Moving Horizon Estimation: Open Problems, Theoretical Progress, and New Application Perspectives

Angelo Alessandri, Giorgio Battistelli

Abstract

The special issue presents results of the current research on moving horizon estimation for linear and nonlinear systems. The special issue includes six papers with two main types of contributions, i.e., those with new theoretical developments and those focused on practical applications such as fault diagnosis, outlier accommodation, and field monitoring. Notably, these two types of contribution are somehow mixed since the combination of theory and application is a fundamental inspiration mechanism, whenever systematic and rigorous algorithm devising enables outstanding experimental performance and, conversely, challenging experimental problems inspire conceptual insights in theoretical understanding.

I. Introduction

Moving horizon estimation (MHE) was originally proposed to deal with uncertainties in the system knowledge. The growing interest in these moving horizon techniques is motivated by some interesting features such as capability of taking explicitly into account constraints on state and disturbances in the filter design, ability of defining a performance criterion that can be designed specifically for the problem under consideration, and stability guarantees. On the other hand, similarly to model predictive control, the computational effort has always been the most critical issue in MHE. In fact, since MHE in its original formulation requires the on-line solution of optimization problems, the impact of MHE in real-time industrial applications has been mainly limited to processes characterized by slow dynamics such as chemical plants. These considerations have motivated the present special issue, which aims at showing recent advances in theory and application of MHE.

Concerning the reduction of the computational issues, possible solutions reported in the literature consist in performing the computation off line as much as possible and/or constructing the estimators that provide the on-line estimates with a reduced real-time effort. Specifically, this last goal is achieved in [1] by means of imperfect optimization with few (or even only one) descent steps based on gradient, conjugate gradient, and Newton algorithms. Linear constraints are explicitly taken into account by projecting the resulting estimates, while always ensuring stability.

A possible strategy to guarantee the feasibility of on-line optimization is to select the performance criterion so that it allows for a simple and efficient numerical implementation. This solution is pursued in [2] where the performance criterion includes convex stage costs as well as a proximity measure, expressed in terms of a Bregman distance between the MHE estimate and a stabilizing a priori estimate. Notably, this formulation enables to incorporate the constraints into the cost function by choosing the so-called relaxed barrier functions as Bregman distances. A proximal MHE scheme based on Bregman distances is designed for a class of nonlinear discrete-time systems that can be transformed into systems that are affine in the unmeasured state, and the stability of the estimation error is investigated. The robustness properties of the proposed MHE approach in the presence of process and measurement disturbances are ensured in terms of an input-to-state stability analysis.

Another strategy to improve efficiency is using distributed methods in such way as to allocate the optimization workload on different computational units, as shown in [3], [4] with applications to fault diagnosis. The approach proposed in [3] is based on the decomposition of the original MHE problem into subproblems, each requiring a limited computational complexity in order to mitigate the effect of the dimensionality, while ensuring scalability. To this end, the overall system is regarded as a network of interacting subsystems in which the faults enter as additive signals on the state or output equations. By exploiting the separability of the performance criterion as well as the system decomposition, an iterative and distributed fault detection algorithm is proposed. Convergence properties of the iterative scheme and stability of the estimation error dynamics are investigated, and an analysis of the algorithm characteristics in case of persistent faults is provided.

Following the distributed paradigm, an application to the distributed monitoring of an amine-based post-combustion plant for CO_2 capture is presented in [4], where the process is decomposed into subsystems by using a community detection scheme. The large-scale interaction network associated with the system dynamics is partitioned into smaller communities so that connections within each community can be strong, while links connecting different communities are relatively sparse. A distributed iterative MHE scheme is proposed in which the number of iterations steps at each sampling time is adapted in real-time according to the innovations for the subsystems. Based on the distributed MHE and computation of suitable residuals, a fault diagnosis mechanism is proposed to detect and identify potential sensor faults.

The availability of low-cost sensors in many today's important applications such as global navigation satellite systems (GNSSs, including GPS, GLONASS, Galileo, and Beidou) has posed the problems of dealing with a large number of measurements that may be affected by sporadic perturbations, usually referred to as outliers. The contribution of [5] regards just the application of MHE to estimation in the presence of outliers. In particular, the on-line optimization aims at selecting a subset of measurements in a sliding window of given length in order to achieve a desirable level of performance, while minimizing the incurred risk.

Among low-cost sensors, threshold sensors are widely used for monitoring and control thanks to the practical advantages in terms of ease of sensor deployment and minimization of communication requirements. On the other hand, these sensors provide the least-possible amount of information, i.e., a binary output indicating whether the sensed variable is below or above a given threshold, and hence pose a challenge in the design of efficient estimators. In [6], a maximum a posteriori probability (MAP) MHE approach is proposed and analyzed for state estimation with threshold sensors. Then, the proposed method is applied to the problem of reconstructing a two-dimensional dynamic field sampled with a network of threshold sensors. In order to deal with the large-scale nature of the system, a suitable solution is proposed with a substantial decrease of the computational complexity.

Let us conclude by discussing some open problems in MHE. The results on the input-to-state stability or exponential boundedness of the estimation error w.r.t. external disturbances for uncertain linear and nonlinear systems usually require assumptions on the boundedness of the state trajectories and/or Lipschitz nonlinearity. The relaxation of such assumptions can the subject of future investigations. Another relevant research topic is the development of distributed MHE algorithms enjoying stability properties and performance as close as possible to their centralized counterparts, in particular in a nonlinear setting. Finally, in view of the emergence of computationally-efficient MHE schemes able to provide in real-time estimates with guaranteed accuracy, the use of such MHE schemes within the context of feedback control systems certainly deserves further research.

REFERENCES

[1] A. Alessandri and M. Gaggero, "Fast moving horizon state estimation for discrete-time systems with linear constraints," *International Journal of Adaptive Control and Signal Processing*, vol. n/a, no. n/a. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.3054

[2] M. Gharbi and C. Ebenbauer, "A proximity moving horizon estimator for a class of nonlinear systems," *International Journal of Adaptive Control and Signal Processing*, vol. n/a, no. n/a. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.3092

[3] M. Lauricella, M. Farina, R. Schneider, and R. Scattolini, "Iterative distributed fault detection and isolation for linear systems based on moving horizon estimation," *International Journal of Adaptive Control and Signal Processing*, vol. n/a, no. n/a. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.3063

[4] X. Yin, B. Decardi-Nelson, and J. Liu, "Distributed monitoring of the absorption column of a post-combustion CO_2 capture plant," *International Journal of Adaptive Control and Signal Processing*, vol. n/a, no. n/a. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.3074

[5] E. Aghapour and J. A. Farrell, "Outlier accommodation in moving-horizon state estimation: A risk-averse performance-specified approach," *International Journal of Adaptive Control and Signal Processing*, vol. n/a, no. n/a. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.3055

[6] G. Battistelli, L. Chisci, N. Forti, and S. Gherardini, "MAP moving horizon estimation for threshold measurements with application to field monitoring," *International Journal of Adaptive Control and Signal Processing*, vol. n/a, no. n/a. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/acs.3049