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13th Scientific Meeting of the Classification and Data Analysis Group Firenze, September 9-11, 2021

edited by

Giovanni C. Porzio Carla Rampichini Chiara Bocci



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Preface

This book collects the abstracts and short papers presented at CLADAG 2021, the 13th Scientific Meeting of the Classification and Data Analysis Group (CLADAG) of the Italian Statistical Society (SIS). The meeting has been organized by the Department of Statistics, Computer Science, Applications 'G. Parenti' of the University of Florence, under the auspices of the University of Florence, the SIS and the International Federation of Classification Societies (IFCS).

CLADAG is a member of the IFCS, a federation of national, regional, and linguistically-based classification societies. It is a non-profit, non-political scientific organization, whose aims are to further classification research.

Every two years, CLADAG organizes a scientific meeting, devoted to the presentation of theoretical and applied papers on classification and related methods of data analysis in the broad sense. This includes advanced methodological research in multivariate statistics, mathematical and statistical investigations, survey papers on the state of the art, real case studies, papers on numerical and algorithmic aspects, applications in special fields of interest, and the interface between classification and data science. The conference aims at encouraging the interchange of ideas in the above-mentioned fields of research, as well as the dissemination of new findings. CLADAG conferences, initiated in 1997 in Pescara (Italy), were soon considered as an attractive information exchange market and became an important meeting point for people interested in classification and data analysis. A selection of the presented papers is regularly published in (post-conference) proceedings, typically by Springer Verlag.

The Scientific Committee of CLADAG 2021 conceived the Plenary and Invited Sessions to provide a fresh perspective on the state of the art of knowledge and research in the field. The scientific program of CLADAG 2021 is particularly rich. All in all, it comprises 5 Keynote Lectures, 26 Invited Sessions promoted by the members of the Scientific Program Committee, 10 Contributed Sessions, and a Plenary Session on Statistical Issues in the COVID-19 Pandemic. We thank all the session organizers for inviting renowned speakers, coming from many different countries. We are greatly indebted to the referees, for the time spent in a careful review of the abstracts and short papers collected in this book. The Conference was planned as an in presence event; unfortunately due to the persistent uncertainty of the COVID-19 epidemic condition, CLADAG 2021 will be completely online.

Special thanks are finally due to the members of the Local Organizing Committee and all the people who collaborated for CLADAG 2021. Last but not least, we thank all the authors and participants, without whom the conference would not have been possible.

Giovanni Camillo Porzio Carla Rampichini Chiara Bocci

Florence, September 2021

ROBUSTNESS METHODS FOR MODELLING COUNT DATA WITH GENERAL DEPENDENCE STRUCTURES

Marta Nai Ruscone ¹ and Dimitris Karlis²

ABSTRACT: Bivariate Poisson models are appropriate for modelling paired count data. However, the bivariate Poisson model does not allow for a negative dependence structure. Therefore, it is necessary to consider alternatives. A natural way is to consider copulas to generate various bivariate discrete distributions. While such models exist in the literature, the issue of choosing a suitable copula has been overlooked so far. Different copulas lead to different structures and any copula misspecification can render the inference useless. In this work, we consider bivariate Poisson models generated with a copula and investigate its robustness under outliers contamination and model misspecification. Particular focus is on the robustness of copula related parameters. English Premier League data are used to demonstrate the effectiveness of our approach.

KEYWORDS: copula, dependence, outliers, robustness.

1 Introduction

Bivariate Poisson models are appropriate for modelling paired count data exhibiting correlation. Paired count data arise in a wide context including, for example, sports (e.g. the number of goals scored by each one of the two opponent teams in soccer). Several models are available that can incorporate different structures and marginal properties, see for example Karlis & Ntzoufras, 2003. See also the work in Nikoloulopoulos, 2013 for defining models with copulas. While several extensions and models have been proposed, up to our knowledge, issues of robustness have been overlooked. Following da Fonseca & Fieller, 2006, there are two kinds of achieved robustness that one should consider. The first one refers to contamination from outlier observations or, better, from observations that are unexpected under a certain model. The second one refers to model deviation, i.e. a researcher would like to fit the model

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with such a method that even if the model is not correct the method would protect from deriving inconsistent results.

In this work, we consider a copula based bivariate Poisson distribution. We apply a minimum distance estimation methodology using Hellinger distance. We investigate its robustness under outliers contamination and model misspecification. Particular focus is given on the robustness of copula related parameters that measure the association exhibited by paired count data. The effectiveness of this methodology is examined on data from English Premier League 2013-2014.

2 Copulas

Copula are functions that join multivariate distributions to their marginal distributions (Nelsen, 2007). They describe the dependence structure existing across marginal random variables. In this way we can consider bivariate distributions with dependency structures different from the linear one that characterizes the multivariate Gaussian distribution.

A bivariate copula $C: I^2 \to I$, with I = [0,1], is the cumulative bivariate distribution function of the random variables (U,V) with uniform marginal distributions in [0,1]. It is define as:

$$C(u, v; \theta) = P(U \le u, V \le v; \theta), \quad 0 \le u \le 1 \quad 0 \le v \le 1$$

where θ is a parameter measuring the dependence between U and V.

Let (Y_1, Y_2) be a bivariate random vector with marginal cdfs $F_{Y_1}(y_1)$ and $F_{Y_2}(y_2)$ and joint cdf $F_{Y_1,Y_2}(y_1,y_2;\theta)$. There always exists a copula function $C(\cdot,\cdot;\theta)$ such that

$$F_{Y_1,Y_2}(y_1,y_2;\theta) = C(F_{Y_1}(y_1),F_{Y_2}(y_2);\theta), \quad y_1,y_2 \in \mathbb{R}.$$
 (2)

This result states that each joint distribution can be expressed in terms of two separate but related issues, the marginal distributions and the dependence structures between them. The dependence structure is explained by the copula function $C(\cdot,\cdot;\theta)$.

When Y_1 and Y_2 are discrete random variables taking values on some lattice, Ω , the copula C is unique in $(y_1, y_2) \in \Omega$ but not elsewhere. Thus, in the discrete case the mapping from two marginals and a copula $\{F_1, F_2, C\}$ to a bivariate distribution $F(Y_1, Y_2)$ is not one-to-one. However, this is not-uniqueness is of no consequence as the region outside Ω is not of interest in the discrete case (Nelsen, 2007).

3 Bivariate count models based on copulas

For count data, a common starting point is to use the Poisson distribution for the marginals:

$$f(y;\mu_j) = \mu_j^y e^{-\mu_j}/y!, \qquad j = 1,2 \quad y = 0,1,...$$
 (3)

where $\mu_j > 0$. Models based on copulas in the case of bivariate counts offer the advantage of allowing easy generalization to several different models which is not easy in general. Take, for instance, the Frank copula:

$$C(u, v; \gamma) = -\gamma^{-1} \log \left[1 + \frac{(\exp^{-\gamma u} - 1)(\exp^{-\gamma v} - 1)}{\exp(-\gamma) - 1} \right], \quad \gamma \in R - \{0\}, \ u, v \in [0, 1].$$
 (4)

Then

$$F(y_1, y_2; \mu_1, \mu_2, \gamma) \equiv C(F(y_1; \mu_1), F(y_2; \mu_2); \gamma), \tag{5}$$

is a well defined distribution function with a dependence structure. It's probability mass function is

$$P(Y_1 = y_1, Y_2 = y_2; \mu_1, \mu_2, \gamma) = F(y_1, y_2; \mu_1, \mu_2, \gamma) - F(y_1 - 1, y_2; \mu_1, \mu_2, \gamma)$$

$$-F(y_1, y_2 - 1; \mu_1, \mu_2, \gamma) + F(y_1 - 1, y_2 - 1; \mu_1, \mu_2, \gamma)$$
(6)

In the present work we focus on bivariate models. For a review of discrete valued models based on copulas see Nikoloulopoulos, 2013.

4 Minimum distance estimation

In discrete data, model robustness and efficiency can be achieved almost at the same time, by defining distances that downweight some observations Lindsay, 1994. The minimum distance estimators can be interpreted as weighted likelihood estimators, the weights are determined by some kind of distance between observed and expected frequencies. For example, consider Minimum Hellinger distance estimators based on minimizing

$$\sum_{x} \left(d(x)^{1/2} - m_{\beta}(x)^{1/2} \right)^{2}$$

where d(x) is the observed relative frequency and $m_{\beta}(x)$ is the probability mass at x with the assumed model with parameters of interest β . It turns out that this quantity leads to estimating equations of the form

$$\sum_{x} \left(\frac{d(x)}{m_{\beta}(x)} \right)^{1/2} \frac{\partial m_{\beta}(x)}{\partial \beta} = 0$$

directly comparable to the ML estimating equations

$$\sum_{x} \frac{d(x)}{m_{\beta}(x)} \frac{\partial m_{\beta}(x)}{\partial \beta} = 0$$

which actually implies that we weight the observations differently (see Lindsay, 1994).

In this work we extend the approach for bivariate count models defined by copulas aiming at deriving robust estimators for both the marginal and the copula parameters. Now x implies a pair of observations. Also, in our case the parameters β to estimate are those of the marginal distribution plus the copula parameter(s). We have also developed an iterative algorithm that facilitates the estimation. In the bivariate case we are interested in the relative frequencies are still reasonable estimators of the underlying probabilities but we need larger sample sizes for that. As we move on higher dimensions, problems similar to that of the regression setting may occur.

5 Application

Bivariate count models are widely used for modelling the outcome of a football game. The two counts refer to the number of goals scored by each team. It seems natural to assume some dependence between the goals to represent the competitive nature of soccer. Our data refer to all scores from English Premier League 2013-2014 where a series of unexpectedly large scores have occurred. We apply a robust approach to estimate the parameters of the model to reduce the effect of the large scores.

References

DA FONSECA, V GRUNERT, & FIELLER, NRJ. 2006. Distortion in statistical inference: the distinction between data contamination and model deviation. *Metrika*, **63**(2), 169–190.

KARLIS, D., & NTZOUFRAS, I. 2003. Analysis of sports data by using bivariate Poisson models. J. R. Stat. Soc., **52**(3), 381–393.

LINDSAY, B. G. 1994. Efficiency versus robustness: the case for minimum Hellinger distance and related methods. *Ann. Stat.*, **22**(2), 1081–1114.

NELSEN, R B. 2007. An introduction to copulas. New York: Springer.

NIKOLOULOPOULOS, A.K. 2013. Copula-based models for multivariate discrete response data. *Pages 231–249 of: Copulae in Mathematical and Quantitative Finance*.