UNIVERSITY OF GENOA

PH.D. THESIS

Mathematical modeling in Quantitative Finance and Computational Economics

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in

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The first part of my PhD Thesis deals with different Machine Learning techniques mainly applied to solve financial engineering and risk management issues.
After a short literary review, every chapter analyzes a particular topic linked to the implementation of these models, showing the most suitable methodologies able to solve it efficiently.
The following topics are therefore covered:

- Data Fitting and Regression
- Forecasting
- Classification
- Outlier Detection and Data Quality
- Pricing

Every chapter provides the theoretical explanation of the model, the description of the implementation in a numerical computing environment and the solution for real case-studies.
Among others, the main technologies discussed in this work are the following:

- Shallow Multi-Layers networks
- Feed-forward and static networks
- Radial Basis Functions (RBF) networks
- Recurrent and Dynamic Neural Networks
- Nonlinear Autoregressive (NAR) networks and Nonlinear Autoregressive networks with exogenous variables (NARX)
- Deep Neural networks
- Convolutional Networks (Conv Net)
- Fuzzy C-Means (FCM) clustering
- Self-Organizing Maps (SOM) and Kohonen networks
- Neural Networks with Circular Neurons
- Auto-Associative Neural Networks (AANN) and Auto-encoders for Nonlinear Principal Component Analysis (NLPCA)

The second part of my PhD Thesis deals with the problem of Optimal Control in Quantitative Finance and Labour Economics.
Even if the fields of application are hugely different, they share the same mathematical instrument for their solution: the Bellman principle of optimality.
After a short literary review that introduces the financial and economic problems solved in this part, the following four chapters show the most popular pricing techniques used to evaluate an option: closed formulas, Partial Differential Equations (PDE), Lattice methods and Stochastic Differential Equations (SDE).
Chapter 6 faces the problem of early-exercise in option pricing and shows how to apply the principle of optimality in the models presented in the previous chapters.
The following pricing methodologies are covered:

- Stochastic Trees and Lattice models (Cox-Ross-Rubinstein, Tian, Jarrow-Rudd, Drifted CRR, Leisen-Reimer, CRR Trinomial, Adaptive Mesh Method (AMM), Pentanomial and Heptanomial Trees)
- PDE numerical schemes (Finite Difference Method - FDM, Finite Elements Method - FEM and Radial Basis Function - RBF)
- SDE numerical solution (Longstaff-Schwartz Monte Carlo)
- Quasi-closed formulas (Roll-Geske-Whaley, Barone-Adesi-Whaley, Bjerksund-Stensland model)

The last two chapters examine two important Labour Economics dynamic problems in the field of Optimal Control Theory: Implicit Contracts and Wage Bargaining.
They share the same procedure for the solution which can be synthesized in these steps:

- Infinite-horizon deterministic optimal control problem formulation. The solution for this kind of problem can be found applying the Hamilton – Jacobi – Bellman (HJB) Equation.
- Design of a Markov Decision Chain for the numerical solution of the previous problem.
- Infinite-horizon stochastic optimal control problem formulation. After the validation of the discretization scheme in the deterministic context, the Markov Decision Chain can be extended in order to solve the stochastic version of the problem. In particular, an Ornstein-Uhlenbeck process has been introduced in the model.

The third part of my PhD Thesis deals with Forecasting and Risk Management in Energy Markets.
The first chapter introduces the two studies presented in this field through a short literary review and the Regulatory framework.
The second chapter suggests some quantitative methods with the aim of managing the main risks of Guarantees of Origin (Gos).
Given that Gos trading is rather recent, it implements an innovative integrated control system in order to handle market and counterparty risks.
The following techniques are covered:

- Market Risk: Historical, parametric and Monte Carlo VaR with a special focus on volatility modeling (historical, implied, GARCH, SABR σ).
- Liquidity Risk: Bid-Ask spread analysis.
- Counterparty Risk: Probability of Default estimation starting from: listed CDS premium, traded bond prices and statement analysis (KMV model).

The third chapter deals with the energy spot prices forecasting problem.
The aim of the study is to establish a time-horizon within which it is reasonable to predict prices.
The state-of-the-art architectures based on Deep Learning methods are implemented in order to solve this econometric issue.
The analyzed techniques are:

- A multi-layered Nonlinear Autoregressive (NAR) network (Endogenous variable: prices).
- A multi-layered Nonlinear Autoregressive with an exogenous variable (NARX) network (Endogenous variable: prices - Exogenous variable: demand).
- A Long Short-Term Memory (LSTM) network with one feature (prices).
- A Long Short-Term Memory (LSTM) network with two features (prices and demand).

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PART I:

ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING TECHNIQUES

PART II:

DETERMINISTIC AND STOCHASTIC OPTIMAL CONTROL

PART III:

FORECASTING AND RISK MEASURES IN ENERGY MARKETS

FIGURE 1: PhD Thesis structure