Harmonic scaling of mode shapes for operational modal analysis

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Abstract

Mode shapes from operational modal analysis are normally unscaled, since the forces acting on the structure are not measured. Several methods for obtaining scaled mode shapes have been proposed in the past, some of them rather elaborate. In this paper we present and investigate using harmonic excitation at (or close to) the natural frequencies to obtain the mode shape scaling. The method is explored through simulated data, and on data from a staircase. Results are validated by comparing the results of applying the harmonic force in different points. The method is found to work very well.

1 Introduction

Modal analysis is a well-established tool for estimating modal parameters of structures and systems. The knowledge of eigenfrequencies and mode shapes is crucial for several different purposes, for example response estimation, model updating, and structural health monitoring. For most of these applications, the mode shape components must be properly scaled. This is one of the most important issues related to operational modal analysis (OMA), which in itself only provides unscaled mode shapes, because the loads acting on the structure are not measured with such an approach.

Different methods were proposed and investigated in the past, with the aim of scaling the mode shapes provided by OMA tests. Most of the proposed methods rely on repeating OMA tests with different system configurations, which in turn means to change, in a controlled way, the amount/distribution of the mass and the stiffness of the structure [1–11]. Other approaches rely on the use of OMAX (operational modal analysis with exogenous inputs) tests, which are tests where the excitation to the structure is partly provided by natural environmental excitation (e.g. wind, traffic) and partly by actuators providing broad band excitation, which is thus measured [12, 13]. Further approaches couple known dynamic systems (e.g. tuned mass dampers, people) to the structure under investigation and this allows to estimate the modal mass and thus to scale the mode shapes [14–16].

The referenced approaches all rely on relatively complicated experimental procedures (e.g. mass changes), or require the perfect knowledge of the dynamics of additional systems, or again ask for using potentially large actuators (on large structures) able to produce different kinds of excitation signals (e.g. chirp or multisine). The aim of the present work is to present an alternative method for computing modal masses and thus scaling mode shapes, using a fast and readily applicable experimental procedure which involves the use of relatively inexpensive, general-purpose, actuators and simple signal processing.

The paper is structured as follows. Section 2 introduces the theory related to this work and the OMAX method proposed. Then, Section 3 explains how we tested the method with some numerical simulations, while Section 4 shows the experimental tests carried out to validate the method proposed herein. Finally, Section 5 discusses the results of the numerical tests and experiments.

2 Theory

A scaled frequency response function of a structure can be formulated generally in receptance form (displacement over force) as

$$H(\mathrm{j}\omega) = \sum_{r=1}^{N} \frac{1}{m_r(\mathrm{j}\omega - s_r)(\mathrm{j}\omega - s_r^*)} \tag{1}$$

where m_r is the modal mass of mode r, s_r the pole of mode r, and * denotes complex conjugation. The expression of s_r is $-\xi_r\omega_r + \omega_r\sqrt{1-\xi_r^2}$, where ω_r and ξ_r are the eigenfrequency and non-dimensional damping ratio of the mode r respectively. In OMA, the poles are obtained by the parameter extraction, and, according to Equation (1), scaling the mode shape thus reduces to finding the modal mass m_r of the modes to be scaled.

The method proposed here is to obtain the modal scaling by applying a known harmonic force in one point, at each natural frequency of the modes to be scaled. This single frequency measurement can then be used to obtain the scaling of the mode by assuming that the single mode is dominating at the excitation frequency considered, which will be equivalent to a single-degree-of-freedom (SDOF) approach. This will, in general, require that the frequency of excitation is very close to the natural frequency of the mode.

Therefore, the first part of the method proposed herein is to carry out an OMA, finding the s_r values and the corresponding unscaled mode shape components $\psi_{r,i}$, where i indicates generic points of the structure where its response has been measured during the OMA test. Then, the second part of the method consists in providing a known mono-harmonic excitation (at a frequency as close as possible to ω_r) to the structure in point E. This part is explained underneath within this section. It is essential that E is a point where $\psi_{r,i}$ has been found by means of OMA (i.e. E is a point where the structural vibration was acquired during the OMA test).

If we consider a SDOF approximation, the frequency response function (FRF) of Equation (1), can be approximated as:

$$H(\mathrm{j}\omega) \simeq \frac{1}{m_r(\mathrm{j}\omega - s_r)(\mathrm{j}\omega - s_r^*)} \tag{2}$$

If the system FRF is measured with a co-located configuration (input and output measured in the same point) in point E at a frequency ω_{ex} as close as possible to ω_r (we call this FRF value H^{est}), the value of m_r can be estimated as:

$$m_r = \frac{1}{H^{est}(\mathrm{j}\omega_{ex} - s_r)(\mathrm{j}\omega_{ex} - s_r^*)} \tag{3}$$

It is noticed that the value of m_r calculated by means of Equation (3) corresponds to an eigenvector component (in the point of excitation E and for mode r) equal to 1. Then, the eigenvector component in the point of excitation E and for mode r scaled to the unit modal mass (named $\phi_{r,i=E}^{exp}$) can be calculated as:

$$\phi_{r,i=E}^{exp} = \frac{1}{\sqrt{m_r}} \tag{4}$$

Finally, the eigenvector component scaled to the unit modal mass in a point $i \neq E$ for mode r (named $\phi_{r,i\neq E}$) can be computed as:

$$\phi_{r,i\neq E} = \frac{\phi^{exp}_{r,i=E}\psi_{r,i\neq E}}{\psi_{r,i=E}} \tag{5}$$

Therefore, the knowledge of $\phi^{exp}_{r,i=E}$ allows to scale the mode shapes coming from the OMA to unit modal mass.

Of course, if higher accuracy is required, or if there are coupled modes so that several modes are contributing to the frequency response at frequency ω_{ex}, then $H(\mathrm{j}\omega_n)$ evaluated at several different frequencies ω_n can be used; at least as many frequencies as the number of modes to be scaled. In fact, to obtain the best accuracy, at least two more frequencies than number of modes in the frequency range of interest should be excited, to allow for residual terms, accounting for the out-of-band modes, to be computed. The requested modal masses can then be computed by using a standard least squares frequency domain method [17, 18].

2.1 Advantages with the proposed technique

The proposed technique has several important advantages over previously proposed techniques. The main advantages are that (i) a device for applying harmonic force can be realized relatively easily and inexpensively for force levels ranging from small to very large, and (ii) a harmonic force and the response due to this force can readily be extracted even in the case of existing natural loads of random character, from, e.g., traffic loads or wind loads, by common signal processing techniques. The technique proposed here, can therefore be implemented for structures such as bridges and highrise buildings without any large hardware cost. The benefits proposed by the present approach are evident if we compare it to the works referenced in Section 1.

If we consider the methods relying on repetitions of modal tests by changing the original structure through changes of mass (or stiffness), it is easy to notice that the presently proposed approach has several advantages: it is much simpler and consequently cheaper since it does not require both changes of the structure and OMA test repetitions. As for the other referenced OMA methods, there are a couple of key points to be taken into account:

- some of the methods mentioned require to provide the structure with a broadband excitation. This in turn means that the actuator must provide high force levels because the structural response must be measured even out of resonances. Conversely, the present approach only requires a mono-harmonic excitation at resonance. This simplifies the test because the maximum force needed from the actuator is lower because we work just at resonance (or predominantly at resonance, in the case of repeated poles, or in the case one wants to take the residual terms into account);
- other methods require to couple the structure with known dynamic systems. This requires to have an accurate knowledge of these additional systems if a high accuracy is needed. Anyway, the additional systems can easily undergo changes of their modal behaviour (e.g. due to thermal shifts) and uncertainty affects the knowledge of their modal parameters. These factors contribute to increase the uncertainty on the final result. Furthermore, the additional systems must be designed on purpose to work in the frequency range of interest while a simple actuator able to work at almost any frequency is used in the present method.

2.2 Signal processing

This section explains the procedure used to finally scale the unscaled mode shapes estimated by means of an OMA test. Indeed, there are a number of ways to apply this approach.

The first task is to perform an OMA test from which it is possible to extract the eigenvalues of the system, as well as the unscaled mode shapes. Then, a mono-harmonic force is applied to the structure and both the input force and the structural response are measured; the response is measured in the same point where the input is exerted. Although not necessary *per se*, we suggest to excite at resonance, which is easily achieved by tuning the frequency of the signal provided to the actuator and monitoring at the same time the phase (between force and response), for example by an oscilloscope, until its value is as close as possible to $-90°$ if the response transducer is displacement. If velocity or acceleration are measured in place of displacement, the phase must be as close as possible to $0°$ or $90°$ respectively.

Then, after having estimated the eigenvalues s_r with OMA tests and after having acquired the excitation at resonance and the consequent structural response for a given amount of time, some easy-to-apply signal processing tasks are carried out:

- the excitation frequency ω_{ex} is estimated with higher accuracy by applying auto-correlation to the force signal. We use the estimated value of ω_{ex} also to cut the input and output signals in order to have an integer number of periods for both the signals;
- then, we need to measure the amplitude of the response at the excitation frequency. There are several ways to do this. In this case we windowed the whole output signal by means of a Flattop window and then performed a digital Fourier transform (DFT) on such a windowed signal, finding the corresponding spectrum $G_{out}(\mathrm{j}\omega)$. The use of the Flattop window allows for an accurate estimation of the height of the spectral lines [19, 20], avoiding the effects of leakage-induced errors (we assume that the output signal produced by the active excitation is much higher than the noise floor and the response due to environmental excitation). Then, the same procedure is applied to the input signal, finally achieving the the estimation of $G_{in}(\mathrm{j}\omega)$ and thus H^{est} as the ratio of $G_{out}(\mathrm{j}\omega_{ex})$ and $G_{in}(\mathrm{j}\omega_{ex})$;
- Equations (3), (4) and (5) are applied to scale the modes.

3 Numerical tests

We tested this method at first by carrying out some numerical simulations on three different SDOF systems. To this purpose, we normalized the model presented in Equation (2) as follows:

$$H(\mathrm{j}\omega) \simeq \frac{1}{m_r(\mathrm{j}\omega - s_r)(\mathrm{j}\omega - s_r^*)} = \frac{1}{\omega_r^2}\frac{1}{-\varphi^2 + 2\mathrm{j}\xi_r\varphi + 1} \tag{6}$$

where $\varphi = \omega/\omega_r$ and m_r is fixed equal to 1. In Equation (6), ω_r is just a multiplicative constant so that it can be neglected (or fixed to 1) in order to test the method, without loss of generality. It is noticed that the resonance is at $\varphi = 1$.

We fixed three different values of ξ_r (i.e. 10^{-4}, 10^{-3} and 10^{-2}), which means that we tested three different systems, as already mentioned.

For each of these systems, we simulated a mono-harmonic input at φ_{ex} close to 1 with amplitude equal to 1 N and we derived the corresponding output. Then, we added noise on the output and we estimated the scaled mode shape component (i.e. $\phi^{exp}_{r,i=E}$) with the procedure described in Section 2.2. In this case we assumed to perfectly know the values of the eigenvalue s_r coming from the OMA. Indeed, we were interested in understanding which errors are introduced by the noise in the computation of the scaled mode shapes when the active exciter is used. This is an important outcome from which the user can understand if it is possible to apply the procedure proposed herein in practical applications.

The value of the normalized excitation frequency $\varphi_{ex} = \omega_{ex}/\omega_r$ has been fixed as a shift s_h from $\varphi = 1$: $\varphi_{ex} = 1 + s_h$. This allows to simulate situations where the excitation is not provided exactly at resonance,

Table 1: Modal data of the staircase identified by means of OMA

Mode	$\omega_r/(2\pi)$ [Hz]	ξ_r [%]
1	7.836	0.222
2	8.880	0.391

which is quite usual in real applications. This causes a decrease of the amplitude of the structural output and thus a worsening of the signal-to-noise ratio.

The noise added to the output was extracted by a Gaussian distribution with null mean value and a variance $V = pV_s$, where V_s is the variance of the output for an input of 1 N at resonance, and p is a positive number. In other words, we compute the amplitude of the response when excited by a mono-harmonic force at resonance with amplitude of 1 N and we calculate the variance of the noise as a percentage of the variance of this signal. Then, this kind of noise is added to all the responses, whatever value of φ_{ex} is used.

Finally, we used different sampling frequencies f_s and different time-length values T_s for the signals. It is expected that all the parameters p, f_s and T_s have influence because their effect is to change the power-spectrum of the noise.

The values used for f_s were 5, 10 and 20 times the resonance frequency of the SDOF system. As for T_s, we used 100, 200, 500, 1000 and 2000 s. The values of p employed here were 0.01, 0.05, 0.1, 0.5, 1 and 2. Finally, the values used for s_h were: $0, \pm 0.05, \pm 0.02, \pm 0.01, \pm 0.005, \pm 0.001$. Each configuration (i.e. fixed values of ξ_r, f_s, T_s, p and s_h) was tested 200 times in order to have a good statistical reliability of the results and check their dispersion.

The results of these numerical tests are provided and discussed in Section 5, after having presented the experiments carried out to test the method in practice in Section 4.

4 Measurements

We applied the method presented herein to a steel staircase (12.03 m length, 1.80 m width and 5.22 m height). This structure was used because its modal behaviour was already analysed by the authors [21, 22].

The structure was instrumented with fifteen (numbered from 0 to 14) seismic piezoelectric accelerometers with a full scale of 4.9 ms^{-2} and a sensitivity of 1.02 V/(ms^{-2}). Thus, such sensors are able to properly collect OMA data (usually vibration levels in OMA applications are low) as well as to have low electrical noise on the signals. The devices where placed as depicted in Fig. 1.

First an OMA test was carried out by measuring the structural response while the structure was in ambient vibration. This allowed to estimate the eigenfrequencies, non-dimensional damping ratios and unscaled mode shape components for the first two modes (see Table 1). The algorithm used to extract the modal data was a multi-reference Ibrahim time domain method, which is essentially identical to the covariance driven stochastic subspace method (SSI), see [23].

A small electro-dynamic shaker was then applied co-located with accelerometer 0 (see Fig. 1), or 10, or 11 (these points are among those showing the highest mode shape components for both mode 1 and 2, as evidenced by the OMA tests), and it was used to provide a mono-harmonic excitation to the structure. Indeed, the shaker was able to provide a force to the structure by moving a steel block (with a cubic shape and a side of approximatively 7 cm). The motion of the mass was measured by a further accelerometer placed on top of it (see Fig. 2). The total mass moved by the shaker (i.e. steel cube + accelerometer) was measured and found to be 3.237 kg. Hence, the force applied by the shaker to the structure can be computed by multiplying this mass by its measured acceleration.

The values of T_s and f_s were 100 s and 256 Hz respectively in all the tests. The mono-harmonic excitation was provided at different frequencies and with different amplitudes, in correspondence of both mode 1 and

Figure 1: Positions of accelerometers on the staircase

Figure 2: Source of external load: shaker moving a mass

2. Table 2 gathers the data related to the different tests carried out. It is noticed that in many cases the actual value of ω_{ex} resulted slightly different from the value of ω_r because it was hard to detect exactly the frequency at which excitation and response acceleration are 90° shifted (see Table 2). This was, however, entirely due to the fact that we did not use any sophisticated equipment to measure the phase; it could easily be implemented in practice. Furthermore, a slight non-linear behaviour of the structure was noticed, mainly resulting in slight shifts of the eigenfrequency values when changing the amplitude of the excitation; conversely the damping was almost not affected by such a non-linear behaviour, as found during the tests carried out on the same staircase for other works [21, 22].

Furthermore, it is remarked that the shaker was not able to provide a pure sine below 10 Hz and the excitation force was basically composed by three harmonic components (one at ω_{ex} and the others at $2\omega_{ex}$ and $3\omega_{ex}$) due to the distortion caused by the actuator. This, however, does not affect the method proposed here, since we only look at the fundamental frequency in the excitation force.

Table 2: Description of the experimental tests

Test #	shaker position	Nominal value of $\omega_{ex}/(2\pi)$ [Hz]	Nominal value of the input force [N]	phase shift between input and output [deg]	mode excited
1	0	7.725	9.53	93.86	1
2	0	7.724	9.53	93.97	1
3	10	7.685	7.94	92.04	1
4	10	7.748	0.79	87.94	1
5	11	7.743	9.53	95.18	1
6	0	8.701	12.70	94.22	2
7	0	8.700	12.70	93.90	2
8	0	8.770	1.27	84.42	2
9	10	8.770	15.88	95.41	2
10	11	8.755	14.29	94.04	2

5 Results and Discussion

In this section we discuss the results from the numerical (see Section 3) and experimental (see Section 4) tests described previously.

5.1 Numerical tests

The accuracy of the method to estimate the scaled mode shapes was checked following the procedure described below. After having estimated the excitation frequency φ_{ex}, the amplitude of the force A^{in} (whose actual value was 1 N, see Section 3) and the modal mass through the procedure described in Section 2.2, Equation (6) was used to compute the estimated amplitude A^{est} of the response of the structure excited by the sine force at φ_{ex} with an amplitude equal to A^{in}. The value of ξ_r employed in Equation (6) was equal to that used to generate the signals (see Section 2.2); indeed, we assumed to perfectly know the eigenvalues, as mentioned in Section 2.2.

Then, the value of A^{est} was compared with the actual amplitude of the response of the structure A^{ref}. Therefore, the error E can be defined as:

$$E = A^{est} - A^{ref} \tag{7}$$

Since each numerical test (i.e. fixed values of ξ_r, f_s, T_s, p and s_h) was repeated 200 times, we have a statistical population describing E for each configuration. Therefore, we computed for each statistical population its mean value $\bar{E}$ and its standard deviation σ_E. Then, $\bar{E}$ and σ_E were normalized on the reference value A^{ref} and expressed as a percentage:

$$\bar{E}_n = 100\frac{\bar{E}}{A^{ref}}, \quad \sigma_n = 100\frac{\sigma_E}{A^{ref}} \tag{8}$$

The results show that the values of $\bar{E}_n$ and σ_n are mainly influenced by the ratio R between the variance of the mono-harmonic signal of the structural output and the power associated to each spectral line of the random noise. Therefore, R can be intended as a sort of signal-to-noise ratio.

Although there are some oscillations of $\bar{E}_n$ and σ_n as function of f_s, T_s, p and s_h for the same value of R, these are not significant and most likely related to the unavoidable statistical variability. If the target is to stay in the area where σ_n is lower than 5%, R must be higher than about 1000. R must instead be higher than about 400 if σ_n must be lower than 10%. In both the cases $|\bar{E}_n|$ is lower than 2%.

5.2 Experimental tests

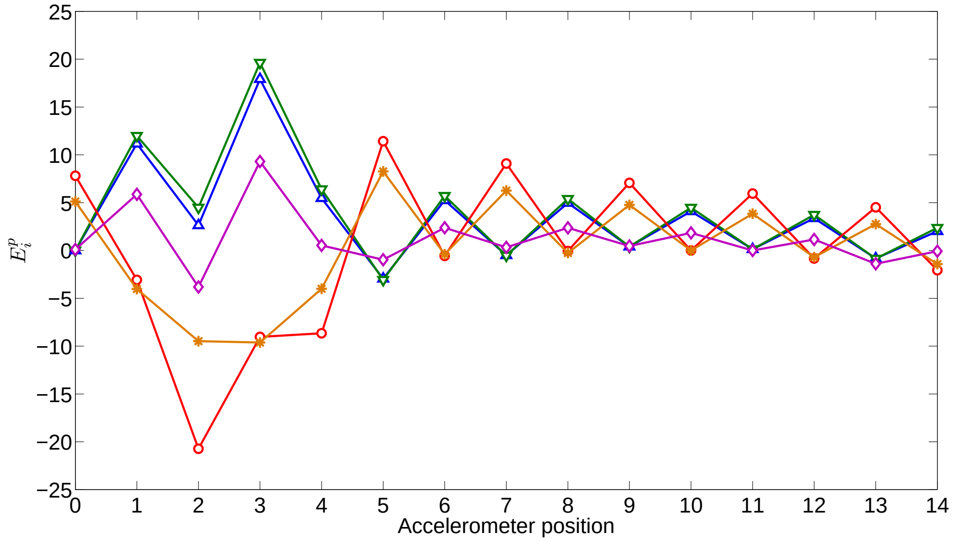

Figure 3: Trend of E_i^p as function of the various accelerometer positions for tests on mode 1 (see Table 2): $\triangle$ for test 1, $\bigtriangledown$ for test 2, $\circ$ for test 3, $*$ for test 4 and $\diamond$ for test 5

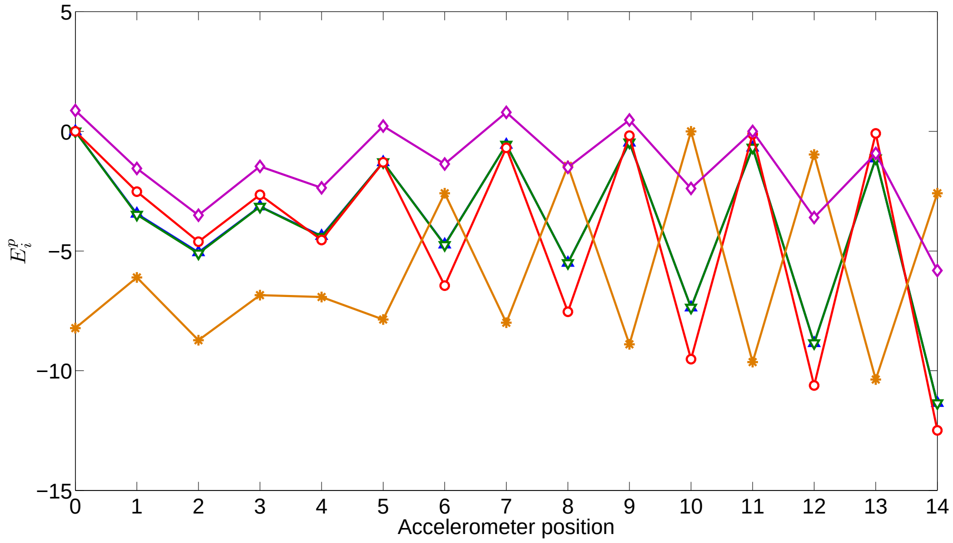

Figure 4: Trend of E_i^p as function of the various accelerometer positions for tests on mode 2 (see Table 2): $\triangle$ for test 6, $\bigtriangledown$ for test 7, $\circ$ for test 8, $*$ for test 9 and $\diamond$ for test 10

In this case we cannot define an estimation error as in Equation (7) because we do not know the value of A^{ref}. Therefore we employed the following approach for each test performed. After having estimated the modal mass and the scaled mode component in the position of the shaker (see Table 2), we calculated the scaled mode shape components in the other points of the staircase where we placed accelerometers. These data were used to compute the expected amplitude of response in these points (named A_i^{exp}, where the subscript i indicates the point considered, $i = 0, 1, ..., 14$) produced by the force provided through the shaker. Then, each A_i^{exp} was compared to the actual amplitude A_i^{meas} measured by the accelerometer placed in the corresponding point. Hence, the error is defined as:

$$E_i^{exp} = A_i^{exp} - A_i^{meas} \tag{9}$$

Figures 3 and 4 show the results in terms of percentage normalized errors E_i^p (i.e. $E_i^p = 100 E_i^{exp}/A_i^{meas}$) for the tests of Table 2. It should be noticed that the points where the shaker was placed in each test present a null error because of the procedure used.

The accuracy provided by the method is good, even in points where the mode component is very low (e.g. points 1 and 2). Therefore, the accuracy of the estimation is found to be satisfactory, particularly considering the advantages provided by this kind of testing approach.

6 Conclusions

This paper has proposed and investigated a new method for scaling mode shapes estimated by means of operational modal analysis, using harmonic excitation at (or close to) the natural frequencies of the tested structure. First, the method is studied numerically in order to understand the effect of different parameters on the accuracy of the final result. Then, experimental tests have been carried out in order to validate the method. The experiments showed that the method provides very satisfactory results. In addition, it is emphasized that applying a harmonic force can relatively easily and inexpensively be done to structures of all sizes, from small scale to large bridges or highrise buildings.

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