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Working Memory Predictors of Written Mathematics in 7-8 Year Old Children

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Abstract: There is extensive evidence for the involvement of working memory in mathematical attainment This study aims to identify the relative contributions of verbal, spatial-simultaneous, and spatial-sequential working memory measures in written mathematics. Year 3 children (7-8 years of age, n=214) in the UK were administered a battery of working memory tasks alongside a standardised test of mathematics. Confirmatory factor analyses and variance partitioning were then performed on the data to identify the unique variance accounted for by verbal, spatial-simultaneous, and spatial-sequential measures. Results revealed the largest individual contribution was that of verbal working memory, followed by spatial-simultaneous factors. This suggests the components of working memory underpinning mathematical performance at this age are those concerning verbal-numeric and spatial-simultaneous WM. Implications for educators and further research are discussed.

Keywords: verbal, visuospatial, working memory, mathematics, children

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Abstract

There is extensive evidence for the involvement of working memory in mathematical attainment This study aims to identify the relative contributions of verbal, spatial-simultaneous, and spatial-sequential working memory measures in written mathematics. Year 3 children (7-8 years of age, n=214) in the UK were administered a battery of working memory tasks alongside a standardised test of mathematics. Confirmatory factor analyses and variance partitioning were then performed on the data to identify the unique variance accounted for by verbal, spatial-simultaneous, and spatial-sequential measures. Results revealed the largest individual contribution was that of verbal working memory, followed by spatial-simultaneous factors. This suggests the components of working memory underpinning mathematical performance at this age are those concerning verbal-numeric and spatial-simultaneous WM. Implications for educators and further research are discussed.

Keywords: verbal, visuospatial, mathematics, children

Introduction

There is some discrepancy in the literature with regard to the proportional influence of components of the Baddeley and Hitch working memory model (1974) on mathematics achievement. Whilst there are suggestions of a stronger influence of visuospatial working memory (e.g., Caviola, Mammarella, Lucangeli & Cornoldi, 2014; Clearman, Klinger & Szucs, 2017; Holmes, Adams & Hamilton, 2008; Li & Geary, 2017), there is also evidence of developmental shifts in the respective contributions and the potential for a cyclical relationship (e.g., Li & Geary, 2013; Soltanlou, Pixner & Nuerk, 2015; Van de Weijer-Bergsma, Kroesbergen & Van Luit, 2015). Additionally, there is some evidence for a greater influence of verbal working memory (e.g., Wilson & Swanson, 2001) on mathematics. Visuospatial working memory is implicated in mathematics performance in a number of areas, including, but not limited to arithmetic (Ashkenazi et al., 2013; Caviola, Mammarella, Cornoldi & Lucangeli; Passolunghi & Cornoldi, 2008;, 2012), word problem solving (Swanson & Beebe-Frankenberger, 2004; Swanson & Sachse-Lee, 2001; Zheng, Swanson & Marcoulides, 2011), and geometry (Giofrè, Mammarella & Cornoldi, 2014; Giofrè, Mammerella, Ronconi & Cornoldi, 2013), as well as mathematical difficulties (Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999; Passolunghi & Cornoldi, 2008; Szucs, Devine, Soltesz, Nobes & Gabriel, 2013). It is, therefore, important to understand the intricacies of this relationship in order to mediate difficulties associated with mathematics to the fullest extent possible.

Some authors argued that the visuospatial working memory system is not unitary (e.g., Logie, 1995). An alternative approach that has recently received some support is one that distinguishes between spatial-sequential tasks requiring the recall of a sequence of spatial locations, and spatial-simultaneous tasks demanding the recall of an array of simultaneouslypresented locations (see Cornoldi & Vecchi, 2003; Mammarella, Borella, Pastore, & Pazzaglia, 2013, Mammarella, Caviola, Giofrè, & Szucs, 2018).

Mammarella et al. (2006, 2018) identified a double dissociation between spatial-simultaneous and spatial-sequential working memory, which has been further investigated for its relationship with mathematics, thus providing reason for differentiating between spatial-simultaneous and spatial-sequential formats of VSWM tasks. Since spatial-simultaneous and spatial-sequential VSWM can be uniquely affected in visuospatial learning difficulties, it is logical that these two components may demonstrate differential relations with mathematics attainment in young children.

Various measures are available for assessing mathematical performance, ranging from single-step calculations to multi-step contextual story problems. A number of these measures have been standardised for their use with children within a particular age range (e.g., Wechsler Individual Achievement Test, Pearson Clinical, 2017), however, a large number of the measures used are measures derived by researchers for the purpose of research. Measures designed for research purposes should be considered carefully when applying the findings to any context other than that it was originally designed for since direct comparisons are not possible from unstandardised data. Furthermore, such measures can lead to concerns regarding reliability and validity since the number of applications of the measures is generally fewer than that of standardised measures. To combat these issues, a standardised written mathematics measure was used in this study to ascertain how children performed compared to age norms. The measure is designed to map on to current England and Wales SATs papers and so is directly related school attainment data.

The principal aim of this study is to examine the relationship between different working memory components and mathematics attainment. Here we aimed to further this knowledge by identifying the unique contributions of verbal, spatial-simultaneous, and spatial-sequential factors to written mathematics in Year 3 children (7-8 years of age). In doing so, this knowledge will allow us to understand more deeply the predictive nature of this relationship and understand where best to target preventative measures for mathematics difficulties, for example by identifying the age group most likely to benefit from an intervention. This age group was chosen based on previous evidence highlighting a stronger influence of visuospatial working memory on mathematics attainment in this age group (Holmes & Adams, 2006, Holmes, Adams & Hamilton, 2008). The age group chosen also aligns with a period of intensive skill acquisition; a time when visuospatial working memory is most likely employed (Andersson, 2008).

Spatial-sequential tasks requiring order during the recall phase, as well as those that do not require order during recall, were used in order to ensure the model was fully crossed. The main research question being asked was “how do the subcomponents of working memory relate to of the performance of written mathematics?”. Previous meta-analytic findings indicate different subcomponents of working memory do not tend to make different contributions to mathematical performance (Peng, Namkung, Barnes, & Sun, 2016). Such a finding, however, might be determined by a heterogeneous number of measures in use in different studies and by the fact that the aforementioned meta-analysis did not distinguish between simultaneous and sequential subcomponents of WM. In addressing this issue, a recent systematic review by Allen, Higgins and Adams (2019) identified no influence of spatial-sequential versus spatial-simultaneous working memory on mathematical performance. Similarly to Peng, Namkung, Barnes and Sun (2016), this review compared studies with a wide range of measures both for mathematics and working memory. Further, verbal components of working memory were not considered, which may have influenced the results. This work will expand on the understanding of previous papers by including the unique contributions of spatial-simultaneous and spatial-sequential measures to children's mathematics.

Method

Participants

The sample initially include of 214 7-8 year children. Some children were absent during the second administration and were excluded from the final sample. The final sample included a total of 197 children (95 male and 102 female, M age = 95.99 months, SD = 3.63). An opportunity sample of Year 3 pupils in each of the five schools was used, using opt-out parental consent to reduce bias in the sample (Krousel-Wood et al., 2006). The study was approved by the School of Education Ethics Committee at the University of Durham. Parental consent was obtained. Children with special educational needs, intellectual disabilities, or neurological and genetic conditions were not included in the study.

Design & Procedure

All children were tested individually in a quiet area of their school. The six working memory measures were administered in a randomised order so as to reduce the influence of rehearsal or fatigue (α = .80). However, the size of the grids used in the derived measures of visuospatial working memory were administered in a fixed order (3 × 3 then 4 × 3, and 4 × 3 then 4 × 4, for spatial-sequential and spatial-simultaneous, respectively). A correlational design was adopted to explore the relationships between visuospatial working memory and maths performance. Measures were administered as per the administration instructions provided with the WMTB-C where standardised measures were used. Where measures were derived for the purposes of the study, administration procedures paralleled those set out for standardised measures. The mathematics test was presented in paper format. Children could ask for a question to be read aloud in order to not place children of lower reading ability at a disadvantage.

Measures

Verbal WM

Working Memory Test Battery for Children (WMBT-C)*:* Three subtests of the WMTB-C were administered to children: digit recall (children recall a list of digits presented to them verbally), backwards digit recall (children recall a list of digits presented to the verbally in reverse order), and counting recall (children count aloud the number of dots on a page then recall the list of totals, in the correct order, once all pages in the sequence have been counted). All subtests were administered in accordance with the instructions set out for the WMTB-C, hence sequences were presented at a rate of one item per second. Blocks of six trials of each sequence length were employed, however, following four correct trials, testing moved on to the next block. Testing was discontinued following three mistakes within one block, however, if this was the first block of trials, the previous block was administered to ascertain the child's span score. The child's raw score was recorded for each subtest.

Visuospatial WM

Spatial-simultaneous*:* A grid was presented to the child (firstly a 4 × 3 grid was used, followed by a 4 × 4 grid; all children completed both grid sizes) containing dots. The dots were displayed for 3s before disappearing to leave a blank grid. Immediately following the disappearance of the dots, children were asked to tap on the screen of the laptop being used to indicate where the dots had been. They were instructed that this could be done in any order or pattern. The number of dots per grid ranged from two to eight dots, with block of six trials of each number of dots. This reflects the procedure of the WMTB-C. Additionally, the same discontinuation rule was applied. Unlike the subtests of the WMTB-C, a moving-on rule was not employed in this test.

Spacial-sequential, no order: The same format of test was used as that for spatial-simultaneous working memory. However, dots were presented one at a time for a period of 1s each on grid sizes 3 × 3 then 4 × 3. All children completed both grid sizes. Again, blocks consisted of six trials, and contained between two and eight dots. Recall in this test was also immediate, with the children being required to tap the screen where the dots had appeared previously. Importantly, children were instructed that they could indicate the location of the dots in any order they wished. This test was designed to determine the role of order during the recall phase in the number of dot positions correctly recalled.

Block recall (Corsi, 1972): The block recall task from the WMTB-C was used to assess spatial-sequential working memory with order. A sequence of blocks are tapped at a rate of one block per second which children must recall in the correct order. Only forwards order recall was required. This test was administered in accordance with the instructions set out by the WMBT-C, as with those used for verbal WM, hence administration and scoring were as described above.

Mathematics

Access Mathematics Test (AMT): The AMT is a standardised measure of mathematics, available for use with children between the ages of 6 and 12 years. As such it provides a comprehensive profile of how children perform when faced with different aspects of maths. The AMT is aligned to the areas of maths taught on the England and Wales national curriculum, with requirements for children to develop an understanding in the areas of number, measurement, geometry, and statistics, hence providing a valid measure. Questions include those concerning using and applying mathematics (e.g. “tick the two division facts that give the same answer”), counting and understanding number (e.g. “one part of the circle is shaded. How many more parts do you need to shade so exactly one half of the circle is shaded?”), knowing and using number facts (e.g., “what is half of 24?”), calculating (e.g. “complete this calculation and show the remainder: 721 ÷ 2 =_ remainder_”), understanding shape (e.g. “shade in the squares to show the reflection of the shape”), measuring (e.g. “what time does this clock show, in digital form?”), and handling data (e.g. “the table gives the ages of the members of a golf club. How many members are 55 or older?”).

Children were read the instructions set out for the AMT, which included a time limit of 45 minutes, clarification of where to write their answer on the paper, and explanation that workings were allowed on the paper, providing their answer was clearly written in the correct space. Typical classroom test conditions were adopted throughout. Children were permitted to request questions be read aloud to them should they have difficulties so as not to disadvantage those with weaker reading abilities, however, no further explanation of the question, or what was required, was given. No discontinuation rule was employed as children were instructed to complete as many questions as they could, but that questions were also included for children much older than they were so not to worry if they could not complete them all. The total number of test items for this test is 60, with a maximum score of 60.

Data analysis

The R program (R Core Team, 2018) with the “lavaan” library (Rosseel, 2012) was used. Model fit was assessed using various indexes according to the criteria suggested by Hu and Bentler (1999). We considered the chi-square (χ^2), the comparative fit index (*CFI*), the non-normed fit index (*NNFI*), the standardized root mean square residual (*SRMR*), and the root mean square error of approximation (*RMSEA*).

Results

Preliminary analyses

Age (in months) was partialled out of all analyses to remove its influence on the data (see Giofrè, Mammarella, & Cornoldi, 2013 for a similar procedure). Descriptive statistics and correlations are presented in Table 1. There is little variation evident between the raw and covaried correlations, with r values of a similar order in both bases, e.g. r = .398 and r = .399 for the raw and covaried correlations between spatial-simultaneous 4 × 4 and knowing and using number facts, respectively. Asymmetry and kurtosis were tested on all variables. “Measuring”, a single component of mathematics, was skewed and presented with extremely high values of kurtosis, therefore, this component was removed from further analysis. All other measures had skewness and kurtosis values lower than 1. All the analyses were performed again including measuring and results were extremely similar.

Table 1 about here

Confirmatory Factor Analysis (CFA)

To confirm the reliability of the structure of the variables, a CFA was conducted. We hypothesized the existence of three separate WM factors, spatial-simultaneous, spatial-sequential and verbal, and one mathematics factor. The fit of the model was acceptable, $\chi^2(71) = 94.23$, $p = .03$, $RMSEA = .04$, $SRMR = .05$, $CFI = .97$, $NNFI = .97$, and so this model was adopted for the remainder of the analysis (Table 2 and Figure 1). The CFA showed that mathematics is highly correlated with both spatial-simultaneous and verbal WM, while the correlation with spatial-sequential WM was moderate. Reliabilities were also calculated from the CFA model using omega, as this is shown to be a more robust measure of reliability at this level (Deng & Chan, 2016; Peters, 2014; Zinbarg, Revelle, Yovel & Li, 2005; verbal: ω = .60, spatial-simultaneous: ω = .81, spatial-sequential: ω = .70)

Table 2 and Figure 1 about here

Variance partitioning

In the final set of analyses, we used variance partitioning to examine the unique and shared portion of the variance of mathematics explained by the spatial-simultaneous, spatial-sequential and verbal factors. A series of regression analyses were conducted to understand the unique and specific contribution of spatial-simultaneous, spatial-sequential, and verbal WM (see Chuah & Maybery, 1999; Giofrè, Donolato & Mammarella, 2018, for a similar procedure). As shown in Figure 2, only verbal (10.8%) and spatial-simultaneous (3.4%) factors were explaining a unique portion of the variance of mathematics. Not surprisingly, the larger portion of the variance was shared by the three predictors (15.3%). The total amount of variance accounted for by the model was 37.8%. These findings suggest that a large portion of the explained variance in mathematics is shared, however, some domains, i.e. verbal and spatial-simultaneous WM, are uniquely predicting mathematics, over and above the effect of the other WM domains.

Figure 2 about here

Additional analyses

All the analyses were replicated also including "measuring" and the results, not reported, changed very little. Alternative models were tested for WM. In particular, we tested a single WM factor, $\chi^2(20) = 71.91$, $p < .001$, *RMSEA* = .12, *SRMR* = .08, *CFI* = .87, *NNFI* = .82, and a three factor solution, $\chi^2(22) = 22.50$, $p = .16$, *RMSEA* = .04, *SRMR* = .05, *CFI* = .99, *NNFI* = .98. These analyses confirm that the fit of the three factor solution, which was adopted in the current paper, was superior when compared to the other two models.

Discussion

This paper aimed to investigate the independent contribution of verbal, spatial-simultaneous, and spatial-sequential working memory to written mathematical performance in 7- and 8-year old children.

From the correlation matrix (Table 1), it is evident that all elements of working memory (besides those correlations between digit recall and spatial-simultaneous 4 × 4, spatial-sequential 3 × 3, spatial-sequential 4 × 3, and block recall) are significantly correlated. All other correlations between each of the measures taken for verbal, spatial-simultaneous, and spatial-sequential working memory were statistically significant, both before and after covarying for age. Results of this nature suggest digit recall may be measuring a different construct to the other measures used to assess working memory, potentially relating to the division of working memory tasks into active and passive tasks (as explained by Passolunghi & Cornoldi, 2008).

In relation to our research question, variance partitioning demonstrates that 15.3% of the variance of maths is shared between the three factors of working memory concerned. The next largest proportion of variance explained is uniquely explained by verbal measures, explaining 10.8% of the variance. This is interpreted as the amount of variance in mathematics accounted for by verbal measures over and above the influence of all other variables measured. This relationship with verbal working memory is consistent with studies suggesting numerically-based verbal tasks are distinguishable from non-numerical verbal tasks and are directly related to children's mathematical performance (see Raghubar, Barnes & Hecht, 2010 for a review of this literature).

Caution must also be exercised that reading was not measured alongside mathematics, though previous research suggests that the relationship with verbal working memory remains after partialling out reading ability (Wilson & Swanson, 2001; see Simmons, Willis & Adams, 2012 for a similar argument in relation to elements of mathematics). In the current study, the extent of the impact of this was limited by allowing children to have questions of the mathematics test read aloud to them if they wished. Whilst uptake of this offer was not recorded explicitly, children did make use of the adults present to read the questions for them. In line with previous findings, 37.8% of the variance of mathematical ability was accounted for by verbal and visuospatial measures in total (see Giofrè et al., 2018 and Kyttälä & Lehto, 2008 for similar results).

Interestingly, the results did not show any unique variance explained by spatial-sequential working memory. We had anticipated a larger involvement of spatial-sequential working memory due to the additional active component, however were unsure what the extent of this involvement would be. There are a number of potential explanations for this. The first possible explanation is the ease with which such young children could perform the tasks they were required to do. Whilst unlikely as the sole explanation, as we did not see a floor effect in the data, the results did show positive skew, indicating that the majority of children were performing at the lower end of the scale, therefore, they may have encountered some difficulties with the instructions of the tasks. Note that the spatial-sequential task did not require order during the recall phase, whereas the block recall task did, which may have contributed to the last of floor effect seen.Secondly, there is evidence that children with high and low mathematical ability are not distinguishable based on their spatial-sequential working memory scores (Bull, Johnston & Roy, 1999; but see Andersson & Lyxell, 2007; D'Amico & Guarnera, 2005; McLean & Hitch, 1999 for a different argument).

With regard to the contribution made by spatial-simultaneous working memory to mathematics performance, a unique contribution of 3.4% is higher than expected, based on previous literature (e.g., Kyttälä & Lehto, 2008; Swanson & Kim, 2007). This result is a potential by-product of the way in which written mathematics questions are presented in a standard testing procedure. In such a procedure, all information is presented to the child at once and so is available to the child at all times, hence presentation is in line with that of simultaneous working memory measures. Future research could seek to mitigate this effect by presenting a selection of mathematics questions in a sequential format (see Szucs and Csépe, 2004a; Szucs & Csépe, 2004b), to ascertain whether this has any influence over the results gathered. It should be noted that the sample here comprised typically developing children attending mainstream primary schools, none of whom had been identified as exhibiting specific mathematical difficulties. Previous research has identified a relationship between the spatial component of working memory and mathematics (Passolunghi & Mammarella, 2010; 2010), however, this effect has been shown to be stronger in those with a mathematical difficulty (e.g. Mammarella, Caviola, Giofre & Szucs, 2018; Peng, Namkung, Barnes, & Sun, 2016). As such, it would be reasonable to suggest that a more distinct profile may have resulted from the current study had children with mathematical difficulties been included in the sample.

There are some limitations inherent in this study that it will be necessary to address in future work. Regarding the measures used, verbal measures involved the use of number words, which could feasibly have altered the predictive relationship between verbal working memory and mathematics performance. This is of particular significance in an age group in which one would expect dramatic developmental changes. However, the use of such measures is in line with previous work suggesting a component of working memory responsible for numerical information (as reviewed by Raghubar, Barnes & Hecht, 2010), hence the results generated are not entirely unexpected.

Continuing on from this, the study concerned only a narrow age group of typically developing children. As such, it is not possible to examine any longitudinal changes relating to age, or to highlight any differences between typical and atypical populations. In fact, from 7 years of age, there is a shift in mnemonic strategy with the emergence of rehearsal in children (Flavell et al., 1966; Gathercole, 1998; Henry, Messer, Luger-Klein, & Crane, 2012). It is possible that some children might have used some sort of mnemonic strategy during WM tasks. For this reason, the relationship between verbal-numeric working memory and mathematic performance may have been underpinned by some sort of a subvocalization process (e.g., rehearsal or other verbal strategies). Overall, it would appear incorrect to assume that children approach the task in the same way (Flavell et al. 1966; Gathercole, 1998). For all these reasons, future studies should be performed to tackle this issue, for example by trying to reduce the use of strategies during WM tasks.

For tasks that require serial recall there is some suggestion that a common order mechanism is at play (Guerard & Tremblay, 2008). For verbal tasks, it is argued that participants use subvocal rehearsal (e.g., speech-based motor-planning) to maintain the order of to-be-remembered items (e.g., Jones, Hughes, & Macken, 2006). Children involved in this study did appear to use sub-vocal/ vocal rehearsal during the presentation stage, in line with these findings. For visuo-spatial material, the sequence could be maintained via ocular movements (Tremblay, Saint-Aubin, & Jalbert, 2006; Morey, Mareva, Lelonkieqicz, & Chevalier, 2017). There is some evidence (through similar error patterns (Guerard & Tremblay, 2008) and susceptibility to interference from secondary tasks (Jones, Farrand, Stuart, & Morris, 1995)) that the two forms of sequential-order memory have similar underpinnings. It is quite surprising that the verbal sequential task correlated with mathematical performance, but the spatial-sequential task did not. If children are indeed sub-vocally rehearsing, then the relationship to mathematical performance may be attributable to some sort of speech-motor planning (inner speech) that participants engage in when attempting to solve mathematical problems (Rohrkemper, 1986), and not a domain general ordering mechanism (otherwise the spatial-sequential task should have been related to mathematical performance). In a similar vein, several studies indicate that the spatial-sequential WM component tends to be more strongly related to the mathematical performance in both children with typical development and children with dyscalculia (Mammarella, et al. 2018; Passolunghi & Mammarella 2010; 2012).

On possible explanation for this contradictory finding is that, in the particular mathematical task used, simultaneous processes might take precedent over sequential processing. The mathematical test that we decided to use encompass several abilities, e.g., geometry and there is some evidence indicating that some simultaneous tasks, tend to have a greater contribution as compared to other sequential tasks (see Giofrè et al., 2013 on this point). This observation is in line with other evidence indicating that the active manipulation of the stimuli tends to be crucial later on in the curriculum, but not in the early stages (see Giofrè, et al., 2014 on this point). This is also coherent with the observation that in some tasks, such as fractions, holistic strategies, which require the simultaneous manipulation of visual objects, seem to be very effective as compared to other strategies (e.g., Fabbri, Caviola, Zorzi, & Butterworth, 2012). Consistently, evidence shows developmental differences indicating that different mathematical training is effective in different age groups (e.g., Caviola, Gerotto, Mammarella, 2016). Finally, there is some evidence that younger children tend to use less reliable and less efficient strategies prior to a declarative shift in strategy use (see Schneider, 2008 for a review of this), which might have influenced the pattern of results we observed (e.g., Caviola, Mammarella, Pastore, Lefevre, 2018). For all these reasons, the present findings should be replicated using a more diverse sample including children at different levels of the mathematical curriculum.

The findings from this study have important implications for educational research. An understanding of the elements of working memory that support mathematics development is fundamental for educators aiming to improve children’s mathematical attainment. Research is currently trying to exploit this relationship to generate working memory training programmes (e.g., Alloway, 2012; Holmes & Gathercole, 2014). However, at present, randomised controlled trials have not identified evidence of transfer of effects onto academic tasks (e.g., Dunning, Holmes & Gathercole, 2013), though evidence is mixed (see Morrison & Chein, 2011 for a review of this literature). A recent randomised controlled trial by the Education Endowment Foundation (Wright, Dorsett, Anders, Buzzeo, Runge and Sanders, 2019) identified a non-significant positive influence of working memory training programmes on working memory capacity and mathematics performance when teaching working memory strategies. Caution should be applied when interpreting these results, however, as measures of working memory capacity involved predominantly numerical recall tasks, though children did show additional progress on mathematics measures. It would be of great benefit to educators to understand the predictive nature of working memory for individual components of mathematics as this would enable educators to highlight potential areas of vulnerability in their students. In which case, there is scope for the provision of appropriate aids and alternative methods to be put in place in an attempt to alleviate some of the child's difficulties in that particular area.

In conclusion, this study confirmed a positive relationship between working memory tasks and mathematics attainment. Further verbal-numeric tasks appear to be more predictive of mathematics performance when compared directly to spatial-simultaneous and spatial-sequential tasks, suggesting numerical information is of higher predictive value than visual information when the two are compared directly.

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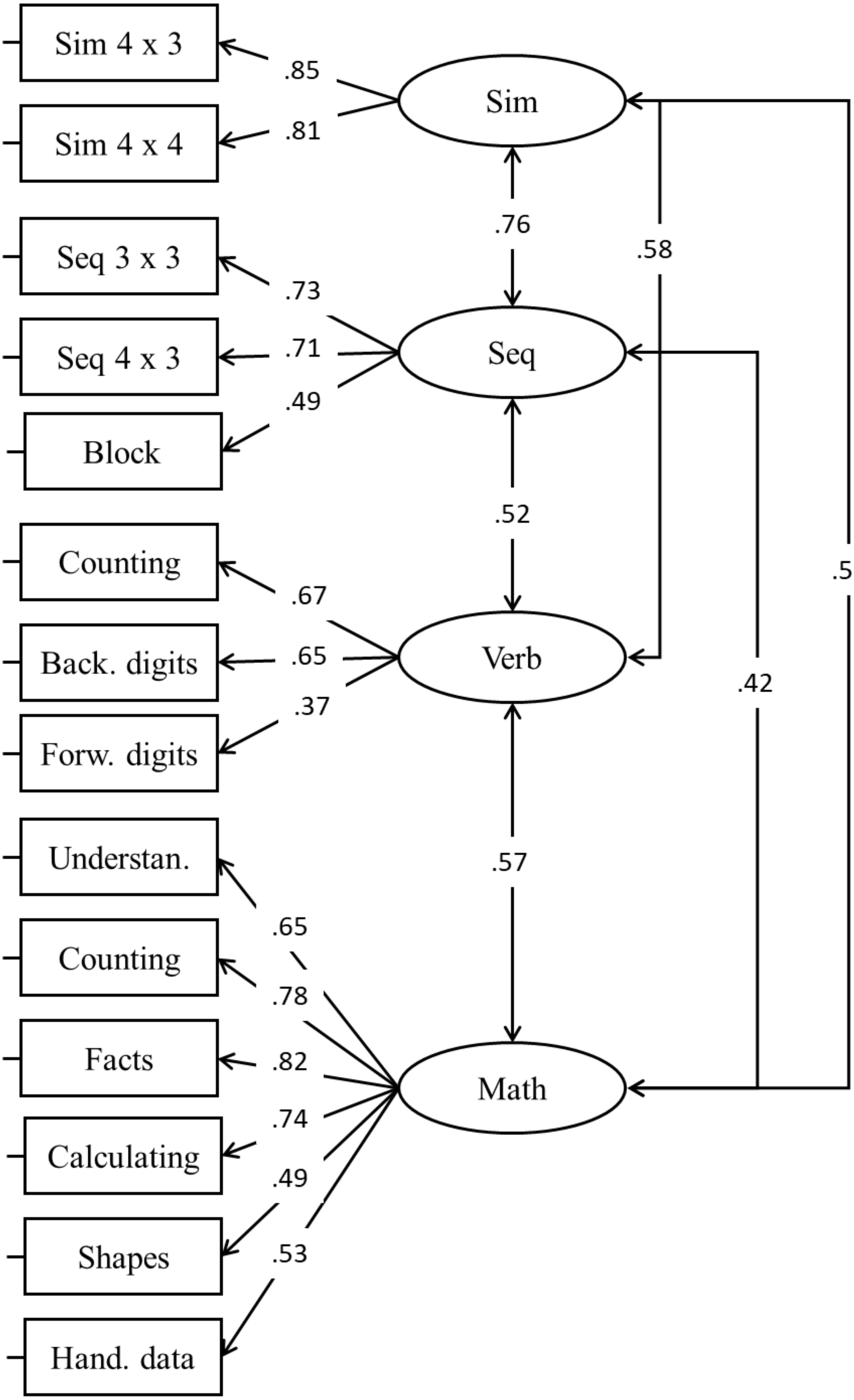

Figure 1. CFA model for spatial-simultaneous, spatial-sequential, verbal and mathematics Coefficients are statistically significant ($p < .05$).

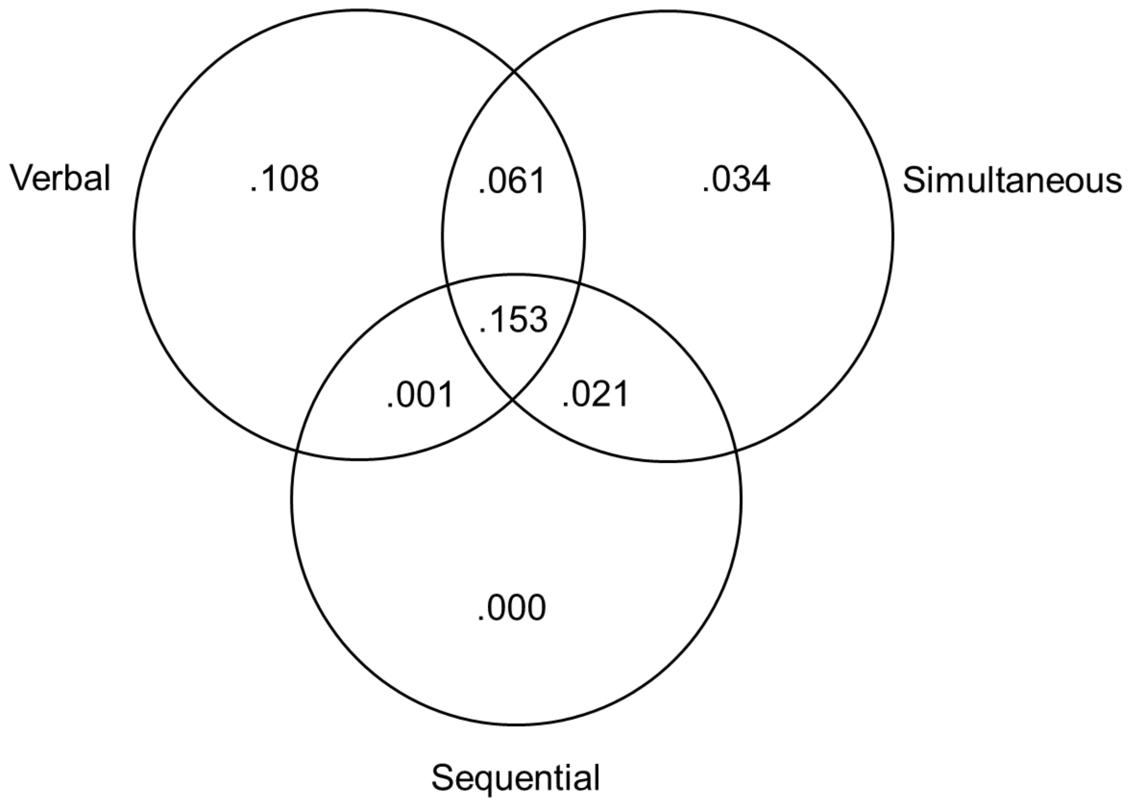

Figure 2. Venn diagram indicating the shared and unique variance explained in mathematics by spatial-simultaneous, spatial-sequential and verbal factors.

Table 1

Correlation matrix with raw score correlations below the leading diagonal and covaried scores above, including means and standard deviations for each measure.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. Simultaneous 4 x 3	—	.685	.471	.420	.375	.339	.330	.143	.302	.312	.363	.238	.323	.315
2. Simultaneous 4 x 4	.681	—	.408	.425	.409	.297	.335	.091	.283	.251	.399	.285	.300	.296
3. Sequential 3 x 3	.470	.408	—	.550	.300	.266	.254	.086	.212	.230	.275	.195	.165	.197
4. Sequential 4 x 3	.418	.425	.550	—	.312	.185	.259	.093	.207	.190	.227	.212	.145	.224
5. Block recall	.379	.409	.301	.313	—	.238	.264	-.026	.224	.116	.175	.159	.030	.140
6. Counting recall	.343	.298	.267	.185	.241	—	.416	.294	.348	.331	.299	.224	.250	.206
7. Backward digit	.333	.336	.255	.259	.266	.418	—	.253	.287	.313	.342	.240	.122	.131
8. Digit recall	.143	.091	.086	.093	-.025	.295	.253	—	.098	.182	.164	.088	-.050	.112
9. Understanding and app	.322	.278	.213	.204	.234	.351	.289	.099	—	.480	.512	.459	.378	.434
10. Count. and underst.	.318	.251	.231	.191	.121	.334	.315	.183	.483	—	.618	.590	.407	.511
11. Knowing and using	.370	.398	.276	.227	.180	.302	.344	.165	.517	.621	—	.645	.378	.470
12. Calculating	.221	.282	.191	.209	.150	.216	.234	.086	.418	.577	.629	—	.359	.403
13. Understanding shape	.322	.300	.165	.145	.031	.250	.123	-.050	.372	.408	.378	.354	—	.200
14. Handling data	.304	.295	.195	.223	.136	.201	.128	.111	.407	.504	.462	.406	.198	—
M	28.5	2.25	18.84	15.42	21.86	16.47	1.56	26.69	1.77	3.18	2.17	1.66	.9	1.36
SD	5.98	6.93	4.76	4.15	3.55	3.92	2.93	3.14	1.24	1.89	1.43	1.24	.98	1.18
Min.	5	1	7	2	6	6	1	17	0	0	0	0	0	0
Max.	42	39	38	29	32	26	19	47	6	9	6	5	4	5

Note. Correlations greater than .14 are statistically significant at the .05 level. All correlations greater than .18 are significant at the .01 level.

Table 2.

Factor loadings, inter-factor and residual correlations for measures included in the model

	Simultaneous	Sequential	Verbal	Math
Simultaneous				
1. Simultaneous 4 x 3	. 845**			
2. Simultaneous 4 x 4	. 811**			
Sequential				
3. Sequential 3 x 3		.727**		
4. Sequential 4 x 3		.705**		
5. Block recall		.486**		
Verbal				
6. Counting recall			.670**	
7. Backward digit recall			.653**	
8. Digit recall			.370**	
Mathematics				
9. Understanding and applying				.648**
10. Counting and understanding number				.778**
11. Knowing and using number facts				.817**
12. Calculating				.742**
13. Understanding shape				.491**
14. Handling data				.593**
Inter-factor correlation matrix				
Simultaneous	1			
Sequential	.758**	1		
Verb	.575**	.518**	1	
Math	.518**	.418**	.568**	1

Note.

** $p < .01$.