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A Discrete-Time Model for Large-Scale Multi-Modal Transport Networks

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Abstract: The purpose of this paper is to provide a method that can be used to perform resilience analyses on large-scale multi-modal transportation networks. In particular, this work proposes an assignment model through which the allocation of mobility demand on a multi-modal transport network is defined at a regional or supra-regional level. The results of the assignment problem are then used as input data of a macroscopic dynamic model specifically developed to represent the dynamics of a multi-modal transport network. Finally, the proposed methodology is applied to represent the behavior of users in the Nguyen-Dupuis test network.

Keywords: Multi-modal transport network, multi-modal assignment, discrete-time network model

1. INTRODUCTION

Since the beginnings of civilization, transportation systems have played a decisive role in the social and economic advancement allowing to connect business activities and provide access to vital resources for everyday activities. The link between productive/social activities and transport activities has been further strengthened in the contemporary era, making all human activities exposed to the criticalities that arise in transport networks. It is a matter of fact that the infrastructures used for transport activities are those which have suffered most from natural and/or man-made disasters. Furthermore, the experience gained after disasters as the recent collapse of the Morandi bridge in Italy in 2018, has shown that, besides the loss of human lives, a critical event of that type can have both immediate and local impacts and wider effects over a long period and on a wide geographical area.

These types of events have disclosed the necessity of analyzing the intrinsic characteristics of transport infrastructures and networks leading to a growing body of literature on performance evaluation of transport systems during disruptive events. Those works are generally aimed at providing estimations of the system operation level in order to assess the resilience of a transportation network, defined as the ability of a system potentially exposed to a risk to resist, adapt or change in order to achieve and maintain acceptable performances. The concept of resilience as a property of a system has been first introduced in [1] in the area of ecological systems. Then, this concept has been extended also to transport and civil infrastructure [2, 3]. More recently, resilience analysis has been applied both to road networks [4, 5, 6] and to railway networks [7, 8].

Other works in the literature are aimed at assessing the impacts produced by events that partially or totally deteriorate the capacity of a transport network. For instance, the study in [9] proposes a methodology aimed at defining the most critical links of a road traffic network, by means of suitable indices which evaluate: risk, reliability, accessibility and vulnerability. A vulnerability analysis has also been conducted in [10], where several indexes are analyzed in order to assess the vulnerability of a dense urban road network. Further works, as for instance in [11], assess the robustness of road networks subject to disruptive events. The resilience analysis of multi-modal transport networks is a very important research topic, particularly useful when the critical event involves several modes of transport at the same time, or when the decision makers intend to use all the mobility capacity of a geographical area by allocating the mobility demand on different modes or by suggesting multi-modal itineraries. However, just few works in the literature are focused on this field of application, such as those reported in [12] and [13].

The aim of this paper is to provide a tool that may be used to conduct a resilience analysis on a large-scale multi-modal transport network. Specifically, this work proposes a macroscopic dynamic model capable of representing a multi-modal transport network at regional or supra-regional level. The flows on the paths of such multi-modal network are obtained through the adoption of a specific assignment model. The multi-modal transport tool developed in this work could be the basis for regulation approaches to determine routing and modal indications for users after a disruptive event. In addition, the proposed tool could be adopted to evaluate the resilience of a multi-modal transportation network under the occurrence of several critical scenarios. In the present work, the proposed methodology is applied to a benchmark network, that is the Nguyen-Dupuis network.

The present paper is organized as follows. In Section 2 the representation of the multi-modal transport network is introduced. The multi-modal traffic assignment model is described in Section 3 while the macroscopic multi-modal transport network model is outlined in Section 4. The application of the proposed methodology to the Nguyen-

Dupuis test network is shown in Section 5. Finally, some conclusive remarks are gathered in Section 6.

2. A GRAPH REPRESENTATION FOR MULTI-MODAL TRANSPORT NETWORKS

As mentioned in the Introduction, the aim of this work is to provide a framework that can be used, at a strategic level, to assess the resilience of a large-scale multi-modal transport network. This framework is composed of two modules: a multi-modal assignment module and a dynamic modelling module. Both modules are referred to a regional multi-modal transport network in which the considered transport modalities are road and rail transport. The multi-modal transport network is represented by means of a unique graph as depicted in Fig. 1.

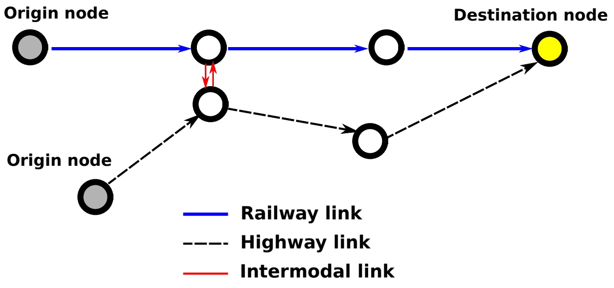

Fig. 1. Sketch of the intermodal transport network.

Let us start by denoting with $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ the adopted oriented graph, in which $\mathcal{N}$ indicates the set of nodes, whereas $\mathcal{A} = \mathcal{A}^{\mathrm{H}} \cup \mathcal{A}^{\mathrm{R}} \cup \mathcal{A}^{\mathrm{I}}$ represents the set of arcs (or links) composing the network. Each subset of links is defined as follows:

- $\mathcal{A}^{\mathrm{H}}$ is the set of highway connections;
- $\mathcal{A}^{\mathrm{R}}$ is the set of railway connections;
- $\mathcal{A}^{\mathrm{I}}$ is the set of intermodal arcs.

Let us also consider the network in an origin-destination-oriented mode in which $J^{\mathrm{O}} \subseteq \mathcal{N}$ represents the set of all possible origin nodes, $J^{\mathrm{D}} \subseteq \mathcal{N}$ represents the set of all possible destination nodes. Let us denote with l, $l = 1, \ldots, L$, a generic path existing in the network and with $\mathcal{L}^{od}$ the set of all possible paths connecting all the origin-destination pairs with $o \in J^{\mathrm{O}}$ and $d \in J^{\mathrm{D}}$. Let us also define, for each $(i,j) \in \mathcal{A}$, the set $\mathcal{L}^{od}_{i,j}$ containing the paths connecting the origin-destination pair od in which the link (i,j) is included. Note that, if an arc (i,j) is not included in any of the paths connecting the origin-destination pair od, the corresponding set $\mathcal{L}^{od}_{i,j}$ is an empty set.

Finally note that, in this network, the intermodal arcs are fictitious links that allow for the modal shift between road transport and rail transport and vice versa. For this reason an origin node $o \in J^{\mathrm{O}}$ cannot be followed by an intermodal link and, in the same way, a destination node $d \in J^{\mathrm{D}}$ cannot be preceded by an intermodal link.

3. THE MULTI-MODAL ASSIGNMENT MODEL

Given a transport network and an origin-destination matrix representing the mobility demand, an assignment model aims to estimate the users' behavior by determining the amount of flow on each arc of the network. In this work, the N-Path restricted User-Equilibrium traffic assignment is used to assign the mobility demand among multiple paths that may also include different transportation modes.

The User-Equilibrium traffic assignment involves finding a pattern of traffic flows on the network such that no user has, unilaterally, an interest in changing his path, since no other alternative can guarantee lower travel times. It can be demonstrated that the pattern of flows satisfying this condition corresponds to the optimal solution of the well-known optimization problem aimed at finding the minimum of the Beckmann's Transformation [14]. The N-Path restricted version of the model [15] forces the assignment of the flows of each origin-destination pair on sets of admissible paths which may have a cardinality lower than the sets of all possible paths. This allows to exclude solutions that are mathematically correct but implausible in a realistic scenario. In this paper, the paths involving more than one modal change are excluded. Therefore, let us denote with $\mathcal{P}^{od} \subseteq \mathcal{L}^{od}$ the set of admissible paths from o to d.

The optimization problem can be defined as follows:

$$\min \; z(x) = \sum_{(i,j)\in\mathcal{A}} \int_0^{x_{i,j}} t_{i,j}(\omega) d\omega \tag{1}$$

subject to

$$\sum_{l\in\mathcal{P}^{od}} f^{od,l} = D^{od} \qquad o \in J^O,\, d \in J^D \tag{2}$$

$$f^{od,l} \geq 0 \qquad o \in J^O,\, d \in J^D,\, l \in P^{od} \tag{3}$$

$$x_{i,j} = \sum_{o\in J^O} \sum_{d\in J^D} \sum_{l\in\mathcal{P}^{od}} f^{od,l} \cdot \delta^{od}_{(i,j),l} \qquad (i,j) \in \mathcal{A} \tag{4}$$

where $f^{od,l}$ is the traffic flow along the l-th path from node o to node d, D^{od} is the mobility demand from origin node o to destination node d and $x_{i,j}$ is the traffic flow on link (i,j) of the network. Constraints (2) impose that all transportation demands are satisfied. Constraints (3), on the other hand, impose that the flows are non-negative. Moreover, constraints (4) define the relation between the flows on the paths and the flows on the arcs of the network where:

$$\delta^{od}_{(i,j),l} = \begin{cases} 1 & \text{if } (i,j) \text{ belongs to path } l \text{ from } o \text{ to } d \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Finally, $t_{i,j}(\cdot)$ are the link performance functions which define the relation between the travel time that users experience passing through a link and the amount of flows on that link. In this work, specific performance functions have been defined for each arc type, i.e. for highway, railway and intermodal arcs. In order to reduce the computational effort, properly linear versions of the performance functions have been used. Excluding the intermodal links, for which constant performance functions have been adopted, for road and railway links hyperbolic functions presenting a divergent behavior in proximity of their theoretical capacity (intended as the maximum number of users that can transit on the link) have been defined.

For road links, the hyperbolic function is linearized by interpolating two points: the free-flow travel time when the arc is empty and the value of the function corresponding to a number of transiting vehicles equal to $\beta \cdot n_{i,j}^{max}$ where $\beta \in [0,1]$ is a suitable coefficient. For railway links a similar approach has been adopted, but the second point of interpolation is in this case the value assumed by the hyperbolic functions corresponding to the technical limit of users transiting on the link, equal to $\frac{C^t \cdot \Delta_{i,j}}{s_{i,j}^{min}}$ (see equation (18)).

In general, it can be demonstrated that if the performance functions are strictly increasing and depend exclusively on the specific arc flows for which they have been defined, the optimization problem is convex relative to the flows on the links. In general, the same cannot be argued with respect to flows on the paths $f^{od,l}$. Since uniqueness is a desirable quality for multiple reasons, researchers have developed several models that ensure uniqueness even with respect to path flows at the equilibrium [16]. One possibility is to compute the flows $f^{od,l}$, consistent with the patterns of flows on the links $x_{i,j}$ found at the equilibrium, which maximize the so-called entropy function [17]. The resulting optimization problem is as follows:

$$\min \ h(x) = \sum_{o \in J^O} \sum_{d \in J^D} \sum_{l \in \mathcal{P}^{od}} f^{od,l} \cdot \ln(f^{od,l}) \tag{6}$$

subject to

$$\sum_{l \in \mathcal{P}^{od}} f^{od,l} = D^{od} \qquad o \in J^O, \, d \in J^D \tag{7}$$

$$x_{i,j} = \sum_{o \in J^O} \sum_{d \in J^D} \sum_{l \in \mathcal{P}^{od}} f^{od,l} \cdot \delta^{od}_{(i,j),l} \qquad (i,j) \in \mathcal{A} \tag{8}$$

The constraints (7)-(8) are essentially the same as the previous ones, because, as already mentioned, the optimal flows $f^{od,l}$ must be consistent with the flows on the arcs $x_{i,j}$ determined at equilibrium. In this case, however, in constraints (8) the flows $x_{i,j}$ are already determined for every link $(i,j) \in \mathcal{A}$. Finally, it is worth noting that the objective function of the problem containing a logarithm implies strict positivity of the flows $f^{od,l}$. This is a consequence of the fact that the search for the optimal solution should be carried out exclusively by considering the flows actually used at the equilibrium, i.e., those that guarantee the shortest travel time to the users of each pair.

4. THE MACROSCOPIC MULTI-MODAL TRANSPORT NETWORK MODEL

The model described in this section has the aim of representing, at a macroscopic level, the dynamic behavior of the multi-modal transport network previously introduced, similarly to what done in [18, 19] in the context of freight logistics in maritime terminals. To this end, a discrete-time model is proposed, in which the time horizon is divided in K time steps, where $k = 1, \ldots, K$ denotes the temporal stage, and T in [h] indicates the sample time interval. Let us also indicate with $\Delta_{i,j}$ in [km] the length of each arc $(i,j) \in \mathcal{A}$. The input data and parameters considered for this model can classified between those common to all types of transport and those associated with road or rail transport only. Let us start by introducing the data and parameters common to all types of transport, that are:

- $f^{od,l}(k)$ is the flow expressed in terms of users entering the network at time step k from the origin node $o \in J^{\mathrm{O}}$, and reaching the destination $d \in J^{\mathrm{D}}$ using the path $l \in \mathcal{L}^{od}$. Such flows are the outputs of the assignment procedure described in Section 3;
- η is the average number of passengers per car.

The parameters related to road transport are:

- $v_{i,j}^{\mathrm{h}}$ [km/h] is the maximum speed allowed in link $(i,j) \in \mathcal{A}^{\mathrm{H}}$;
- $w_{i,j}$ [km/h] is the congested wave speed associated with link $(i,j) \in \mathcal{A}^{\mathrm{H}}$;
- $n_{ij}^{\max}$ is the maximum number of vehicles that can travel in link $(i,j) \in \mathcal{A}^{\mathrm{H}}$.

The parameters related to rail transport are:

- $v_{i,j}^{\mathrm{r}}$ [km/h] is the maximum speed allowed in link $(i,j) \in \mathcal{A}^{\mathrm{R}}$;
- $h_{i,j}$ [h] is the average time headway associated with link $(i,j) \in \mathcal{A}^{\mathrm{R}}$;
- $s_{ij}^{\min}$ [km] is the minimum average space headway allowed in link $(i,j) \in \mathcal{A}^{\mathrm{R}}$.

Finally, in order to ensure a correct time discretization, the length of the time interval T must allow a proper dynamic evolution of the system, therefore the length of the time step is chosen in order to verify, for any arc $(i,j) \in \mathcal{A}^{\mathrm{H}} \cup \mathcal{A}^{\mathrm{R}}$, the condition

$$T \leq \min \left\{ \frac{\Delta_{i,j}}{v_{i,j}^{\mathrm{h}}}, \frac{\Delta_{i,j}}{v_{i,j}^{\mathrm{r}}} \right\} \tag{9}$$

The system dynamic evolution is described by means of aggregate variables, that, for each link $(i,j) \in \mathcal{A}$ and for each time step k, $k = 0, \ldots, K$, are listed as follows:

- $n_{i,j}^{od,l}(k)$ is the number of users in arc $(i,j) \in \mathcal{A}$ in path $l \in \mathcal{L}_{i,j}^{od}$ and associated with the *od* pair, with $o \in J^{\mathrm{O}}$, $d \in J^{\mathrm{D}}$. If an arc is of railway type, i.e., $(i,j) \in \mathcal{A}^{\mathrm{R}}$, the users are expressed in terms of passengers, while they are expressed in terms of vehicles for highway links $(i,j) \in \mathcal{A}^{\mathrm{H}}$. Finally, on an intermodal arc, $(i,j) \in \mathcal{A}^{\mathrm{I}}$, the users can be both passengers and vehicles in accordance with the type of arc preceding the intermodal one;
- $n_{i,j}(k)$ is the total number of users in the arc $(i,j) \in \mathcal{A}$ at time step k;
- $I_{i,j}^{od,l}(k)$ is the number of users entering node i in path $l \in \mathcal{L}_{i,j}^{od}$ and associated with the *od* pair, with $o \in J^{\mathrm{O}}$, $d \in J^{\mathrm{D}}$;
- $O_{i,j}^{od,l}(k)$ is the number of users exiting node j in path $l \in \mathcal{L}_{i,j}^{od}$ and associated with the *od* pair, with $o \in J^{\mathrm{O}}$, $d \in J^{\mathrm{D}}$.

Therefore, the dynamic evolution of the system is described, for each k, with $k = 0, \ldots, K$, by the following dynamic equation:

$$n_{i,j}^{od,l}(k+1) = n_{i,j}^{od,l}(k) + I_{i,j}^{od,l}(k) - O_{i,j}^{od,l}(k) \tag{10}$$

for all $(i,j) \in \mathcal{A}$, $o \in J^{\mathrm{O}}$, $d \in J^{\mathrm{D}}$, $l \in \mathcal{L}_{i,j}^{od}$, in which $I_{i,j}^{od,l}$ is given by:

$$I_{i,j}^{od,l}(k) = c_{p,i} \cdot O_{p,i}^{od,l}(k) + \varsigma_{i,j} \cdot f^{od,l}(k) \cdot T \qquad (11)$$

where (p,i) is the arc preceding arc (i,j) in the path $l \in \mathcal{L}_{i,j}^{od}$. Note that $f^{od,l}(k)$ represents the flows entering the network at time step k from each origin node $o \in J^{\mathrm{O}}$, and it is set equal to 0 for all nodes $i \notin J^{\mathrm{O}}$. In order to correctly quantify the number of passengers or vehicles present in a link, the two conversion factors $c_{p,i}$ and $\varsigma_{i,j}$ are introduced. Specifically, considering that the switch between two different modes of transport can only occur in an intermodal link, the conversion factor $c_{p,i}$ is given by

$$c_{p,i} = \begin{cases} 1 & \text{if } (p,i) \in \mathcal{A}^{\mathrm{H}} \cup \mathcal{A}^{\mathrm{R}} \text{ and } (i,j) \in \mathcal{A} \\ \eta & \text{if } (p,i) \in \mathcal{A}^{\mathrm{I}} \text{ and } (i,j) \in \mathcal{A}^{\mathrm{R}} \\ \frac{1}{\eta} & \text{if } (p,i) \in \mathcal{A}^{\mathrm{I}} \text{ and } (i,j) \in \mathcal{A}^{\mathrm{H}} \end{cases} \qquad (12)$$

The conversion factor $\varsigma_{i,j}$ is instead defined considering that the flow $f^{od,l}(k)$ is given in terms of users and considering that an origin node cannot be followed by an intermodal link, therefore

$$\varsigma_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in \mathcal{A}^{\mathrm{R}} \\ \frac{1}{\eta} & \text{if } (i,j) \in \mathcal{A}^{\mathrm{H}} \end{cases} \qquad (13)$$

Finally, the number of vehicles or passengers exiting from a link $(i,j) \in \mathcal{A}$ in time step k is computed by

$$O_{i,j}^{od,l}(k) = \frac{T}{\tau_{i,j}(k)} n_{i,j}^{od,l}(k) \qquad (14)$$

where $\tau_{i,j}(k)$ is the transfer time required to cover the link (i,j). That transfer time is computed according to the type of link (i,j), according to

$$\tau_{i,j}(k) = \begin{cases} \frac{\Delta_{i,j}}{V_{ij}(n_{ij}(k))} & \text{if } (i,j) \in \mathcal{A}^{\mathrm{H}} \cup \mathcal{A}^{\mathrm{R}} \\ \alpha_{i,j} \cdot T & \text{if } (i,j) \in \mathcal{A}^{\mathrm{I}} \end{cases} \qquad (15)$$

In order to explain (15), let us start from the intermodal connections. For this kind of arc the transfer time $\tau_{i,j}(k)$ is considered constant and equal to $\alpha_{i,j}$ times the sample time T, with $\alpha_{i,j} \geq 1$ for each $(i,j) \in \mathcal{A}^{\mathrm{I}}$.

With regard to highway and railway links, it should be noted that, for both types of arc, the transfer time is estimated as a function of the total number of vehicles or passengers $n_{ij}(k)$ present in that connection. For a generic arc (i,j), $n_{ij}(k)$ is given by

$$n_{i,j}(k) = \sum_{o \in J^{\mathrm{O}}} \sum_{d \in J^{\mathrm{D}}} \sum_{l \in \mathcal{L}_{i,j}^{od}} n_{i,j}^{od,l}(k) \qquad (16)$$

More in details, for each highway link $(i,j) \in \mathcal{A}^{\mathrm{H}}$, the transfer time $\tau_{i,j}(k)$ is computed according to the current traffic conditions, i.e.

$$V_{ij}(n_{ij}(k)) = \min \left\{ v_{i,j}^{\mathrm{h}}, \frac{w_{i,j}}{n_{ij}(k)} \Delta_{i,j} \left[\frac{n_{ij}^{\max}}{\Delta_{i,j}} - \frac{n_{ij}(k)}{\Delta_{i,j}} \right] \right\} \qquad (17)$$

where $V_{ij}(n_{ij}(k))$ is the steady-state relationship between speed and number of vehicles. The relation (17) has been derived from a traffic triangular fundamental diagram, as the one proposed in [20], and expressed in terms of number of vehicles.

Analogously, also for rail transportation, a steady-state speed-number of passengers function has been developed. This relationship has been derived from a hypothetical graphical train timetable, from which it is possible to deduce that the average space headway s is given by $s = h \cdot v + L$, where h is the average time headway, v is the train speed and L is the average length of trains. Hence the steady-state speed relationship $V_{ij}(n_{ij}(k))$ for all $(i,j) \in \mathcal{A}^{\mathrm{R}}$ may be formulated as

$$V_{ij}(n_{ij}(k)) = \begin{cases} v_{i,j}^{\mathrm{r}} & \text{if } \frac{n_{i,j}(k)}{C^{\mathrm{t}}\Delta_{i,j}} \leq \frac{1}{h_{i,j}v_{i,j}^{\mathrm{r}}+L} \\ \frac{1}{h_{i,j}} \left(\frac{C^{\mathrm{t}}\Delta_{i,j}}{n_{i,j}(k)} - L \right) & \text{if } \frac{1}{h_{i,j}v_{i,j}^{\mathrm{r}}+L} < \frac{n_{i,j}(k)}{C^{\mathrm{t}}\Delta_{i,j}} \leq \frac{1}{s_{i,j}^{\min}} \end{cases} \qquad (18)$$

where C^{t} is the train capacity in terms of passengers that a train can host.

It is worth noting that, for each highway or railway arc, condition (9) with (17) and (18) imply that the transfer time $\tau_{i,j}(k)$ is never less than T, ensuring the validity of the conservation equations.

5. SIMULATION RESULTS

This section shows the application of the proposed methodology to a simple test network with the final goal of illustrating the potential of this tool that can be adopted both for resilience analysis and for decision-making purposes allowing, for instance, the development or evaluation of different mobility regulation policies. Specifically, the adopted test network has been obtained by slightly modifying the well-known Nguyen-Dupuis network (for further details see [21]). The resulting network, depicted in Fig. 2, is represented by an oriented graph consisting of 14 nodes and 20 links whose main parameters are listed in Tables 1 and 2.

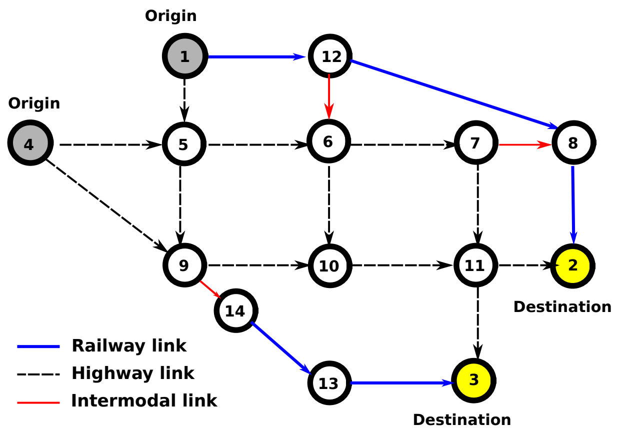

Fig. 2. Sketch of the test multi-modal transport network.

The other parameters have been set as follows: the conversion factor η is equal to 1.45, the congestion wave speed $w_{i,j}$ is equal to 30 [km/h], $\forall(i,j) \in \mathcal{A}^{\mathrm{H}}$, the average time headway $h_{i,j}$ is 15 minutes and the minimum average space headway $s_{ij}^{\min}$ is 2 [km], $\forall(i,j) \in \mathcal{A}^{\mathrm{R}}$, while for each $(i,j) \in \mathcal{A}^{\mathrm{I}}$ the constant parameter $\alpha_{i,j}$ is equal to 2.

Table 1. Values of the main parameters referred to the highway arcs

	$\Delta_{i,j}$ [km]	$v^{\mathrm{h}}_{i,j}$ [km/h]
Link 1-5	9	100
Link 4-5	7	100
Link 4-9	10	80
Link 5-6	9	100
Link 5-9	50	70
Link 6-7	8	100
Link 6-10	50	100
Link 7-11	2	110
Link 9-10	3	100
Link 10-11	10	80
Link 11-2	10	130
Link 11-3	9	100

Table 2. Values of the main parameters referred to the railway arcs

	$\Delta_{i,j}$ [km]	$v^{\mathrm{r}}_{i,j}$ [km/h]
Link 1-12	50	140
Link 8-2	40	140
Link 12-8	40	140
Link 13-3	10	140
Link 14-13	80	140

As for the multi-modal assignment module, the transportation demand has been defined in terms of the number of users per hour and expressed through an origin-destination matrix in which the non-zero elements correspond to the *od* pairs: 1-2 = 360, 1-3 = 300, 4-2 = 500 and 4-3 = 400. According to the methodology outlined in Section 3, in order to discourage the choice of impractical routes, all paths with more than one modal shift have been excluded. Hence the flows on each link resulting from the solution of the assignment problem are depicted in Fig. 3, while the paths used at the equilibrium and the flows of users associated to each path are reported in Table 3. Note that, among the used paths, one adopts only the rail mode of transport, two are highway routes, while the remaining four require the use of both modes of transport.

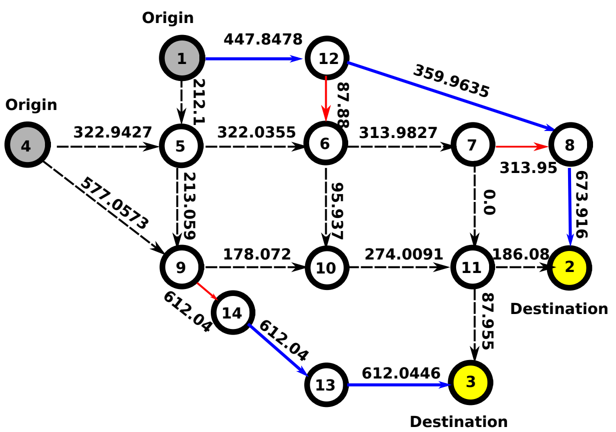

Fig. 3. Flows of users assigned to each link in the multi-modal transport network.

The outputs generated by the assignment procedure, i.e. the paths $\mathcal{L}^{od}$ and the flows $f^{od,l}$ for each *od* pair, are used to feed the module including the dynamic model.

Table 3. Paths used at the equilibrium and flows assigned to each path.

od **pair**	$\mathcal{L}^{od}$ **paths**
1-2	[1 12 8 2]
1-3	[1 12 6 10 11 3], [1 5 9 14 13 3]
4-2	[4 9 10 11 2], [4 5 6 10 11 2], [4 5 6 7 8 2]
4-3	[4 9 14 13 3]

od **pair**	$f^{od,l}$ **flows**
1-2	359.96
1-3	87.90, 212.10
4-2	177.76, 8.04, 313.96
4-3	399.27

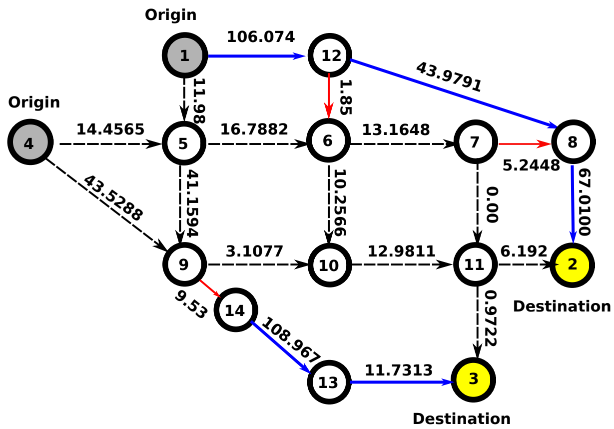

Fig. 4. Total travel time in each link of the multi-modal transport network.

Table 4. Total travel time for each path.

Paths	*TTT*
[1 12 8 2]	217.06
[1 12 6 10 11 3]	132.14
[1 5 9 14 13 3]	183.37
[4 9 10 11 2]	65.81
[4 5 6 10 11 2]	60.67
[4 5 6 7 8 2]	116.66
[4 9 14 13 3]	173.76

In particular, for the macroscopic multi-modal transport network model, a sample time T equal to one minute has been adopted, while the simulation horizon has been set equal to one hour (corresponding to $K = 60$ time steps). In order to evaluate the performance on the multi-modal transport network, the *Total Travel Time* referred to each link is computed as follows

$$TTT_{i,j} = T \sum_{k=1}^{K} n_{i,j}(k) \tag{19}$$

The *Total Travel Time* is an indicator that computes the total time spent by users on a connection, considering that an arc can belong to multiple paths at the same time. Having said that, Fig. 4 shows the values of the *Total Travel Time* for each arc, while Table 4 shows the same indicator referred to each of the adopted routes. From these results it can be easily observed that the highest travel times are found for longer arcs with higher user loads.

6. CONCLUSION

In this paper, a method for the analysis of large-scale multi-modal transport networks is presented. The proposed technique consists of two modules: an assignment module and a dynamic simulation module. In the first module, the problem concerning the assignment of user flows to a set of admissible paths, that may also include a modal shift, is solved. To this end, the N-Path restricted User-Equilibrium problem is applied by considering different performance functions associated to the different types of arcs of the multi-modal network. Furthermore, to ensure the uniqueness of the solution, the problem is reformulated in order to maximize the entropy function. The dynamic module, on the other hand, uses the outputs of the assignment procedure to replicate, at a macroscopic level, the dynamic evolution of the flows on the multi-modal network. Hence, the model allows to evaluate the effects produced by the redistribution of the traffic demand on different routes and using different means of transport. The combined adoption of these two modules can be a useful decision support tool, aimed at orienting strategic decisions after the occurrence of disruptive events (such as the interruption of railway links, the collapse of bridges, etc.) whose recovery requires a long time horizon and the redistribution of mobility demand by making the best use of the remaining capacity of a transport network.

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