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Human-structure interaction in pedestrian bridges: a probabilistic approach

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Abstract

This paper deals with the quantification of the effects of human-structure interaction in the vertical direction for footbridges. Analyses are based on the study of a coupled system of the footbridge, modeled as a continuous dynamic system, and pedestrians, schematized as moving single-degree-of-freedom systems, characterized by random dynamic properties. The equivalent dynamic properties of the coupled system are estimated based on a state-space approach. The dimensionless parameters governing the problem are identified and numerical simulations are carried out varying such dimensionless parameters. Results of Monte Carlo simulations allow to quantify the effects of human-structure interaction on the dynamic properties of the footbridge, identifying the ranges of the governing dimensionless parameters where such effect is determinant.

Keywords: Coupled system; Footbridges; Human-structure interaction; Monte Carlo simulation.

1. Introduction

Modern footbridges are very slender structures with low damping characteristics, and they are often characterized by natural frequencies falling within the range of typical human step frequencies. Thus, they can be very sensitive to human-induced vibrations and their serviceability assessment is becoming a central step in their design. Recent guidelines and research papers (e.g. [1], [2]) provide simplified procedures to deal with their serviceability analyses. All these procedures are based on simplified expressions of pedestrian-induced forces, which neglect human-structure interaction ([3], [4]). Recent literature has recognized that human-structure interaction could have a determinant role in the assessment of human-induced vibrations of footbridges in the vertical direction. Experimental measurements on

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real structures seem to demonstrate that human-structure interaction is beneficial, providing an equivalent additional damping to the footbridge ([5]- [7]). Thus, tests in laboratory conditions have been carried out on scaled footbridges, confirming the results on real structures [8]. Analogous findings have been reported for staircases ([9], [10]). Different models have been proposed in the literature to deal with human-structure interaction in the vertical direction, mainly representing pedestrians as equivalent single-degree-of-freedom or multi-degree-of-freedom systems ([11], [12]), as an inverted pendulum [13], or as a bipedal walking model with damped compliant legs ([14], [15]). In the analysis of grandstands and stadia occupied by humans, some researchers have been modeling sitting or standing humans as single- or multi- degree-of-freedom systems and analyzed the properties of the coupled system from a deterministic [16] or probabilistic [17] point of view. A similar approach has been adopted for footbridges in [18] and [19], where pedestrians have been modelled as single-degree-of-freedom systems with random dynamic properties at uniformly-distributed fixed positions along the deck. Recently, vibration serviceability of footbridges modelling pedestrians as moving SDOF systems with random dynamic properties has been studied ([20], [21]). In [20], the interaction among pedestrians is neglected, and an application to a specific footbridge with fixed dynamic characteristics is shown. In [21], a modelling framework based on a microscopic model of multiple pedestrian traffic and schematizing pedestrians as moving single-degree of freedom systems has been introduced.

The objective of this paper is to provide a general overview of the effect of human-structure interaction on the dynamic properties of footbridges. A reliable model able to schematize also crowded conditions should take interaction among pedestrians into account. However, since accounting for interaction among pedestrians would require a microscopic model of multiple pedestrian traffic and thus to fix the geometric dimensions of the footbridge, it would hardly allow an extensive and general overview of the problem. Thus, interaction among pedestrians is neglected in this paper. The dynamic properties of the coupled system of the footbridge, modeled as a continuous dynamic system, and pedestrians, schematized as moving single-degree-of-freedom systems with random dynamic properties, are studied. In order to get general results valid for any specific structure, the equations of motion of the coupled system are written in dimensionless form and the main dimensionless parameters governing the behavior of the coupled system are identified. The equivalent dynamic properties of the coupled system (damping ratio and vibration frequency) are estimated based on a state-space approach. Monte Carlo simulations are carried out taking into account the randomness of the dynamic properties of pedestrians, for different combinations of the dimensionless parameters identified.

2. Equivalent human model

In the literature, different models have been proposed with the aim of studying the dynamic interaction between humans and structures in the vertical direction ([11]-[15]). In this paper, the simplest model proposed in the literature is adopted: it is represented by a single degree-of-freedom system whose dynamic properties have been evaluated in laboratory in different postures ([11], [12]).

Experimental tests aiming to identify the equivalent dynamic properties of walking humans have been recently carried out ([8], [22]). Results presented in [8] are in accordance with the ones published in [12] for pedestrians standing on one leg. Table 1 lists the model parameters obtained in [12] for a normal standing and one leg postures (stiffness k_p, viscous damping c_p, mass m_p, normalized with respect to pedestrian mass M), together with the resulting natural frequency n_p and damping ratio ξ_p and the results reported in [8].

Table 1. Human SDOF model parameters ([8], [12]).

Posture	k_p/M (N m^{-1} kg^{-1})	c_p/M (N s m^{-1} kg^{-1})	m_p/M	n_p (Hz)	ξ_p
Standing [12]	$1.34\ 10^3$	51.6	1.03	5.8	0.7
One leg [12]	$5.08\ 10^2$	17.5	0.898	3.6	0.39
Walking [8]				2.75-3	0.27-0.3

Toso et al. [22] obtained very dispersed values for the dynamic properties of test subjects (Table 2), very far from the ones in [12].

Table 2. Human SDOF model parameters [22].

	k_p (N m^{-1})	c_p (N s m^{-1})	m_p (kg)	n_p (Hz)	ξ_p
Minimum	2544.79	308.99	11.34		
Maximum	18019.79	1713.12	106.71		
Mean	8052.46	577.87	46.73	2.2	0.53

When dealing with multi-pedestrian traffic, all the models introduced should take into account the randomness of pedestrian characteristics. Due to the lack of a complete statistical characterization of the equivalent pedestrians' dynamic properties, a normal distribution is assumed in this paper.

3. Analytical formulation

Let us consider a footbridge, schematized as a linear mono-dimensional damped dynamical system, crossed by an arbitrary number N_p of pedestrians, modelled as moving single-degree-of-freedom systems, exerting also an external force on the footbridge (Figure 1). Neglecting interaction among pedestrians, pedestrians' arrivals are modelled as Poisson events, and their velocities c_i are modelled as Gaussian random variables.
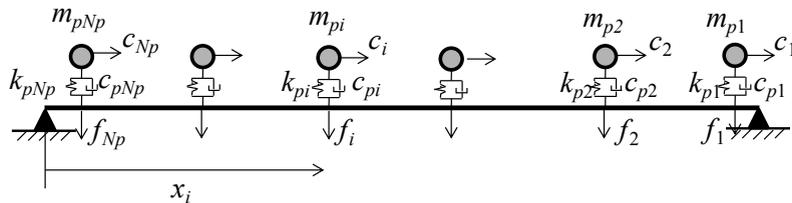

Fig. 1. Schematization of a footbridge crossed by pedestrians.

Assuming that the structural dynamic response is dominated by the j-th vibration mode, the equation of motion of the coupled footbridge-pedestrians system can be written in the following dimensionless matrix form:

$$\mathbf{M}(\tilde{t})\ddot{\mathbf{q}}(\tilde{t}) + \mathbf{C}\dot{\mathbf{q}}(\tilde{t}) + \mathbf{K}\mathbf{q}(\tilde{t}) = \mathbf{f}(\tilde{t}) \tag{1}$$

where the dot denotes derivative with respect to time, and the matrixes are defined as follows:

$$\mathbf{M}(\tilde{t}) = \begin{bmatrix} 1 + \sum_{i=1}^{N_p} \mu_{pi}\varphi_j^2\left[\tilde{x}_i(\tilde{t})\right] & \mu_{p1}\varphi_j\left[\tilde{x}_1(\tilde{t})\right] & \mu_{p2}\varphi_j\left[\tilde{x}_2(\tilde{t})\right] & \cdots & \mu_{pN_p}\varphi_j\left[\tilde{x}_{N_p}(\tilde{t})\right] \\ \varphi_j\left[\tilde{x}_1(\tilde{t})\right] & 1 & 0 & \cdots & 0 \\ \varphi_j\left[\tilde{x}_1(\tilde{t})\right] & 0 & 1 & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ \varphi_j\left[\tilde{x}_{N_p}(\tilde{t})\right] & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{2}$$

$$\mathbf{C} = \begin{bmatrix} 2\xi_j & 0 & \cdots & 0 \\ 0 & 2\xi_{p1}\tilde{\omega}_{p1} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 2\xi_{pN_p}\tilde{\omega}_{pN_p} \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \tilde{\omega}_{p1}^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{\omega}_{pN_p}^2 \end{bmatrix} \quad \mathbf{q}(\tilde{t}) = \begin{Bmatrix} p_j(\tilde{t}) \\ q_{p1}(\tilde{t}) \\ \vdots \\ q_{pN_p}(\tilde{t}) \end{Bmatrix} \quad \mathbf{f}(\tilde{t}) = \begin{Bmatrix} \dfrac{F_j(\tilde{t})}{m_j\omega_j^2} \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

In Eq.(2), φ_j represents the *j*-th mode shape and p_j the corresponding principal coordinate, m_j, ξ_j, ω_j represent the structural *j*-th modal mass, damping ratio and natural circular frequency, respectively; F_j is the modal force, q_{pi} (*i*=1,..N_p) is the displacement of the SDOF system equivalent to the *i*-th pedestrian. Furthermore, the following dimensionless parameters appear:

$$\tilde{t} = \omega_j t \quad \tilde{x}_i = \frac{x_i}{L} \qquad \tilde{\omega}_{pi} = \frac{\omega_{pi}}{\omega_j} \quad \mu_{pi} = \frac{m_{pi}}{m_j} \quad \xi_{pi} = \frac{c_{pi}}{2m_{pi}\omega_{pi}} \qquad \omega_{pi}^2 = \frac{k_{pi}}{m_{pi}} \tag{3}$$

where *t* is the time, x_i is the position along the deck of the *i*-th pedestrian, *L* is the deck span length, m_{pi}, c_{pi}, k_{pi} are, respectively, the mass, the damping and the stiffness of the SDOF system equivalent to the *i*-th pedestrian. In the following, normal random distributions are assumed for the pedestrian equivalent mass m_{pi}, damping ratio ξ_{pi} and natural circular frequency ω_{pi}.

The position $x_i(t)$ of the *i*-th pedestrian can be expressed as a function of its arrival time τ_i and velocity c_i:

$$x_i(t) = c_i(t - \tau_i)\left(H[t - \tau_i] - H\left[t + \frac{L}{c_i} - \tau_i\right] \right) \tag{4}$$

being H the Heaviside step function. Being pedestrians' arrivals modelled as Poisson events, their arrival times assume an exponential distribution.

The coupled bridge-pedestrians system is thus characterized by non-proportional damping. The effect of pedestrians on the bridge dynamic properties can be obtained from the complex eigenvalue λ_1 of the state matrix **G** defined as follows:

$$\mathbf{G}(\tilde{t}) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}(\tilde{t})^{-1}\mathbf{K} & -\mathbf{M}(\tilde{t})^{-1}\mathbf{C} \end{bmatrix} \qquad \tilde{\omega}_c(\tilde{t}) = |\lambda_1(\tilde{t})| \qquad \xi_c(\tilde{t}) = \frac{\mathrm{Im}[\lambda_1(\tilde{t})]}{\tilde{\omega}_c(\tilde{t})} \tag{5}$$

where λ_1 is the complex eigenvalue of **G** corresponding to the eigenvector dominated by the structural motion (the eigenvector with maximum first component), $\tilde{\omega}_c$ and ξ_c represent the dimensionless frequency and the damping ratio of the coupled system. A direct inspection of Eq. (5) clearly shows that the dynamic properties of the coupled bridge-pedestrians system vary in time due to the time variation of the mass matrix **M**.

4. A reference test case

In this Section, a reference test case is considered. A value of the structural damping ratio typical of footbridges is assumed ($\xi_j = 0.005$), and an average pedestrians flow is considered ($N_p = 15$). Pedestrians are modelled as single-degree-of-freedom systems with Gaussian-distributed mass, damping ratio and natural frequency. In particular, the following non-dimensional parameters have been set: μ_{pm}=0.0014 (it could be representative of a bridge with model mass m_j=50000 kg and an average pedestrian mass μ_{pm} = 70 kg), $\tilde{\omega}_{pm}$ =2 (which corresponds to assuming that the average pedestrians' frequency is double the natural frequency of the footbridge). Furthermore, according to Tables 1 and 2, the average pedestrians' damping ratio is set ξ_{pm}=0.5. Based on the literature [2], the coefficient of variation of pedestrian mass is set $V_{mp} = 0.17$. To the author's knowledge, no detailed information about the statistical distribution of pedestrians' equivalent dynamic properties is available in the literature; thus, the coefficients of variation of pedestrians' circular frequency and damping ratio are assumed $V_{\omega p} = V_{\xi p} = 0.1$.

Figure 2 plots the results of a reference simulation of the coupled pedestrians-bridge system. Both the circular frequency of the coupled system and its equivalent damping have significant variations in time. An average pedestrians flow of 15 persons can cause an average reduction of the natural frequency lower than 1%. More significant variations of the damping of the coupled system can be observed: on the average, an increase of the damping of the coupled system from 0.55 % up to 0.65 % occurs, but instantaneous values up to 0.68 % can be reached.
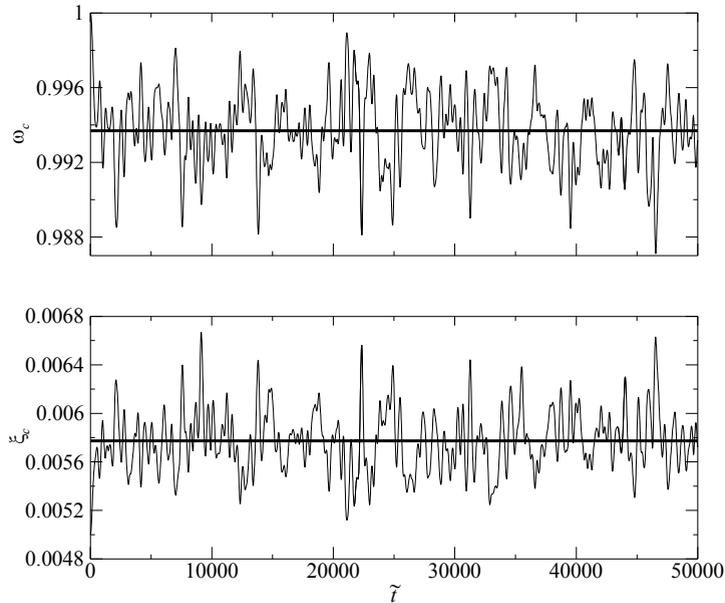
Fig. 2. Time variation of the dimensionless circular frequency and of the damping ratio of the coupled system..

5. Parametric analysis

A wide range of Monte Carlo simulations have been carried out fixing the structural damping ratio $\xi_j = 0.5$ %, considering N_p between 1 and 100 and ξ_{pm} between 0.3 and 0.7, and varying μ_{pm} and $\tilde{\omega}_{pm}$.

Figure 3 plots, as a test case, the results of Monte Carlo simulations for $N_p = 100$, $\xi_{pm}=0.4$. The average damping ratio of the coupled bridge-pedestrians system of a footbridge with a modal damping ratio $\xi_j = 0.5$ %, $\tilde{\omega}_{pm}=1$, $\mu_{pm}=0.01$ crossed by 100 pedestrians can increase up to 40 times, reaching the value of 20%. However, it should be pointed out that, since the average natural frequency of pedestrians is roughly comprised between 3 Hz and 6 Hz (see Table 1), this effect may be significant for footbridges with a natural frequency in this interval, and thus sensitive to the second loading harmonic, which generally provides a small excitation. The typical average frequency ratio for footbridges sensitive to human-induced vibrations is in the interval $\tilde{\omega}_{pm}=[1.5\ 3]$: for such values of $\tilde{\omega}_{pm}$, the effect of human-structure interaction may be relevant only for large numbers of pedestrians.
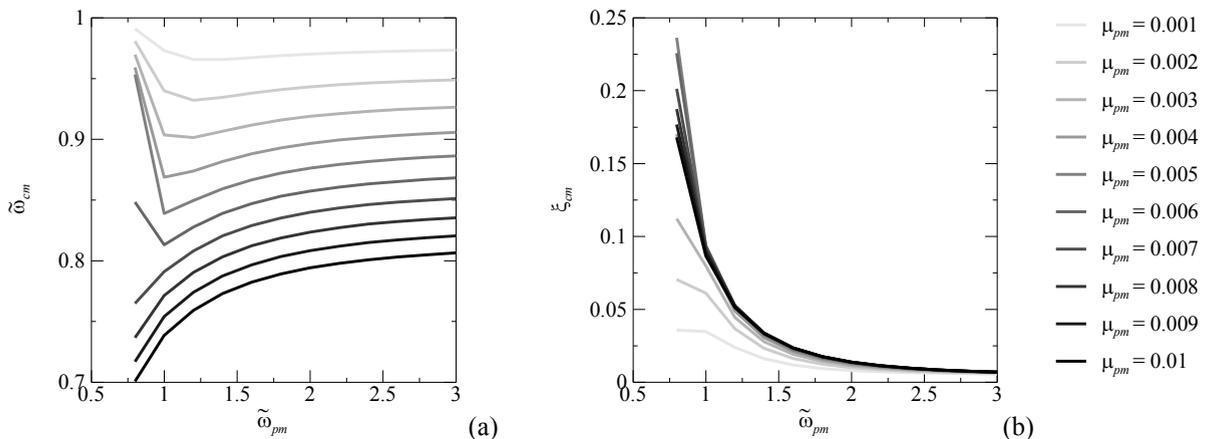

Fig. 3. Equivalent dimensionless circular frequency (a) and damping ratio (b) of the coupled system ($N_p = 100$, $\xi_{pm}=0.4$).

6. Conclusions and Prospects

In this paper, the effect of human-structure interaction in the vertical direction for footbridges has been studied through a probabilistic approach as a function of the mean dimensionless pedestrian natural frequency (i.e. the ratio between the average pedestrian natural frequency and the footbridge natural frequency) and the mean pedestrian mass ratio (i.e. the ratio between the mean pedestrian mass and the bridge modal mass). Results show that human-structure interaction can cause a significant increase in the structural damping for light footbridges crossed by significant pedestrian flows and characterized by a natural frequency close to the mean pedestrian frequency.

The scarce and largely dispersed data on the equivalent dynamic properties of pedestrians does not allow to clearly identify the structures for which such effect may be significant. This observation clearly points out the need of reliable experimental characterization of walking pedestrians' dynamic properties. Furthermore, a simplified model for pedestrian interaction would allow to get more reliable results also in crowded conditions.

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