

Smooth MT Transfer Function Estimation by An Inverse Scheme

Daniele Rizzello¹, Egidio Armadillo², Claudio Pasqua³, Riccardo Balsotti³, Paolo Pisani³, Biagio Giorgio³, Eliyasi Charleson⁴, Taramaeli Mnjokava⁵, Jonas Mwano⁵, Didas Makoye⁵, Lucas Tumbu⁵

¹ Tellus sas, Via Pozzetto, 2, 17146 Sassello, Italy

² DISTAV, Universita' di Genova, Corso Europa, 26, 16132 Genova, Italy

³ ELC-Electroconsult spa, Via Marostica, 1, 20146 Milano, Italy

⁴ Ministry of Natural Resources, Energy and Mining, Private Bag 350, Capitol Hill, Lilongwe 3, Lilongwe – Malawi, Malawi

⁵ Tanzania Geothermal Development Company (TGDC) P.O.box 14801, House No. 25 Ursino, Mwai Kibaki Road, Dar es Salaam-Tanzania.

rizzello@tellus-explora.eu

Keywords

Magnetotellurics, artificial EM noise, transfer functions, smoothness

ABSTRACT

Magnetotellurics (MT) is the main geophysical tool applied to geothermal exploration. The method exploits the measurement of the low-power natural EM field at the Earth's surface, and through the estimation of the MT transfer functions (TF), the subsurface resistivity distribution can be inferred.

Due to the diffusive nature of the low-frequency EM waves in the Earth, the MT TF's are inherently smooth, and this characteristic is the main criterion adopted by EM scientists to evaluate the quality of the TF and the presence of noise.

If noise is Gaussian, the frequency-domain the least squares (LSQ) estimation provides the best possible TF estimate; natural MT data, however, contains a significant amount of non-stationary data that constitute outliers. This fact makes the TF's sharp at several frequencies, according to the nature of the noise.

If non-stationarity occurs at a minor portion of the record, this problem is circumvented by robust estimation methods. However, it is expectable that geophysical investigation of geothermal resources in the next future will be carried out in densely populated and industrialised areas, where high-power artificial EM noise, typically at the **the power line frequency and its harmonics** is superimposed to the weak natural signal. In this case, preliminary analog filtering aimed at mitigate this issue can be ineffective due to the high power of the noise, and robust methods can

fail since the disturbance signals can be persistent. Moreover, **EM noise is often coherent, making coherence pre-sorting ineffective**. Therefore, the application of MT is severely hampered.

Current practice can foresee to artificially smooth the MT TF, by means of **splines or numerical smoothing procedures**, and smoothness can also be part of estimation methods. However, these approaches lack of physical foundation or imply the adoption of some arbitrary assumptions, **such as the representability of the TF by polynomials and the validity of the dispersion relationship between its real and imaginary parts**.

We propose a new heuristic and effective algorithm which goal is to get the smoothest MT TF's, via an inverse method applied to event selection. The frequency domain selection is done through variable power thresholds that reject powerful events making the TF sharp. The TF smoothness represents the objective function, and the infinite set of threshold vectors constitutes the model space.

After the completion of the process, the distribution of the event powers is closer to Gaussian, and the LSQ residuals are consequently closer to a Rayleigh distribution.

We found that the algorithm greatly reduces the effects of artificial strong-power signal on the MT acquisitions. Moreover, it can be combined with the MT remote-reference technique. Physical consistency of the TF has been checked by 1D inversion.

Successful application of the technique to MT data collected over a geothermal prospect in the East African Rift System is here presented, **where strong coherent noise at the 50 Hz fundamental and its upper harmonics strongly biased the MT impedance. The method was also successful at reducing bias in the low-frequency band**.

1. Introduction

Due to its inherent advantages, magnetotellurics (MT) has become the standard geophysical method for geothermal exploration since the '80s (Berketold, 1983). This technique exploits the low-amplitude variations of the natural magnetic field, produced by many natural sources. Its successful application for geothermal investigation arises from the low-cost, logistic effectiveness and high production rates, allowing to effectively depict the subsurface resistivity structure down to several kilometres depth.

However, as geothermal development proceeds on a global scale, a magnetotelluric investigation can involve densely populated and industrialised areas. In these zones, high-power artificial sources acting at a few specific frequencies, are more likely present. Since these signals can be persistent, robust methods can fail, and preliminary filtering can be ineffective. This hampers the applicability of MT in many areas or limits its reliability. The robust method can also fail since the high-power noise signal can constitute a high S/N portion of the data (see, e.g. Egbert, 1996, Weckmann, 2005).

A possible solution can be the manual selection of good data sections or frequency-domain events if the disturbance signal is easily detectable. In case this noise is persistent, this approach can be tricky or impossible, and therefore an automated scheme is preferable.

2. Outline of the algorithm

The rationale for the algorithm is that strong-power (respect to the natural EM signal) noise causes severe bias in the TF at particular frequencies, according to its origin. In industrialised areas, the main noise source comes from the power network, acting at a 50 Hz (or 60Hz) and some harmonics and sub-harmonics (Szarka,1988, Junge,1996). The pre-acquisition filters, embedded in

MT equipment, can be ineffective to eliminate this issue. Moreover, any filter will equally attenuate the natural signal to be analysed, causing an information loss.

The frequencies at which the TF is biased are immediately recognisable since it is dramatically sharpened. It is assumed that the smooth remaining portion of the TF is unbiased. The bias at these points can be removed by an inverse approach aiming at eliminating the outliers from the frequency domain events used for the impedance estimation. The roughness of the resulting TF is then reduced. The smoothness of the MT TF is often used as a quality indication (Jones, 1989), and is also artificially introduced by a-posteriori smoothing (e.g. Wannamaker, 1989), but this approach lacks physical consistency. It can also be part of estimation methods but implying the adoption of some arbitrary assumptions (e.g. Larsen, 1996).

The MT impedance single-station estimation by LSQ is given by (e.g. Chave and Thompson, 1989)

$$\mathbf{Z} = (\mathbf{E} \mathbf{B}^h)(\mathbf{E} \mathbf{B}^h)^{-1} \quad (1)$$

where, \mathbf{E} and \mathbf{B} are frequency-dependent $n \times 2$ (two-components) field matrices, where n is the number of observations; h denotes the Hermitian conjugate.

The assumptions to be made when applying (1) is that the errors are only affecting the E-field, their distribution is nearly-Gaussian and that they are independent. These conditions can be approached if a proper event selection is made before the LSQ estimate. To reduce the influence of the noise in the B channel, the Remote Reference technique is applied (Gamble et al., 1979); in the real case discussed hereafter, we only deal with RR estimate, since we noticed from our experience that in the vast majority of the cases some noise affects the magnetic field acquisitions.

However, in our approach the event selection is performed by applying power thresholds to the local EM field, implying that the computed \mathbf{Z} impedance is function of the applied thresholds. This approach is for example used in GDS (Geomagnetic Depth Sounding) to prevent the influence of strong auroral electrojet field in Antarctic regions (Armadillo, 2001) and is somewhat analogous to the method by Jones (2002) for northern auroral zones.

The implemented algorithm seeks for minimisation of the following functional

$$U(\mathbf{t}) = \|\delta_R \mathbf{R}(\mathbf{t})\| + \mu \|\delta_\phi \Phi(\mathbf{t})\| \quad (2)$$

where, $\mathbf{R}(\mathbf{t})$ is the $p \times 2$ - matrix of the apparent resistivity, $\Phi(\mathbf{t})$ is the $p \times 2$ - vector of the phase, and δ_R and δ_ϕ are the $p \times p$ difference matrices to be applied to the apparent resistivity and phase matrices used to compute their differences; p is the length of the estimation frequency vector. The factor μ is a trade-off between resistivity and phase smoothing. Variable \mathbf{t} is a threshold vector. We only consider xy and yx - mode; therefore, the first column of matrix $\mathbf{R}(\mathbf{t})$ or $\Phi(\mathbf{t})$ is referred to the xy-polarization, while the second one to the yx. Note that the minimisation of (2) aims at producing a smooth MT transfer function.

The frequency-dependent thresholds are automatically applied to the power of the electric field events until the maximum TF smoothness is reached. The set of all these thresholds form a p -vector \mathbf{t}' that, from the classical inversion theory standpoint, constitutes the sought model. In other words, the model space in our algorithm is constituted by the infinite set of frequency-dependent power threshold vectors. The optimal model will, therefore, correspond to the threshold vector that will allow to exclude or minimise the influence of the outliers from the final LSQ estimate. We found that the optimisation is much more effectively carried out employing a derivative-free algorithm. We therefore used a direct-search algorithm implemented in MATLAB[®]. The initial model of the inversion is usually chosen to compute the median or mean value of the electric field

power of the event sets at the various estimation frequencies; this is computed separately for E_x and E_y .

3. Application to data from geothermal exploration

We applied our algorithm to a MT dataset from a geophysical survey carried out over the Kasitu geothermal field, Malawi (Fig.1). The data was acquired in the period June - July 2017, using a Remote Reference site located about 20 Km south of the survey area. The area is interested by the presence of a 50 Hz power line located along the main road; from our data analysis, we found that the stations lying along it are affected by noise up to a distance of about 700 m.

With the purpose to test the capabilities of our algorithm under controlled conditions, we first computed the synthetic E-field from measured B-data, through a synthetic MT one-dimensional impedance Z_{1D}

$$E = Z_{1D} * B$$

As well known (e.g. Simpson and Bahr, 2005) the impedance tensor in 1D environment is $Z_{1D} = [0 \ Z_{xy}; -Z_{xy} \ 0]$. Z_{xy} has been retrieved by computation of the MT response of a model consisting of two layers of 1000 and 10 Ohm m resistivity (1000 and 200 m thick), overlying a half space of 100 Ohm. The real (not synthetic) **B**-data have been collected in an EM quiet site, located at about 3.5 km from the power line (station MT51, Fig.1). We therefore applied our algorithm to the synthetic data set after having contaminated it by uniform noise, which standard deviation was set to 500 times the average signal. The noise was added to the E_x and E_y components, at the estimation frequencies of 338, 117, 57, 14 and 6.9 Hz. These frequencies are close to the 7th, 3rd harmonics, the fundamental f_0 , and $f_0/4$ and $f_0/8$ subharmonics of a 50 Hz power line signal, and have been established according to Junge (1996).

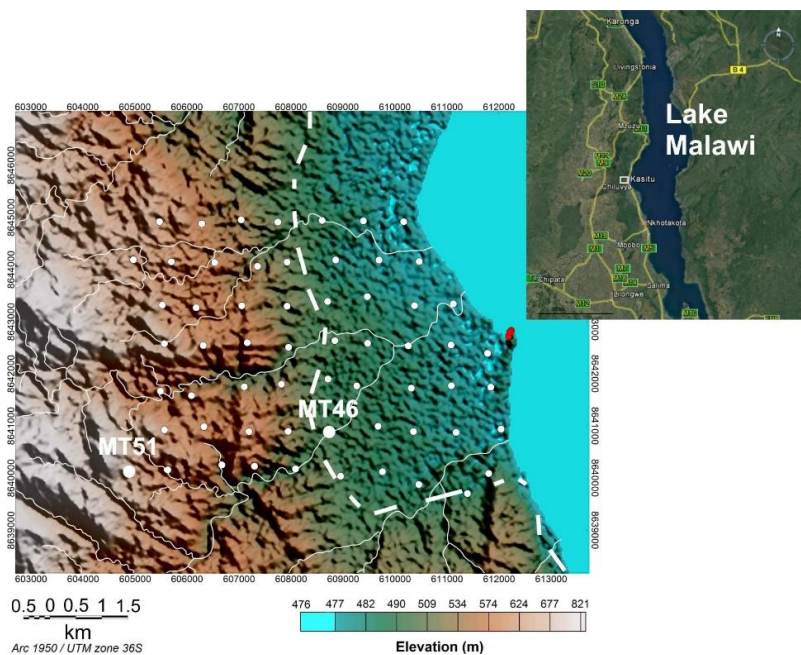


Figure 1: The Kasitu geophysical survey area. White dots indicate the MT stations. The dashed white line marks the main road, along which the power lines is located. Red dots mark the Kasitu hot springs.

The standard deviation of the noise signal was set to 500 times the average signal. In Fig.2 are shown the Ex events at 57 Hz, after noise addition. It must be remarked that 80% of the events have been affected by artificial noise, simulating a persistent disturbance.

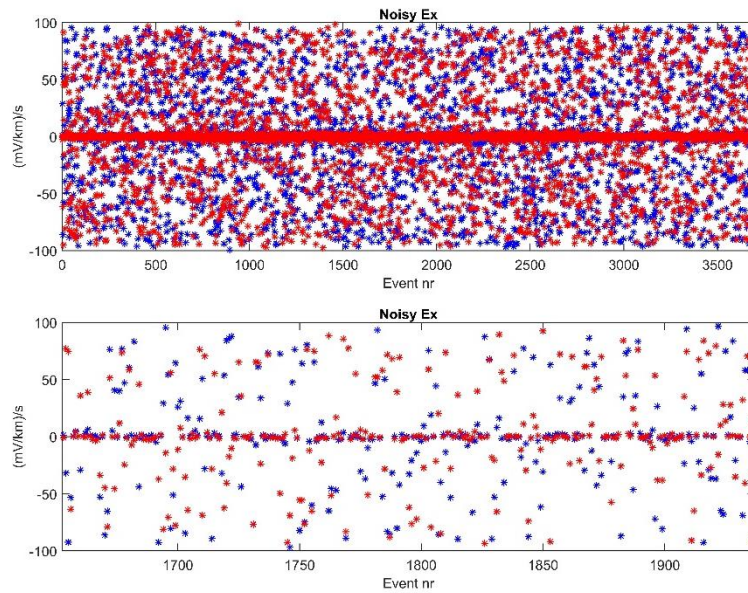


Figure 2. Example of data (Ex field) contaminated by synthetic gaussian noise.

Our inverse TF estimation was set to search for apparent resistivity and phase smoothing, using equal weight ($\mu=1$). The inversion started from the average of the E - field and was stopped after 68 iterations; the frequency range was limited between 338 Hz and 0.14 Hz. The results are shown in Fig.3, where they are compared with a robust estimate (according to the Chave and Thompson, 1989, algorithm); it can be observed that this latter fails to find an acceptable result, due to the fact that most of the data are contaminated by noise.

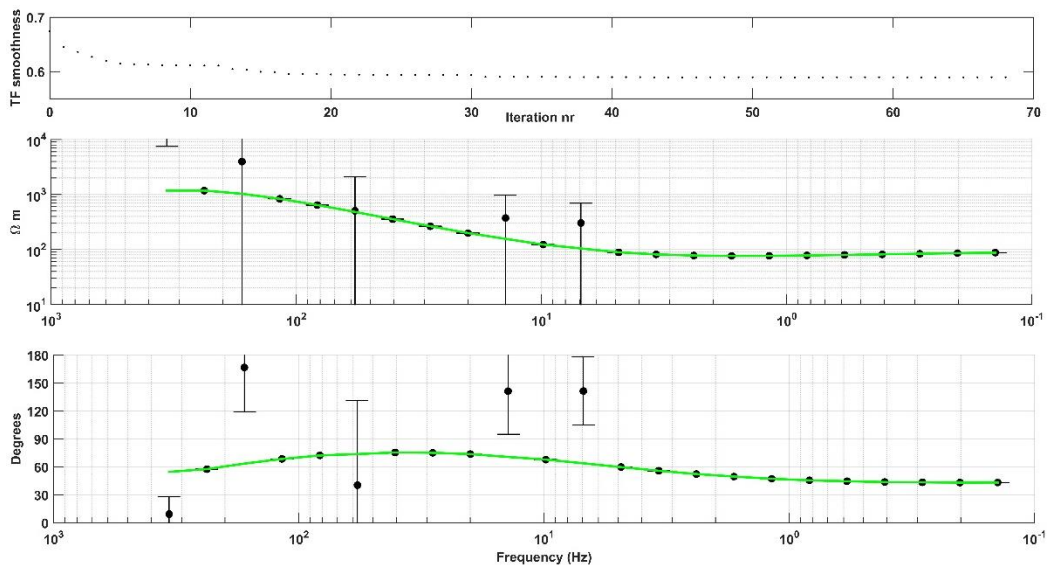


Figure 3. Smooth estimation of real data contaminated by synthetic noise. The smooth estimate is marked by green dashed lines, the robust one by black dots. Top: iteration progress.

Hereafter, we process the data from station MT46, lying at about 250 m from the power line (Fig.1). The used MT equipment recorded the EM data using three different sampling rates: 2400, 150 and 15 Hz. The noise is well evident in the 2400 Hz - sampling record, and much less in the 150 Hz; it is not visible in 15 Hz - sampling. Noise is ubiquitous in all channels except Hx. An example of the 2400 Hz - band data is shown in Figure 4.

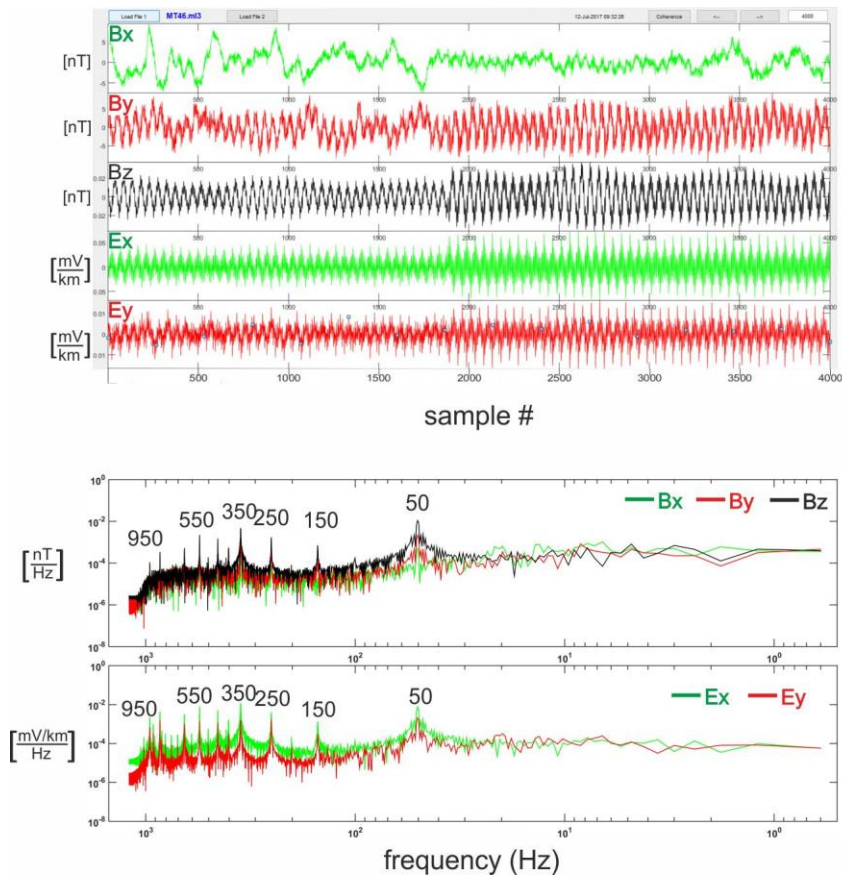


Figure 4. The EM signal at station MT46 (2400 Hz band). Top, time series, bottom power spectra.

Data shows strong and modulated 50 Hz coherent noise. The inspection of other data windows shows that the noise is nearly continuous across the whole record length. Spectral analysis reveals the 50 Hz fundamental harmonic and the odd upper harmonics up to 950 Hz. The hodogram analysis (Fig.5) shows an E-polarization coherent with the orientation of the power line (about NNW). The H-field hodograms indicate the quasi-verticality of the field. The strong polarisation shown by these diagrams indicates a predominant linear EM source.

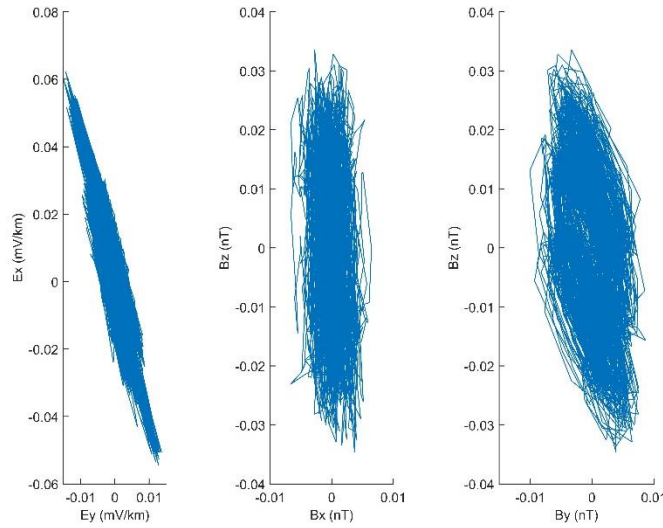


Figure 5. Hodograms of the EM signal at station MT46 (2400 Hz-band).

In Fig. 6, we show the results of our smooth inverse estimation, compared with the robust estimate. We adopted phase and apparent resistivity smoothing (with phase weight = 1) and Remote Reference (RR) estimate. The RR site was chosen at about 20 km south of the MT46 station. The initial model for the inversion was computed from the average E-field power of the event dataset. The inversion was stopped after 170 iterations.

As can be seen, the smooth estimation eliminates the noisy (sharp) points in the upper band (> 10 Hz), both in the xy/yx apparent resistivity and phase data. Again, it can be noticed that the robust method failed to find an acceptable result since the power line noise is persistent. In fact, from the robust curves, it can be seen noise in the Z at the estimation frequencies of 57, 237 and 338 Hz; these frequencies are the closest to the fundamental, fifth and seventh harmonics of the power line signal. The robust curves appear also biased in the lower band (< 10 Hz); the origin of the noise in this case is not clear, but it can be observed that the inverse estimation provides a much smoother MT impedance.

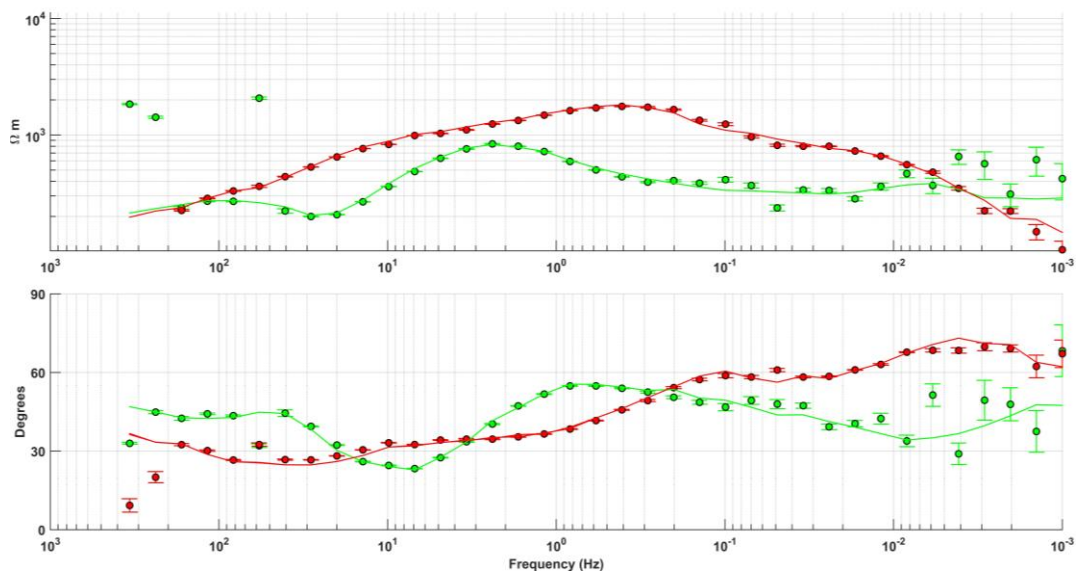


Figure 6. Results of the smooth transfer function MT estimation (continuous lines) for station MT46. The robust estimate is shown by dots.

From the analysis of the QQ (quantile-quantile) plots of the power of the Ex-events (Fig.7), it can be observed that after the selection process is completed, the distribution of the event powers are more gaussian, and then, more suitable to LSQ estimation (Fig.7, top); the corresponding residuals are consequently closer to a Rayleigh distribution (Fig.7, bottom).

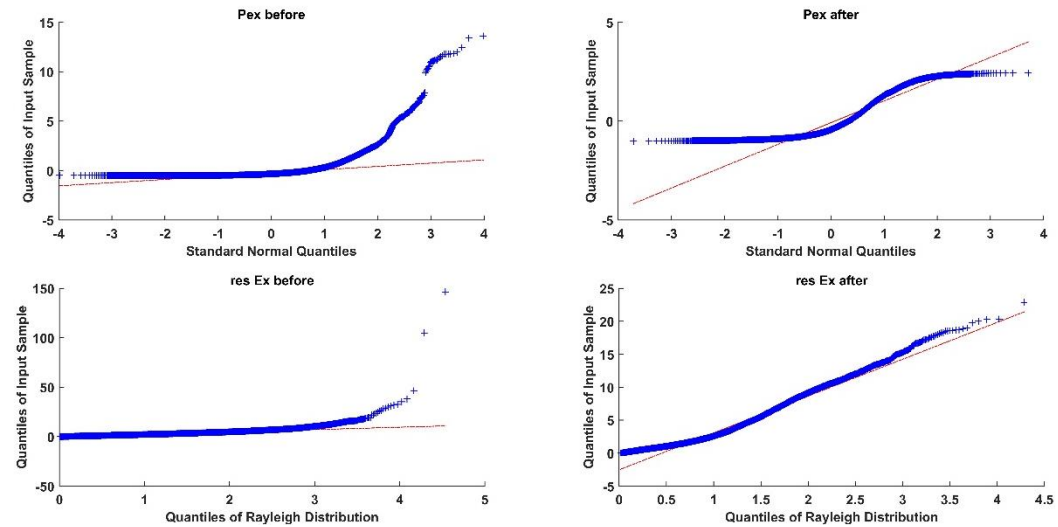


Figure 7. Ex power and residual QQ plots at 57 Hz: left column before smooth algorithm, right column after it. Top Ex power, bottom Ex residuals. Data are more gaussian (or Rayleigh) distributed after event selection.

4. 1D modelling

With the aim to test the physical consistency of our smoothly estimated MT TF, we performed the 1D inversion of the effective impedance. Static shift factor to be applied to the MT apparent resistivity was estimated by joint inversion of effective MT phase and TDEM data and resulted to be 0.6. The results of the modelling are shown in Fig. 8. Considering the noise level, the inverted model explains reasonably well the data.

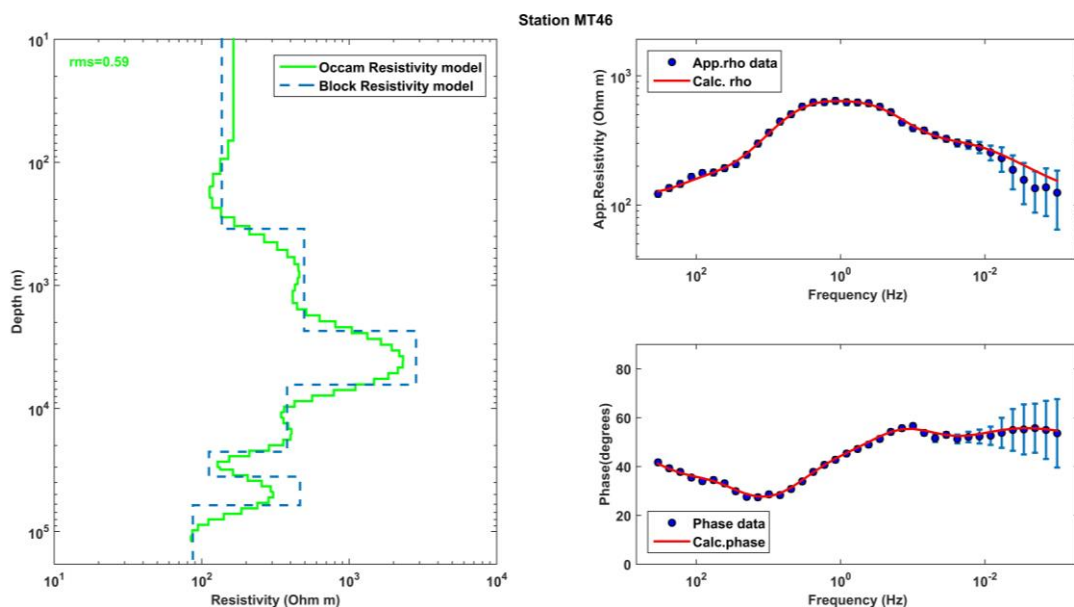


Figure 8. Occam 1D inversion of the smooth effective impedance for station MT46. The green line marks the Occam model, the blue dashed line the blocky one.

5. Conclusions

We have presented the capabilities of a new MT impedance estimation algorithm, conceived to obtain the smoothest possible TF through an inverse scheme. It has been successfully tested with synthetic noise applied to quiet MT station data, where 80% of the data was contaminated.

The algorithm demonstrated to be effective in eliminating ubiquitous and continuous power line noise at the base frequency (50 Hz), and its odd harmonics, at one geothermal survey site located in Malawi. It also effectively reduced the bias of the MT impedance at low frequencies. The physical consistency of the obtained MT transfer function has been successfully tested by 1D inverse modelling.

Although we only applied our method to noisy data coming from an electric power line, we deem that it could be successfully applied to those cases where strong and persistent artificial noise sources are present in a survey area. This could aid the MT exploration even in areas interdict so far, and, therefore, it could be considered as a promising tool for the further development of geothermal energy.

REFERENCES

- Armadillo, E., Bozzo E. and Caneva G.: Deep Electrical Resistivity Investigation across the Rennick Graben and Oates Land by Geomagnetic Depth Sounding, *Terra Antarctica*, **10(3)**, (2003), 171-178.
- Berkthold A.: Electromagnetic Studies In Geothermal Regions, *Geophysical Surveys*, **6** (1983), 173-200.
- Chave A., and Thomson D.: Some Comments on Magnetotelluric Response Function Estimation, *Journal of Geophysical Research*, **vol. 94, No. B10**, (1989), 14,215-14,225.
- Egbert G.D. and Livelybrooks D.W.: Short note - Single station magnetotelluric impedance estimation: Coherence weighting and the regression M-estimate, *Geophysics*, **Vol. 61, No. 4** (1996), 964-970.
- Gamble, T.D., Goubau, W.M. and Clarke, J.: Magnetotellurics with a remote magnetic reference, *Geophysics*, **Vol. 44, No.1** (1979), 53-68.
- Jones, A.G and Spratt, J.: A simple method for deriving the uniform field MT responses in auroral zones, *Earth Planets Space*, **54** (2002), 443–450.
- Junge A.: Characterization of and correction for cultural noise, *Surveys in Geophysics*, **17** (1996), 361- 391.
- Larsen, J., Mackie, R., Manzella, A., Fiordelisi, A. and Rieven, S.: Robust smooth magnetotelluric transfer functions, *Geophys. J. Int.*, **124** (1996), 801–819.
- Szarka, L.: Geophysical Aspects of Man-made Electromagnetic Noise in the Earth - A Review, *Surv. in Geophys.* **9** (1988), 287-318.
- Wannamaker P.E., Booker J.R., Filloux J.H., Jones A.G., Jiracek G.R, Chave A.D., Tarits P., Waff H. S., Egbert G.D., Young C.T., Stodt J.A., Martinez G., L.K. Law, Yukufake T., Segawa J.S., White A. and Green A.W. Jr.: Magnetotelluric Observations Across the Juan de Fuca Subduction System in the EMSLAB Project, *Journal of Geophysical Research*, **vol. 94, No. B10**, (1989), 14,111-14,125.
- Weckmann, U., Magunia, A. and Ritter, O.: Effective noise separation for magnetotelluric single site data processing using a frequency domain selection scheme, *Geophys. J. Int.*, **161** (2005), 635-652.