Passive multi-mode piezoelectric shunt damping: an approach based on matrix inequalities

M. Berardengo[1], S. Manzoni[2], M. Vanali[1], A.M. Conti[2]

[1] Università degli Studi di Parma - Department of Engineering and Architecture
Parco Area delle Scienze, 181/A – 43124 Parma (Italy)
e-mail: **marta.berardengo@unipr.it**

[2] Politecnico di Milano - Department of Mechanical Engineering
Via La Masa, 34 – 20156 Milan (Italy)

Abstract

Piezoelectric shunt damping is a well-known technique for suppressing vibrations in light mechanical systems. The method is based on the connection of a properly designed electrical network (shunt circuit) to a piezoelectric actuator bonded to the vibrating structure. This network can be either passive (i.e. made from resistances, capacitances and inductances) or active. When active shunts are used, possible problems related to instability of the system can raise. This paper addresses a new approach for designing shunt electrical circuits allowing to damp more than one mechanical mode of the structure at the same time with a single piezoelectric actuator. Moreover, the method assures to design passive shunt impedances, thus avoiding instability problems. Starting from a state space description of the electro-mechanical system, the definition of the shunt circuit is achieved using an approach based on matrix inequalities, which allows to design shunt circuits with different goals by expressing the desired target as a single or a system of matrix inequalities.

1 Introduction

The use of piezoelectric shunt is a renewed approach for damping vibrations [1]. Many approaches are possible: from the use of passive shunt networks (e.g. [2–6]), to the use of negative capacitances for enhancing the damping performance (e.g. [7–13]), from the employment of non-linear impedances (e.g. [14–16]), to the use of networks of piezoelectric actuators (e.g. [17–22]).

Here, the discussion is focused on shunt methods which allow to use even a single piezoelectric actuator for controlling more than one mode at the same time and ensure the passivity of the electro-mechanical system (EMS, composed by the vibrating structure, the piezoelectric actuator bonded to the structure and the shunt electric impedance). Indeed, such an approach, based on the EMS passivity, plays an important role, especially in the industrial and aerospace fields, thanks to its passivity, lack of instability, and lack of additional devices.

When the aim of the control system is to act on several modes, using a single piezoelectric actuator, there are different methods to design proper passive shunt electrical networks.

Hollkamp firstly proposed an impedance design technique employing a single piezoelectric patch working on different modes at the same time [23]. The proposed network design is made from as many branches connected in parallel as the number of modes to be damped. The main problems related to this method are the cross-talk between the branches of the circuit, which requires perfectly decoupled modes, and the complexity of the procedure to fix the values of all the electric components.

Wu [24] and Behrens et al. [25] introduced the current blocking (CB) and current flowing (CF) methods, respectively. Again, the shunt impedance is made from as many branches as the number of modes to control, and all the branches are connected in parallel.

Although the CB was proposed to simplify somewhat the design of the shunt impedance, the complexity of the circuit is evident, especially when the number of modes to be damped increases. Another problem with the CB method is that there are some degrees of freedom in the tuning of the network: the values of some electric components must be fixed arbitrarily without any guidelines, which often results in non-optimal damping actions.

As for the CF technique, each branch is made from three elements: a resistance, a capacitance, and an inductance and the values of the resistances in the shunt impedance must be fixed by numerical minimization. Furthermore, there are some degrees of freedom that lead to a non-optimal solution in this case as well, although some guidelines to overcome this problem were proposed [26]. There are also cross-talk effects that are not accounted for in the tuning procedure and can be solved just by using numerical minimizations.

Fleming et al. [27] proposed a method that can be seen as a mix of CB and CF. The shunt network is a combination of cells connected in series and parallel but, like in other approaches, the main drawback of the method is that there are some degrees of freedom in the tuning of the network wich have to be arbitrarily fixed.

Despite all the described tuning strategies provide good damping performances, the values of some electric components must be fixed arbitrarily without any guidelines; as mentioned, this often results in non-optimal damping actions. Moreover, these approaches often do not allow for a specific control target to be set. Therefore, the aim of this paper is to present a new general approach to multi-mode vibration reduction which can be applied to any generic structure. Furthermore, the proposed approach is aimed at finding the optimal controller for given control applications, and it relies on the matrix inequality (MI) theory. The MI approach allows for a control target to be set (e.g. H_2 or H_∞ on acceleration, velocity, or displacement) and for constraints to be imposed on the control features. The particular focus is on passive control. Therefore, the method allows an expression of the shunt impedance to be found that satisfies the fixed control target, assuring that it can be realized by a passive physical network.

The paper structure is as follows: Section 2 presents the model used to describe the EMS and its state space expression. Such a way to express the system behavior is needed for using the matrix inequality approach, as explained in Section 3. Finally, Section 4 presents some experimental tests to show the reliability and effectiveness of the proposed approach.

2 Model of the system

The model of the EMS describes both the mechanical and the electrical dynamics. A generic structure excited by an external forcing $\mathbf{F}_{\text{ext}}$ is taken into account; the piezoelectric actuator used for damping is shunted by an impedance Z (see Figure 1a). Q_{p} is the charge in the upper electrode ($-Q_{\text{p}}$ in the lower electrode), and V_{p} is the voltage between the electrodes. The displacement W of any point x of the structure at time t can be expressed as a modal summation:

$$W(x,t) = \sum_{r=1}^{N} \Phi_r(x) q_r(t) \tag{1}$$

where q_r is the r^{th} modal coordinate, N is the number of modes considered (theoretically $N \to \infty$), and Φ_r is the rth eigenmode, scaled to the unit modal mass, of the structure. The modal coordinates are the solutions of the following problem [10,28–30]:

$$\ddot{q}_r + 2\xi_r\omega_r\dot{q}_r + \omega_r^2 q_r - \chi_r V_{\mathrm{p}} = F_r \qquad \forall\, r \epsilon\, \{1, \dots, N\} \tag{2}$$

$$C_\infty V_{\mathrm{p}} - Q_p + \sum_{l=1}^{N} \chi_l q_l + \frac{\int V_{\mathrm{p}}}{R_{\mathrm{p}}} = 0 \tag{3}$$

where ω_r is the r^{th} eigenfrequency of the EMS in short-circuit (SC, i.e. $Z = 0$), ξ_r is the associated non-dimensional damping ratio, and F_r is the harmonic modal force. χ_r is a modal coupling coefficient that describes the energy transfer between the piezoelectric patch and the r^{th} mode. Therefore, the behaviour of the EMS is described by two equations: Equation (2) and Equation (3). Equation (2) describes the equations of motion of the system. The term χ_r couples these equations of motion to Equation (3), which models the electric behaviour of the EMS (see Figure 1b, where the term $\sum_{l=1}^{N} \chi_l \dot{q}_l$ is abbreviated as $\dot{Q}_{\mathrm{cs}}$). R_{p} is the resistance associated with the piezoelectric patch, which is usually very high [31]. $\int V_{\mathrm{p}}$ is intended as an integral in time (i.e. $\int V_{\mathrm{p}} \mathrm{d}t$). C_∞ is the electrical capacitance of the piezoelectric patch with blocked structure, which also corresponds to the value of the capacitance at infinite frequency [10].

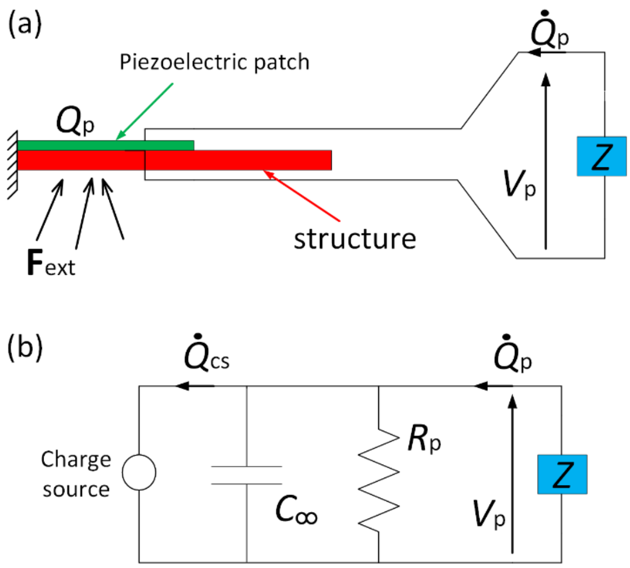

Figure 1: A generic structure with a shunted piezoelectric patch (a) and electric model of the EMS (b).

The terms χ_r can be found analytically [30], through a finite element model [29], or experimentally by measuring the effective coupling coefficients associated with each mode [32]. In case of low modal density, if only the modes between the u^{th} (i.e. $r = u$) and the h^{th} modes (i.e. $r = h$, with $h > u$) are considered, Equations (2) and (3) can be rearranged as:

$$\ddot{q}_r + 2\xi_r\omega_r\dot{q}_r + \omega_r^2 q_r - \chi_r V_{\mathrm{p}} = F_r \qquad \forall\, r \epsilon\, \{u, \dots, h\} \tag{4}$$

$$C_\infty V_{\mathrm{p}} - Q_{\mathrm{p}} + \sum_{l=u}^{h} \chi_l q_l + \frac{\int V_{\mathrm{p}}}{R_{\mathrm{p}}} + M V_{\mathrm{p}} = 0 \tag{5}$$

where M is a term that accounts for the contribution of the modes higher than the h^{th} mode. According to[10,11]:

$$M = \sum_{n=h+1}^{N} \frac{\chi_n^2}{\omega_n^2} \tag{6}$$

Therefore, Equation (5) can be written as:

$$C_\mathrm{p} V_\mathrm{p} - Q_\mathrm{p} + \sum_{l=u}^{h} \chi_l q_l + \frac{\int V_\mathrm{p}}{R_\mathrm{p}} = 0 \tag{7}$$

with:

$$C_\mathrm{p} = C_\infty + \sum_{n=h+1}^{N} \frac{\chi_n^2}{\omega_n^2} \tag{8}$$

C_p can be found by measuring the value of the capacitance of the piezoelectric actuator midway between ω_h and ω_{h+1} [32]. Thus, the dynamics of the EMS in the frequency range of interest is described by Equations (4) and (7). To use the matrix inequality approach to solve a given control problem, the EMS model needs to be represented in terms of state space variables. The state space representation provided in this paper plays a key role since it also allows the shunt impedance to be seen as a controller. Indeed, expressing Z as a controller allows the passivity requirement to be imposed directly on the impedance Z in the matrix inequality problem, which is one of the targets of the proposed control approach.

According to Equations (4) and (7), the following system of equations can be written:

$$\begin{cases} \ddot{q}_r + 2\xi_r \omega_r \dot{q}_r + \omega_r^2 q_r - \chi_r V_\mathrm{p} = F_r & \forall\, r \epsilon \{u, \dots, h\} \\ \dot{q}_r = \dot{q}_r & \forall\, r \epsilon \{u, \dots, h\} \\ C_\mathrm{p} V_\mathrm{p} - Q_\mathrm{p} + \sum_{l=u}^{h} \chi_l q_l + \frac{\int V_\mathrm{p}}{R_\mathrm{p}} = 0 & \end{cases} \tag{9}$$

By defining:

$$\bar{V} = V_\mathrm{p}\sqrt{C_\mathrm{p}} \quad \text{and} \quad \bar{Q} = \frac{Q_\mathrm{p}}{\sqrt{C_\mathrm{p}}} \tag{10}$$

and defining the vector **g** of the state variables:

$$\mathbf{g} = \begin{bmatrix} \dot{q}_u \\ q_u \\ \vdots \\ \dot{q}_h \\ q_h \\ \int \bar{V} \end{bmatrix} \tag{11}$$

the system described by Equation (9) can be written in state space notation:

$$\begin{cases} \dot{\mathbf{g}} = \mathbf{A}\mathbf{g} + \mathbf{B}_{\mathrm{w}}\bar{Q} + \mathbf{B}_{\mathrm{f}}\mathbf{F}_{\mathrm{ext}} \\ z_{\mathrm{o}} = \mathbf{C}_{\mathrm{z}}\mathbf{g} + \mathbf{D}_{\mathrm{zw}}\bar{Q} + \mathbf{D}_{\mathrm{zf}}\mathbf{F}_{\mathrm{ext}} \\ y = \mathbf{C}_{\mathrm{y}}\mathbf{g} + \mathbf{D}_{\mathrm{yw}}\bar{Q} + \mathbf{D}_{\mathrm{yf}}\mathbf{F}_{\mathrm{ext}} \end{cases} \tag{12}$$

where z_{o} is the target variable of the control (e.g. the displacement W, velocity $\dot{W}$, and acceleration $\ddot{W}$ of the system computed at a given point x_{m} of the structure); y is the output of the system, which is $-\int \bar{V}$ in this case ($\int \bar{V}$ is intended as an integral in time $\int \bar{V}\mathrm{d}t$). Finally, $\bar{Q}$ is seen as a control action provided to the vibrating system. More details on the way to express the matrices of the state space expression can be found in [33].

The state space model describes the EMS as a controlled system via a feedback loop. The transfer function of the controller K can be expressed as:

$$K = \frac{\bar{Q}}{-\int \bar{V}} = \frac{\dot{Q}_{\mathrm{p}}}{-C_{\mathrm{p}}V_{\mathrm{p}}} = \frac{1}{C_{\mathrm{p}}Z} = \frac{Y}{C_{\mathrm{p}}} \tag{13}$$

where the admittance Y is $1/Z$. Thus, the model presented can describe the EMS as a system controlled by a feedback loop, and the controller is the electric admittance shunted to the piezoelectric actuator (divided by C_{p}). This control problem is an output feedback problem. It is now possible to set the control targets and the constraints on the controller structure. The next section discusses how to translate these requirements to matrix inequality problems and how to find the most suitable transfer function of Z.

3 The matrix inequality approach

In this section, the H_∞ control (chosen as an example) as well as the passivity of the controller are formulated as linear matrix inequality (LMI) problems. Control specifications for the closed-loop transfer function T_{zF} linking the disturbance F_{ext} (considering a single force F_{ext} acting as disturbance; however, this assumption does not cause any loss of generality) and the target variable of the control z_{o} are taken into account. Referring to Equation (12), T_{zF} can be expressed in the Laplace domain as:

$$T_{\mathrm{zF}} = \mathbf{C}_{\mathrm{z}}(s\mathbf{I} - [\mathbf{A} + \mathbf{B}_{\mathrm{w}}K\mathbf{C}_{\mathrm{y}}])^{-1}\mathbf{B}_{\mathrm{f}} + \mathbf{D}_{\mathrm{zw}}K\mathbf{C}_{\mathrm{y}}(s\mathbf{I} - [\mathbf{A} + \mathbf{B}_{\mathrm{w}}K\mathbf{C}_{\mathrm{y}}])^{-1}\mathbf{B}_{\mathrm{f}} + \mathbf{D}_{\mathrm{zf}} \tag{14}$$

where s is the Laplace operator, and $\mathbf{I}$ is the identity matrix. All the matrices that are always null in Equation (11) (i.e. regardless of whether the target variable is displacement, velocity, or acceleration) are neglected in Equation (14).

Formulating the controller K with a state space representation, the state space representation of the closed-loop system is then obtained:

$$\begin{cases} \dot{\mathbf{g}}_{\mathrm{cl}} = \mathbf{A}_{\mathrm{cl}}\mathbf{g}_{\mathrm{cl}} + \mathbf{B}_{\mathrm{cl}}F_{\mathrm{ext}} \\ z_{\mathrm{o}} = \mathbf{C}_{\mathrm{cl}}\mathbf{g}_{\mathrm{cl}} + \mathbf{D}_{\mathrm{cl}}F_{\mathrm{ext}} \end{cases} \tag{15}$$

where the closed-loop state vector is:

$$\mathbf{g}_{\mathrm{cl}} = \begin{Bmatrix} \mathbf{g} \\ \mathbf{g}_{\mathrm{k}} \end{Bmatrix} \tag{16}$$

$\mathbf{g}_{\mathrm{k}}$ is the vector of state variables of the controller. Refer to [33] for more details about the way to calculate the matrices $\mathbf{A}_{\mathrm{cl}}$, $\mathbf{B}_{\mathrm{cl}}$, $\mathbf{C}_{\mathrm{cl}}$, and $\mathbf{D}_{\mathrm{cl}}$. Therefore, the transfer function T_{zF} of Equation (14) can be expressed as a function of the closed-loop matrices using Equation (15):

$$T_{\mathrm{zF}} = \mathbf{C}_{\mathrm{cl}}(s\mathbf{I} - \mathbf{A}_{\mathrm{cl}})^{-1}\mathbf{B}_{\mathrm{cl}} + \mathbf{D}_{\mathrm{cl}} \tag{17}$$

In the MI approach, each control target or specification is expressed as a constraint on the admissible Lyapunov functions for the internally stable closed-loop system [34]:

$$\exists \quad V(\mathbf{g}_{\mathrm{cl}}) = \mathbf{g}_{\mathrm{cl}}^{\mathrm{T}}\mathbf{P}\mathbf{g}_{\mathrm{cl}}, \quad \mathbf{P} > 0 \quad : \quad \mathbf{A}_{\mathrm{cl}}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathrm{cl}} < 0 \tag{18}$$

where V is a quadratic Lyapunov function, $\mathbf{P}$ the Lyapunov matrix, $\mathbf{A}_{\mathrm{cl}}$ is the state matrix of the system, and $\mathbf{g}_{\mathrm{cl}}$ is the closed-loop state-space vector. The superscript T indicates the transposed matrix.

Since the additional constraints are related to targets on the transfer function T_{zF} of Equation (17), the related MI formulation will be thus expressed as a function of the closed-loop matrices $\mathbf{A}_{\mathrm{cl}}$, $\mathbf{B}_{\mathrm{cl}}$, $\mathbf{C}_{\mathrm{cl}}$, and $\mathbf{D}_{\mathrm{cl}}$ (see Equation (19) further in the paper). Therefore, the solution of the MI problem will provide state matrices that satisfy the given specifications. Then, the controller K transfer function can be derived [33].

As for H_∞ control, the objective is to find a controller K such that $||T_{\mathrm{zF}}||_\infty$ is minimised. As shown in [35] and [34], there exists a controller K such that $||T_{\mathrm{zF}}||_\infty < \gamma$ and $\mathbf{A}_{\mathrm{cl}}$ is stable if and only if the following problem is feasible for some symmetric $\mathbf{P} > 0, \gamma > 0$, and $\mathbf{A}_{\mathrm{k}}$, $\mathbf{B}_{\mathrm{k}}$, $\mathbf{C}_{\mathrm{k}}$, and $\mathbf{D}_{\mathrm{k}}$ of compatible dimensions [36]:

$$\begin{bmatrix} \mathbf{A}_{\mathrm{cl}}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathrm{cl}} & \mathbf{P}\mathbf{B}_{\mathrm{cl}} & \mathbf{C}_{\mathrm{cl}}^{\mathrm{T}} \\ \mathbf{B}_{\mathrm{cl}}^{\mathrm{T}}\boldsymbol{P} & -\gamma\mathbf{I} & \mathbf{D}_{\mathrm{cl}}^{\mathrm{T}} \\ \mathbf{C}_{\mathrm{cl}} & \mathbf{D}_{\mathrm{cl}} & -\gamma\mathbf{I} \end{bmatrix} < 0 \tag{19}$$

It follows that the solution of the problem requires minimising γ.

Other objectives can be set, alternatively or in parallel with the previous H_∞ control requirement (e.g. H_2 [34]). In this paper, special focus is related to the passivity of the controller and thus on the passivity of the shunt impedance (see Equation (13)), as previously mentioned.

Passivity can be expressed as an LMI as well, where the unknown variables are the controller matrices $\mathbf{A}_{\mathrm{k}}$, $\mathbf{B}_{\mathrm{k}}$, $\mathbf{C}_{\mathrm{k}}$, and $\mathbf{D}_{\mathrm{k}}$. If passivity is satisfied, then the transfer function of the controller is positive real.

Therefore, the controller passivity can be expressed as an LMI constraint using the positive real lemma [35]. It follows that a controller $K(s)$ is positive real if and only if there exists $\mathbf{P} > 0$ such that:

$$\begin{bmatrix} \mathbf{A}_{\mathrm{k}}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathrm{k}} & \mathbf{P}\mathbf{B}_{\mathrm{k}} - \mathbf{C}_{\mathrm{k}}^{\mathrm{T}} \\ \mathbf{B}_{\mathrm{k}}^{\mathrm{T}}\mathbf{P} - \mathbf{C}_{\mathrm{k}} & -\mathbf{D}_{\mathrm{k}}^{\mathrm{T}} - \mathbf{D}_{\mathrm{k}} \end{bmatrix} \leq 0 \tag{20}$$

where $\mathbf{P}$ is the Lyapunov matrix.

Actually, Equations (19) and (20) are not LMIs since the dependence on the decision variables is not linear. As an example, referring to Equation (19), the expression $\mathbf{A}_{\mathrm{cl}}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{A}_{\mathrm{cl}}$ is not affine in the variables $\mathbf{P}$ and $\mathbf{A}_{\mathrm{cl}}$ because it involves the product between the Lyapunov matrix $\mathbf{P}$ and the controller variables which are included in the $\mathbf{A}_{\mathrm{cl}}$ expression. Hence, these problems are non-linear and cannot be solved by LMI optimisation (e.g. using the ellipsoid method or the interior-point method [37,38]). However, for the state feedback case and even for the output feedback systems (which is the case considered in this paper), there exists a change of variables that makes all the inequalities affine in a new set of unknowns, making the constraints linear and easily solvable [34].

Even multi-objective problems [34,39] can be solved, e.g. H_∞ control together with controller passivity. In this case Equations (19) and (20) must be fulfilled at the same time. This makes the multi-objective output feedback control problem a non-linear matrix inequality problem; more specifically, it becomes a bilinear matrix inequality (BMI) problem. One suitable approach to solve such a problem is the use of non-linear solvers [40,41]; refer to [33] for more details about the procedure to be used and the algorithms, and to the next section for the experimental validation of passive multi-objective controllers (i.e. BMI controllers).

4 Experimental tests

The experimental setup was made from an aluminum cantilever beam with one piezoelectric patch bonded at its clamped end. The beam was 25 mm wide, 161 mm long, and 1.1 mm thick; the piezoelectric patch was 25 mm wide, 51 mm long, and 0.38 mm thick. The structure was excited by means of a contactless actuator composed of a coil and a magnet bonded to the beam near its tip. The current flowing in the coil results in a proportional force exerted on the beam [42]. The current was measured using a current clamp, and the response of the structure was measured through a laser Doppler velocimeter in a co-located position with the exerted force (on the other side of the beam). The third and fourth eigenmodes were considered, and their modal data are gathered in Table 1. The eigenfrequencies and non-dimensional damping ratios were estimated by an experimental modal analysis with the piezoelectric patch short-circuited. χ_r was estimated by means of measurements of the r^{th} coupling coefficient $k_r^{\mathrm{eff}} = \sqrt{(\omega_{r,\mathrm{oc}}^2 - \omega_r^2)/\omega_r^2}$ ($\omega_{r,\mathrm{oc}}$ is the r^{th} eigenfrequency with the piezoelectric patch in open-circuit (OC)) and then performing the following computation [10]:

$$\chi_r = k_r^{\mathrm{eff}} \omega_r \sqrt{C_{\mathrm{p}r}}. \tag{21}$$

where $C_{\mathrm{p}r}$ is the measured piezoelectric capacitance value after the r^{th} eigenfrequency.

Mode number	$\omega_r/(2\pi)$ [Hz]	ξ_r [%]	k_r^{eff}
3	500.98	0.40	0.0400
4	1004.39	0.44	0.0924

Table 1: Modal parameters identified experimentally

Many tests have been carried out to validate the proposed approach based on matrix inequalities, but only two are described here for brevity. The first test was carried out without synthetizing a real passive shunt impedance (i.e. one made from resistances, capacitances, and inductances). Instead, a synthetic device was used to simulate the shunt impedance [43]. This approach allowed the authors to avoid any possible uncertainty on the values of the electric elements composing the shunt impedance and thus enabled to test the MI approach without additional errors from external factors (i.e. uncertainty on the values of the electrical parameters). Thus, the use of this synthetic system was useful for testing the reliability of the MI approach. The test was carried out on the third and fourth modes, having displacement as target variable to be decreased. The type of control imposed was a mixed H_2/H_∞ control, together with the passivity constraint. Figure 2a shows both the experimental and numerical results. They match satisfactorily, and the damping action is effective, evidencing the reliability of the proposed method.

The second test was instead a H_2 control on displacement, together with the passivity constraint. Again, the modes taken into account are the third and fourth. This time the shunt impedance was built physically. The way to synthetize the impedance from the controller layout resulting from the matrix inequality problem was based on the Brune's method [44,45]. Again, the control action is evident and experimental and numerical results match (see Figure 2b).

More details about these tests can be found in [33].

5 Conclusion

This paper has dealt with vibration attenuation by means of piezoelectric shunt. The paper proposed an approach for designing passive shunt impedances to be used for multi-mode vibration damping. Such an approach is based on matrix inequality problems, which allow to set both performance targets and impedance passivity.

The experimental tests carried out on a cantilever beam confirmed the reliability of the approach and the accuracy of the analytical model used to develop the work.

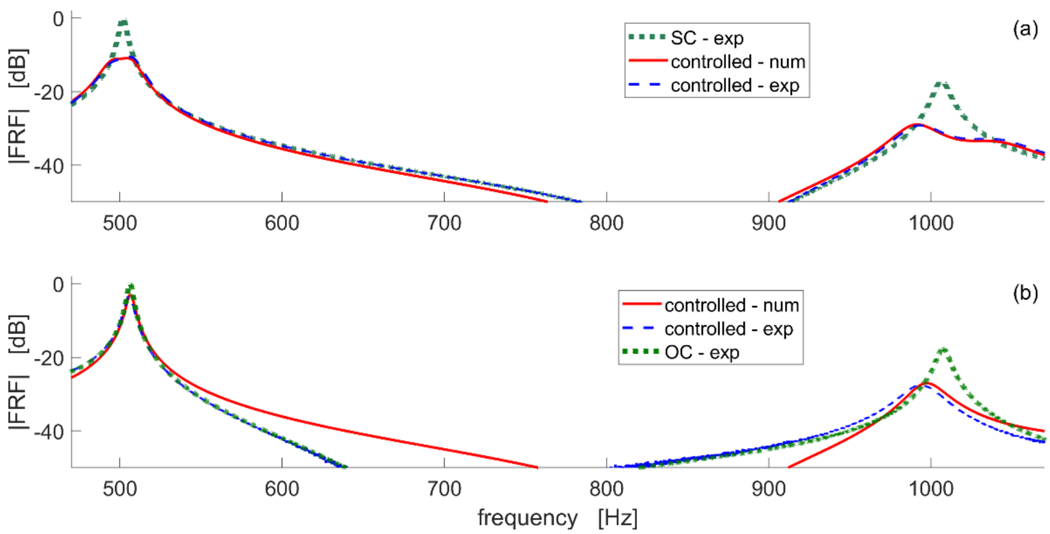

Figure 2: Amplitude of Frequency Response Functions (FRF) for the first test (that performed simulating the shunt impedance with a synthetic device) (a) and for the second test (that carried out building the shunt impedance) (b)

References

[1] N. Hagood, A. von Flotow, Damping of structural vibrations with piezoelectric materials and passive electrical networks, J. Sound Vib. 146 (1991) 243–268.

[2] G. Caruso, A critical analysis of electric shunt circuits employed in piezoelectric passive vibration damping, Smart Mater. Struct. 10 (2001) 1059–1068. doi:10.1088/0964-1726/10/5/322.

[3] M. Berardengo, A. Cigada, S. Manzoni, M. Vanali, Vibration Control by Means of Piezoelectric Actuators Shunted with LR Impedances: Performance and Robustness Analysis, Shock Vib. 2015 (2015) ID 704265. doi:10.1155/2015/704265.

[4] M. Berardengo, S. Manzoni, M. Vanali, The Behaviour of Mistuned Piezoelectric Shunt Systems and Its Estimation, Shock Vib. 2016 (2016) 9739217. doi:10.1155/2016/9739217.

[5] V.P. Matveenko, N.A. Iurlova, D.A. Oshmarin, N. V Sevodina, M.A. Iurlov, N.A. Iurlova, D.A. Oshmarin, N. V Sevodina, An approach to determination of shunt circuits parameters for damping vibrations, Int. J. Smart Nano Mater. 9 (2018) 135–149. doi:10.1080/19475411.2018.1461144.

[6] J. Høgsberg, S. Krenk, Balanced calibration of resonant piezoelectric RL shunts with quasi-static background flexibility correction, J. Sound Vib. 341 (2015) 16–30. doi:10.1016/j.jsv.2014.12.006.

[7] B. de Marneffe, a Preumont, Vibration damping with negative capacitance shunts: theory and experiment, Smart Mater. Struct. 17 (2008) 035015. doi:10.1088/0964-1726/17/3/035015.

[8] O. Heuss, R. Salloum, D. Mayer, T. Melz, Tuning of a Vibration Absorber with Shunted Piezoelectric Transducers, Arch. Appl. Mech. 86 (2016) 1715–1732. doi:10.1007/s00419-014-0972-5.

[9] M. Pohl, Increasing the performance of negative capacitance shunts by enlarging the output voltage to the requirements of piezoelectric transducers, J. Intell. Mater. Syst. Struct. 28 (2017) 1379–1390. doi:10.1177/1045389X16666181.

[10] M. Berardengo, O. Thomas, C. Giraud-Audine, S. Manzoni, Improved resistive shunt by means of negative capacitance: new circuit, performances and multi-mode control, Smart Mater. Struct. 25 (2016) 075033. doi:10.1088/0964-1726/25/7/075033.

[11] M. Berardengo, O. Thomas, C. Giraud-Audine, S. Manzoni, Improved shunt damping with two negative capacitances: an efficient alternative to resonant shunt, J. Intell. Mater. Syst. Struct. 28 (2017) 2222–2238. doi:10.1177/1045389X16667556.

[12] M. Berardengo, S. Manzoni, O. Thomas, C. Giraud-Audine, A new electrical circuit with negative capacitances to enhance resistive shunt damping, in: Proc. ASME 2015 Conf. Smart Mater. Adapt. Struct. Intell. Syst. - SMASIS 2015 - Sept. 21-23 2015, Colorado Springs (CO, USA), 2015: p. ID 8836.

[13] B. Götz, R. Platz, T. Melz, Effect of static axial loads on the lateral vibration attenuation of a beam with piezo- elastic supports, Smart Mater. Struct. 27 (2018) 035011.

[14] M. Lallart, A. Badel, D. Guyomar, Nonlinear Semi-active Damping using Constant or Adaptive Voltage Sources: A Stability Analysis, J. Intell. Mater. Syst. Struct. 19 (2008) 1131–1142. doi:10.1177/1045389X07083612.

[15] A. Badel, G. Sebald, D. Guyomar, M. Lallart, E. Lefeuvre, C. Richard, J. Qiu, Piezoelectric vibration control by synchronized switching on adaptive voltage sources: Towards wideband semi-active damping, J. Acoust. Soc. Am. 119 (2006) 2815–2825.

[16] B. Bao, W. Tang, Semi-active vibration control featuring a self-sensing SSDV approach, Meas. J. Int. Meas. Confed. 104 (2017) 192–203. doi:10.1016/j.measurement.2017.03.018.

[17] L. Yan, B. Bao, D. Guyomar, M. Lallart, Periodic structure with interconnected nonlinear electrical networks, J. Intell. Mater. Syst. Struct. 28 (2017) 204–229. doi:10.1177/1045389X16649448.

[18] M. Collet, M. Ouisse, M.N. Ichchou, Structural energy flow optimization through adaptive shunted piezoelectric metacomposites, J. Intell. Mater. Syst. Struct. 23 (2012) 1661–1677. doi:10.1177/1045389X12449915.

[19] B. Lossouarn, J. Deü, M. Aucejo, K.A. Cunefare, Multimodal vibration damping of a plate by piezoelectric coupling to its analogous electrical network, Smart Mater. Struct. 25 (2016) 115042. doi:10.1088/0964-1726/25/11/115042.

[20] U. Andreaus, F. Dell'Isola, M. Porfiri, Piezoelectric passive distributed controllers for beam flexural vibrations, J. Vib. Control. 10 (2004) 625–659. doi:10.1017/CBO9781107415324.004.

[21] I. Giorgio, A. Culla, D. Del Vescovo, Multimode vibration control using several piezoelectric transducers shunted with a multiterminal network, Arch. Appl. Mech. 79 (2009) 859–879. doi:10.1007/s00419-008-0258-x.

[22] L. Airoldi, M. Ruzzene, Wave Propagation Control in Beams Through Periodic Multi-Branch Shunts, J. Intell. Mater. Syst. Struct. 22 (2011) 1567–1579. doi:10.1177/1045389X11408372.

[23] J.J. Hollkamp, Multimodal passive vibration suppression with piezoelectric materials and resonant shunts, J. Intell. Mater. Syst. Struct. 5 (1994) 49–57.

[24] S. Wu, Method for multiple-mode shunt damping of structural vibration using a single PZT transducer, in: Proc. SPIE 3327, Smart Struct. Mater. 1998 Passiv. Damping Isol., San Diego (USA), 1998.

[25] S. Behrens, S.O.R. Moheimani, a. J. Fleming, Multiple mode current flowing passive piezoelectric shunt controller, J. Sound Vib. 266 (2003) 929–942. doi:10.1016/S0022-460X(02)01380-9.

[26] A. Cigada, S. Manzoni, M. Redaelli, M. Vanali, Optimization of the current flowing technique aimed at semi-passive multi-modal vibration reduction, J. Vib. Control. 18 (2012) 298–312. doi:10.1177/1077546311407537.

[27] A.J. Fleming, S. Behrens, S.O.R. Moheimani, Reducing the Inductance Requirements of Piezoelectric Shunt Damping Systems, Smart Mater. Struct. 12 (2003) 57–64. doi:10.1088/0964-1726/12/1/307.

[28] O. Thomas, J. Ducarne, J.-F. Deü, Performance of piezoelectric shunts for vibration reduction, Smart Mater. Struct. 21 (2012) 015008. doi:10.1088/0964-1726/21/1/015008.

[29] O. Thomas, J. Deü, J. Ducarne, Vibrations of an elastic structure with shunted piezoelectric patches: efficient finite element formulation and electromechanical coupling coefficients, Int. J. Numer. Methods Eng. 80 (2009) 235–268. doi:10.1002/nme.

[30] J. Ducarne, O. Thomas, J.-F. Deü, Placement and dimension optimization of shunted piezoelectric patches for vibration reduction, J. Sound Vib. 331 (2012) 3286–3303. doi:10.1016/j.jsv.2012.03.002.

[31] E. Doebelin, Measurement Systems: application and design, McGraw-Hill, New York, 2003.

[32] M. Berardengo, S. Manzoni, O. Thomas, M. Vanali, Piezoelectric resonant shunt enhancement by negative capacitances: Optimisation, performance and resonance cancellation, J. Intell. Mater. Syst. Struct. 29 (2018) 2581–2606. doi:10.1177/1045389X18770874.

[33] M. Berardengo, S. Manzoni, A.M. Conti, Multi-mode passive piezoelectric shunt damping by means of matrix inequalities, J. Sound Vib. 405 (2017) 287–305. doi:10.1016/j.jsv.2017.06.002.

[34] C. Scherer, P. Gahinet, M. Chilali, Multiobjective Output-Feedback, IEEE Trans. Automat. Contr. 42 (1997) 896–911.

[35] G.-R. Duan, H.-H. Yu, LMIs in control systems - Analysis, design and applications, Press, CRC, Boca Raton (FL, USA), 2013.

[36] C. Papageorgiou, M. Smith, Positive real synthesis using matrix inequalities for mechanical networks: application to vehicle suspension, IEEE Trans. Control Syst. Technol. 14 (2006) 423–435.

[37] S. Boyd, L. El Ghaoui, E. Feron, BalakrishnanV., Linear matrix inequalities in system and control theory, Society for Industrial and Applied Mathematics, 1994.

[38] F.A. Potra, S.J. Wright, Interior-point methods, J. Comput. Appl. Math. 124 (2000) 281–302. doi:10.1016/S0377-0427(00)00433-7.

[39] F. Li, M. de Oliveira, R. Skelton, Integrating information architecture and control or estimation design, SICE J. Control. Meas. Syst. Integr. 1 (2011) 120–128.

[40] M. Stingl, On the solution of nonlinear semidefinite programs by augmented Lagrangian methods, PhD thesis, University of Erlangen-Nurnberg, 2006.

[41] A. Conn, N. Gould, P. Toint, Trust region methods, Society for Industrial and Applied Mathematics, 2000.

[42] O. Thomas, C. Touzé, a. Chaigne, Asymmetric non-linear forced vibrations of free-edge circular plates. Part II: experiments, J. Sound Vib. 265 (2003) 1075–1101. doi:10.1016/S0022-460X(02)01564-X.

[43] S. Moheimani, A. Fleming, Piezoelectric transducers for vibration control and damping, Springer, London, 2006.

[44] O. Brune, Synthesis of a Finite Two-terminal Network whose Driving-point Impedance is a Prescribed Function of Frequency, J. Math. Phys. 10 (1931) 191–236.

[45] E.A. Guillemin, Synthesis of Passive Networks, John Wiley & Sons, New York, 1957.