve and maintain an acceptable level of operation. Initially introduced in the study of ecological systems by Holling (1973), the concept of resilience has been applied to multiple areas of study, transportation and civil infrastructure thanks to Murray-Tuite (2006) and shortly after Tierney and Bruneau (2007).

The work presented in this paper is based on the model detailed by Siri et al. (2020), a convex optimization model designed to represent the dynamics of a transportation network shortly after a disruption and up to a new equilibrium. The choice to proceed using an optimization model instead of a topological model or a simulation model, lies in the fact that unlike the former it is possible to capture the performance of the system by representing the behavior of users on the network and its evolution under changed circumstances, without the excessive computational load of the latter, which obtains the required measurements by simulating the physical dynamics of vehicles on the network. In Zhou et al. (2019), an extensive literature review can be found on metrics and measurement approaches more suited for a wide range of different needs in the research filed of transport systems.

Many researchers have been interested in modeling the evolution of a traffic network immediately after the occurrence of a critical event. A classic assignment model such as User-Equilibrium (UE), firstly proposed by Wardrop (1952), is in fact inadequate to represent these circumstances, since it heavily relies on the hypothesis of users' perfect information on the state of the network, which might be valid only long after the event's initial occurrence. Asakura (1999) uses the Stochastic User Equilibrium (SUE) model to represent route choices under more uncertain conditions than in the normal situation. SUE models assume probability distribution of perceived travel times of drivers instead of objective travel times linked to each arc. The concept of Partial User Equilibrium, introduced by Sumalee and Watling (2008) to represent user's behavior in post-disaster circumstances, is incorporated in He and Liu (2012) within a bi-level model to quantify and optimize the travel time resilience of a roadway network.

The paper is organized as follows. Section II is divided into three sub-sections. The first one introduces the problem of traffic assignment and presents the notation, in the second the model and the algorithm proposed are discussed in detail, while a performance-based metrics is suggested in the third one. Implementation and some results are outlined in Section III whereas some conclusions are finally drawn in Section IV.

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2. THE PROGRESSIVE ASSIGNMENT PROCEDURE

Given a traffic network and an origin-destination matrix, our work aims to describe its evolution at a macroscopic level, in the period immediately following the occurrence of a disruptive event, proposing a model based on the concept of traffic assignment. Traffic assignment models are often used as a basis for the resilience evaluation of a traffic network. This is because they are capable of representing the interaction between users and infrastructure by estimating the patterns of vehicle flows on each arc of the network, something that a topological evaluation alone is not able to do. Moreover, the proposed analysis, being however macroscopic, does not require the effort of simulation models, which can be quite high.

The proposed model makes iteratively use of the UE traffic assignment model. UE traffic assignment involves finding a pattern of traffic flows on the network such that no user has, unilaterally, an interest in changing his path, since no other alternative can guarantee him lower travel times. It can be demonstrated that this pattern of traffic flows, or assignment, corresponds to the optimal solution of the minimization problem known as Beckmann's Tranformation, see Beckmann et al. (1956). This type of minimization problems can be solved using the convex combination algorithm originally suggested by Frank and Wolfe (1956), a procedure for solving quadratic programming problems with linear constraints. This is precisely the case of an assignment problem. In Sheffi (1985), a detailed explanation on the formulation and resolution of traffic assignment problems can be found, while in Siri et al. (2020) the conditions to be met in the problem definition as well as the implications of adopting this methodology for addressing the issue presented in this work are outlined and discussed in detail. It is worth mentioning however, how the uniqueness of the solution to the minimization problem is guaranteed if (1) the domain is convex and (2) the objective function is strictly convex. Condition (1) is fulfilled in the case of an assignment problem since the constraints are generally linearly defined. The fulfilment of condition (2) instead requires that the travel time over one link does not depend on traffic flows on other links in the network and that performance functions are defined as strictly increasing. A performance function relates travel times experienced by users going through a link with the amount of traffic flow over the link itself. Not surprisingly, in an assignment problem, performance functions are always strictly increasing. As the congestion on a link becomes more intense, traffic conditions deteriorate and users experience higher travel times.

The main ideas behind the proposed model are two: the former concerns the specific behaviour of users, the latter regards the dynamics of the system as a whole. Regarding the behaviour of users, the model accounts for the fact that they consider alternative roads to those already in use only when the travel times they experience have increased significantly. By "significantly" we mean more than a certain percentage value expressed in the model by the *user tolerance index* Ω, which will be defined rigorously in the following. For each iteration, the proposed algorithm, comparing for each origin-destination pair the increase in time that users experience with the maximum threshold they tolerate, verifies whether users are satisfied or not with the current situation. If they are not, in the next iteration they will be assigned considering a set of usable paths to which a new path has been added among all those connecting their origin with their destination, obviously not yet used. The assignment that is obtained from this process is defined as *target assignment* and represents at each iteration the direction towards which the system tends to move. Therefore, considering the dynamics of the system as a whole, it is not expected to jump instantly from one new solution to another, but because of its intrinsic inertia, represented by the *inertia coefficient* β, the system should evolve through a series of states that are somewhere in between the best possible assignment given the current circumstances and the previous situation.

2.1 Notation

The notation used in the model is as follows. First of all, the topological quantities and sets are defined:

- $G(\mathcal{V}, \mathcal{A})$: *graph* denoting the transportation network consisting of a set of nodes $\mathcal{V}$ and directed arcs $\mathcal{A}$, where $|\mathcal{A}| = A$
- $\mathcal{R} \subseteq \mathcal{V}$ and $\mathcal{S} \subseteq \mathcal{V}$: *set of origin and destination nodes* respectively
- Q_{rs}: *set of all possible paths* from origin node $r \in \mathcal{R}$ to destination node $s \in \mathcal{S}$
- L_{rs}^{UE}: set of paths, obtained through the UE traffic assignment, that are used by each origin-destination pair rs at the equilibrium before the occurrence of the disruption, $r \in \mathcal{R}$, $s \in \mathcal{S}$
- L_{rs}^{SA}: set of paths from origin node $r \in \mathcal{R}$ to destination node $s \in \mathcal{S}$ on which the shock-assignment is performed the first time after the occurrence of the disruption
- L_{rs}^{n}: *set of available paths* from origin node $r \in \mathcal{R}$ to destination node $s \in \mathcal{S}$ on which the assignment is performed at iteration n
- od_{rs}: *traffic demand* from origin node r to destination node s

Traffic flows and traffic assignment variables are defined as follows:

- x_a^n: *traffic flow* on link $a \in \mathcal{A}$ at iteration n
- z_a^n: *target traffic flow* for link $a \in \mathcal{A}$ to which the system is tending at iteration n
- $\underline{x}^n = [x_1^n, x_2^n, \ldots, x_{\mathrm{A}}^n]$: *assignment vector* at iteration n
- $\underline{x}^{\mathrm{UE}} = [x_1^{\mathrm{UE}}, x_2^{\mathrm{UE}}, \ldots, x_{\mathrm{A}}^{\mathrm{UE}}]$: *User-Equilibrium assignment vector* of flows before the disruption
- $\underline{x}^{\mathrm{SA}} = [x_a^{\mathrm{SA}}, x_2^{\mathrm{SA}}, \ldots, x_{\mathrm{A}}^{\mathrm{SA}}]$: *shock assignment vector* of flows immediately after the occurrence of the disruption
- $\underline{x}^{\mathrm{FE}} = [x_1^{\mathrm{FE}}, x_2^{\mathrm{FE}}, \ldots, x_{A}^{\mathrm{FE}}]$: *final assignment vector* representing the new equilibrium reached by the system after a while from the disruption
- $\underline{z}^n = [z_1^n, z_2^n, \ldots, z_{\mathrm{A}}^n]$: *target assignment vector* at iteration n, pattern of link flows towards which the system is tending
- $f_{k,rs}^n$: *traffic flow* on path $k \in L_{rs}^n$ of origin-destination pair rs, $r \in \mathcal{R}, s \in \mathcal{S}$, at iteration n

The travel times on links and paths are defined as follows:

- $t_a^n = t_a(x_a^n)$ *travel times on link* a at iteration n, $a \in \mathcal{A}$
- $TT_{k,rs}^n$: *travel times on path* $k \in L_{rs}^n$ of origin-destination pair rs, $r \in \mathcal{R}$ and $s \in \mathcal{S}$, at iteration n
- TTT^n: network Total Travel Time at iteration n

Finally, the coefficients of the model are:

- $\Omega \in [0, +\infty)$: *user tolerance index*
- $\beta \in [0, 1]$: *inertia coefficient*

Travel times on paths at each iteration are determined as follows:

$$TT_{k,rs}^n = \sum_{a \in \mathcal{A}} t_a(x_a^n) \cdot \delta_{a,k}^{rs} \quad r \in \mathcal{R}, s \in \mathcal{S}, k \in L_{rs}^n \qquad (1)$$

where $\delta_{a,k}^{rs}$ equals 1 if link a belongs to path k from r to s or 0 otherwise.

2.2 The Procedure

In the following, the main elements of the assignment model are briefly illustrated, according to the flow chart of Fig. 1.

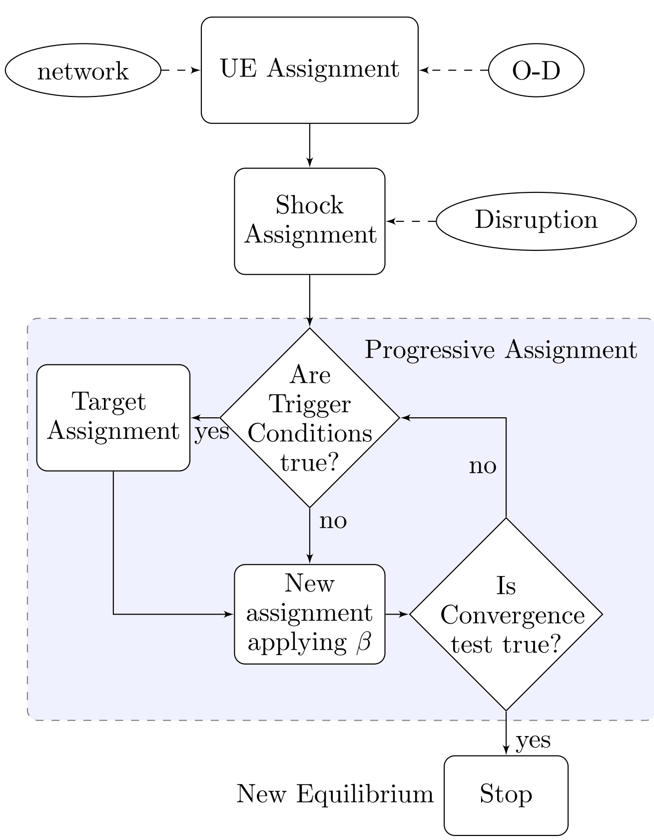

Fig. 1. The assignment model.

1) *User-Equilibrium Assignment.* In the first place, given a network and an origin-destination matrix (OD), an UE assignment is performed. This allows to obtain the pattern of traffic flows $\underline{x}^{\text{UE}}$, the travel times on links t_a^{UE}, the paths used by each pair rs at the equilibrium L_{rs}^{UE} and the corresponding travel times $TT_k^{rs}(\underline{x}^{\text{UE}})$ before the occurrence of the disruptive event.

Progressive Assignment Algorithm

Input: $\underline{x}^{\text{SA}}$, $t(\underline{x}^{\text{SA}})$, L_{rs}^{SA}
Output: $\underline{x}^{\text{FE}}$

1 Initialization:
$\underline{x}^0 = \underline{x}^{\text{SA}}$, $\underline{t}^0 = t(\underline{x}^{\text{SA}})$, $L_{rs}^0 = L_{rs}^{\text{SA}}$
Checking Trigger Conditions for the first time

$$\text{if } \exists \text{ a path } k \in L_{rs}^0 : \frac{TT_{k,rs}(\underline{x}^{\text{SA}}) - TT_{k,rs}(\underline{x}^{\text{UE}})}{TT_k^{rs}(\underline{x}^{\text{UE}})} > \Omega$$

proceed to the next step and set counter $n = 1$

2 Target UE assignment:
Given for each pair rs, the actual sets of paths L_{rs}^{n-1}, perform a User Equilibrium traffic assignment by applying the Frank-Wolfe algorithm. This yields a set of flows:

$$\underline{z}^n = [z_1^n, z_2^n, \ldots, z_{\text{A}}^n]$$

3 Applying the inertia coefficient β:
The flows on links are computed as:

$$\underline{x}^n = \beta \underline{x}^{n-1} + (1 - \beta)\underline{z}^n$$

4 Updating of times:
update travel times on links : $t_a^n = t_a(x_a^n)$
update travel times on paths : $TT_{k,rs}^n = TT_{k,rs}(\underline{x}^n)$

5 Check Trigger Conditions:
Conditions 1 and 2 are checked. For only those origin-destination pairs for which both conditions are true at the same time, add a path from Q_{rs} to the set of available paths at the next iterations. This yields a set of paths L_{rs}^n. If conditions 1 and 2 are true at the same time for at least one origin-destination pair after step (6) restart from step (2), otherwise after step (6) restart from (3) and the new target assignment will remain the one previously calculated:

$$\underline{z}^{n+1} = \underline{z}^n$$

6 Convergence test:
if a convergence test criterion is met, stop. The current solution is the new equilibrium:

$$\underline{x}^{\text{FE}} = \underline{x}^n$$

otherwise set $n = n + 1$ and go to step (2) or (3) depending on the outcome of step (5).

2) *Shock-Assignment.* Once the disruption has occured, a so-called Shock-Assignment is performed. Only the flows on the routes directly involved are reassigned considering a number of paths, among the shortest ones, equal to the number of paths used by the respective pairs rs at the equilibrium. All the other users suffer passively from the increase in travel times. This leads to a pattern of flows $\underline{x}^{\text{SA}}$, representing the state of the system immediately after the disruption.

3) *Progressive Assignment.* This section of the model is shown in the Progressive Assignment algorithm box. Three are the main components of the algorithm.

(I) The *Trigger Conditions* are responsible for managing the sets of paths on which each assignments is performed at each iteration. Two conditions are evaluated:

condition 1 verifies if someone on the network is experiencing higher travel times than acceptable. i.e.

$$\exists \text{ a path } k \in L^n_{rs} : \frac{TT_{k,rs}(\underline{x}^n) - TT_{k,rs}(\underline{x}^{\text{UE}})}{TT_{k,rs}(\underline{x}^{\text{UE}})} > \Omega \quad (2)$$

condition 2 verifies if any origin-destination pair still has an unused path available, i.e.

$$|Q_{rs}| > |L^n_{rs}| \quad (3)$$

where (3) simply states that **condition 2** is met when the cardinality of the set of all possible paths between r and s is greater than the cardinality of the set of paths used for assignments up to the iteration n. Only for those origin-destination pairs for which **conditions 1** and **2** are true at the same time a path from set Q_{rs} is added to the set of currently available paths. This leads to the set of paths L^n_{rs}.

(II) The *Target Assignment* is triggered only if **condition 1** and **2** are true at the same time for at least one origin-destination pair. An EU assignment is performed constrained by the use of sets L^n_{rs} for each rs pair. This leads to an "ideal" assignment $\underline{z}^n$.

(III) *Applying the inertia coefficient* β, the actual pattern of flows $\underline{x}^n$ is obtained at each iteration n by a linear application as shown in the box referred to the algorithm.

2.3 Performance metrics

Similar to what discussed by Omer et al. (2012), Faturechi and Miller-Hooks (2014) and Bhavathrathan and Patil (2015), to assess the level of system operativity, the following performance measures based on users' travel times are proposed.

The global system performance is defined as follows:

$$P^n = r^n / r^{\text{UE}} \quad (4)$$

where, r^n and r^{UE} are the inverse of the total travel time at iteration n (TTT^n) and total travel time in pre-disruption scenario (TTT^{UE}) respectively. As a consequence, the performance during the evolution of the system is expressed as a percentage of the pre-disruption performance, that is, when the system was operating under normal conditions.

Similarly, it is possible to define the quality of the network as perceived by the users of each origin-destination pair as follows:

$$P^n_{rs} = r^n_{rs} / r^{\text{UE}}_{rs} \quad (5)$$

where in accordance with what has been defined for the overall system, r^n_{rs} and r^{UE}_{rs} are the inverse of the travel time experienced by users of the rs origin-destination pair at iteration n (TT^n_{rs}) and travel time in a pre-disruption scenario experienced by the same users (TT^{UE}_{rs}) respectively.

3. IMPLEMENTATION AND RESULTS

The progressive assignment model is evaluated on the Nguyen-Dupuis test network, see Nguyen and Dupuis (1984). This network is represented by an oriented graph consisting of 13 nodes and 19 links. The transport demand is expressed by the origin-destination matrix OD, a sparse 13 by 13 matrix, whose only elements other than zero are: $od_{12} = 50$, $od_{13} = 10$, $od_{42} = 40$ and $od_{43} = 20$. The performance functions specific to each link $a \in \mathcal{A}$ are assumed linear. User-Equilibrium assignments are solved using the convex combination algorithm by Frank and Wolfe (1956).

Fig. 2 and Table 1 show respectively the network with the traffic flows assigned to each link and the paths used by each origin-destination pair at the equilibrium before the occurrence of the disruption.

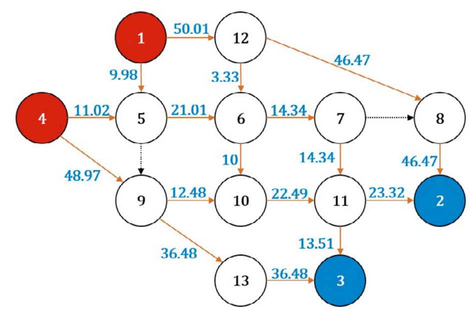

Fig. 2. Nguyen-Dupuis network and pre-disruption assignment.

Table 1. Paths used at the equilibrium.

O-D pair	paths
1-2	[1, 12, 8, 2], [1, 12, 6, 7, 11, 2]
1-3	[1, 5, 6, 10, 11, 3]
4-2	[4, 9, 10, 11, 2], [4, 5, 6, 7, 11, 2]
4-3	[4, 9, 13, 3], [4, 9, 10, 11, 3]

Once the system state is determined under normal conditions, the disruption is obtained by removing the link between nodes 12 and 8. The link has been chosen as peripheral as possible, in order to emphasize any propagation phenomenon in the performance deterioration and to avoid a dynamics excessively fast to be appreciated.

Fig. 3 shows the evolution of the global system performance as defined in (4) having set the *user tolerance index* Ω to 0.2 and the *inertia coefficient* β to 0.6. This means that users are insensitive to increases in travel times of less than 20% while at each iteration 60% of the flows of the current assignment are affected by the previous one. As it can be seen, immediately after the disruption, the performance of the system deteriorates dramatically by approximately 52%. After this spike, as the flows of each origin-destination pair are progressively reassigned over a larger set of paths, the performance of the system gradually improves. At approximately the 8th iteration, the new equilibrium is reached settling around 77% of the initial performance, resulting in a definitive performance loss of about 23%. This means that, if we consider the fact that Ω has been set to 0.2, definitely some users remain unsatisfied even at the new equilibrium, yet they are not able to do any better because of the new network topology.

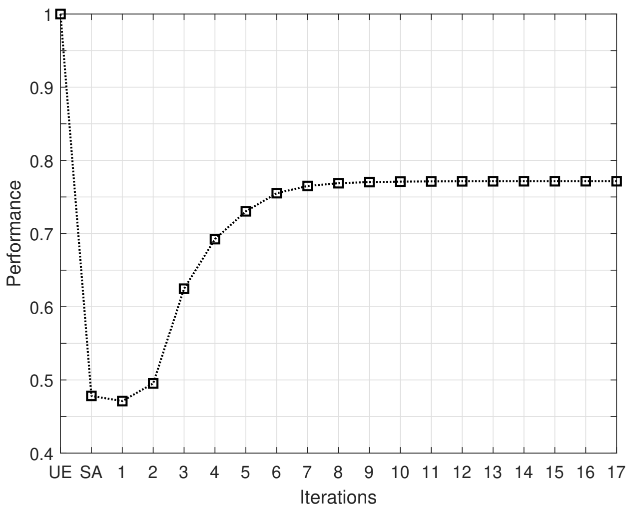

Fig. 3. Global system performance ($\Omega = 0.2$, $\beta = 0.6$).

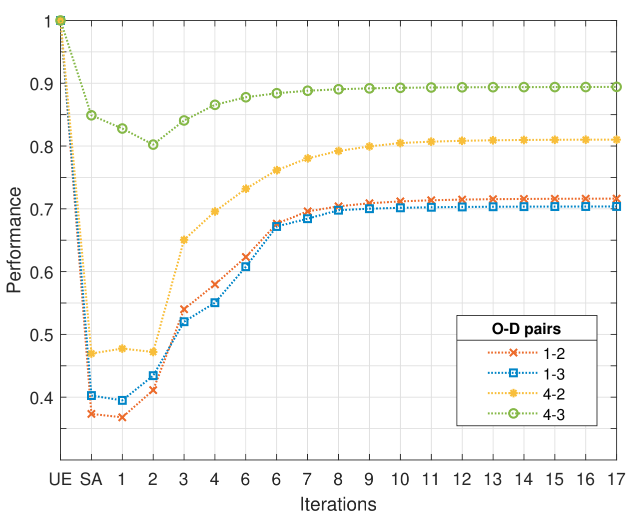

Fig. 4. Performance on paths ($\Omega = 0.2$, $\beta = 0.6$).

Considering instead the metrics defined in (5), Fig. 4 shows the evolution of the performances as they are experienced by the users of each origin-destination pair travelling on their respective paths. In this case, a cascading effect in the deterioration of the performance can be noticed which, starting from the area directly involved in the cancellation of the link, expands with less magnitude as it involves the remaining parts of the network. Looking at Table 1, it can be noticed that $1-2$ is the only one of the origin-destination pairs to be directly involved in the disruption and for this reason it is the one that suffers the most. The path [1, 12, 8, 2] used by the users of the $1-2$ pair at equilibrium is no longer available and as a consequence this transport demand spreads, in the iterations following the disruption, on the network influencing other users. Among all, the $4-3$ pair is the least affected by the disturbance, showing a maximum deterioration in performance on its paths of about 20%. This is consistent with the fact that the users of this pair use paths that are not closely connected with those used by the users of the other pairs, especially those of the $1-2$ pair.

In the following, Fig. 5 and Fig. 6 show how the evolution of the system is influenced respectively by the inertia coefficient β and user tolerance index Ω. Consistently with expectations, as shown in Fig. 5 the inertia coefficient β influences the speed at which the system converges to the new equilibrium. The higher the number of users willing to use alternative paths, the faster the system evolves towards a new stable state. By contrast, the variation of β has no influence on determining what the value of this new equilibrium will be, except in the extreme case of $\beta = 1$. In this case, the state of the system of the $n+1$-iteration does not actually evolve further, once the disruption has occurred.

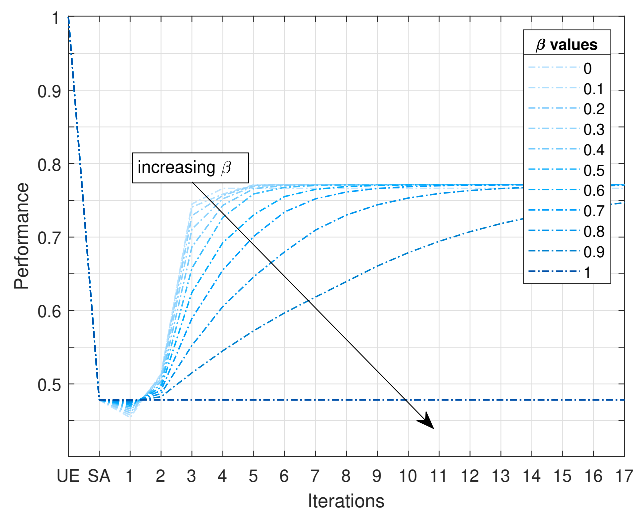

Fig. 5. Impact of β on system performance evolution ($\Omega = 0.2$).

Fig. 6 shows how the global system performance is influenced by the user tolerance index Ω. The response of the system is evaluated for values of Ω ranging from 0 to 2. More specifically, starting from a scenario in which users do not tolerate any increase, however small, in travel times ($\Omega = 0$) to one in which they are insensitive to increases in travel times lower than 200% of pre-disruption values ($\Omega = 2$), we evaluated the response of the system by applying values of the coefficient Ω obtained by discretizing the interval in 0.2 units. As expected, the values on which performance stabilizes, once the new equilibrium is reached, are partly influenced by user preferences. In detail, the more we increase the tolerance of users to increases in travel times the higher the new equilibrium will be. In other words, highly tolerant users have less incentive to use new paths to improve their travel times.

It is worth noting that, the nature of the relationship between the level that the new equilibrium will reach and the values of Ω is strongly discontinuous. Even with major increases in the coefficient, the final equilibrium achieved by the system may not change. On the contrary, sometimes it can happen that for small variations of Ω the new equilibrium changes considerably. This is a consequence of how the model is designed. The user tolerance coefficient affects the set of paths on which the flows of users can be loaded at each next iteration. As long as the higher travel time experienced by the users in the network does not imply percentage increases in times greater than Ω, the set of paths on which the assignment will be performed will not change regardless of Ω and, as a consequence, not even the dynamics of the system. However, when Ω has varied enough to match this critical value for at least the users of one origin-destination pair, the set of paths associated with them changes. As a result, traffic flow of these users will be spread over a larger portion of the network, changing the travel times of other users, eventually causing a cascading phenomenon that results in a large traffic flow re-assignment. This is clearly visible in Fig. 6 if we look at the performance trends for $\Omega = 1.4$ and $\Omega = 1.2$.

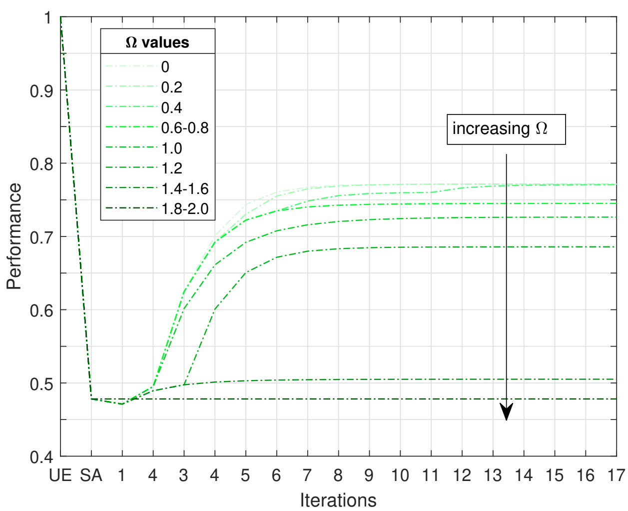

Fig. 6. Impact of Ω on system performance evolution ($\beta = 0.6$).

Finally, it is interesting to note that even if we set $\Omega = 0$, there is still a considerable gap between the original performance of the system and that given by the new equilibrium. This is due to the fact that, as much as users strive, the new network topology does not allow them to get better conditions than the original ones. For this reason, we can conclude that according to the model part of the performance differences between the two equilibriums, the one before and the one reached after the disruption, can be influenced by the users and their preferences, while the remaining part is determined exclusively by the topology of the network and by the location where the disruption takes place.

4. CONCLUSIONS

This article aims to present an assignment model capable of representing the evolution of a traffic network in the short term after the occurrence of a critical event. The Progressive Assignment, taking into account the users tolerance to increases in travel times and the intrinsic inertia of the system, controls the sets of paths on which the vehicle flow will be assigned at each iteration. From the results presented, the model appears to be able to represent some aspects of the evolution of the system in a reasonable way. However, future improvements as well as validation of the model on real data will be necessary.

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