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Chasing the cuprates with dilatonic dyons

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ABSTRACT: Magnetic field and momentum dissipation are key ingredients in describing condensed matter systems. We include them in gauge/gravity and systematically explore the bottom-up panorama of holographic IR effective field theories based on bulk Einstein-Maxwell Lagrangians plus scalars. The class of solutions here examined appears insufficient to capture the phenomenology of charge transport in the cuprates. We analyze in particular the temperature scaling of the resistivity and of the Hall angle. Keeping an open attitude, we illustrate weak and strong points of the approach.

KEYWORDS: Holography and condensed matter physics (AdS/CMT), Gauge-gravity correspondence

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1 Introduction and motivation

The ubiquitous presence of a magnetic field in experimental setups and — more importantly — its direct rôle in investigating the strange metal criticality calls for a solid theoretical account. We refer in particular to the recent experimental proposal highlighting that the strange metal physics (and specifically the transport response) could be fruitfully studied focusing on the competition of the scales dictated by the magnetic field and the temperature [1]. One of the main results of our analysis consists in underlying and specifying explicitly the importance of the magnetic field B in affecting equilibrium and transport properties of a strongly coupled field theory with momentum dissipation modeled through gauge/gravity techniques.

The holographic framework we adopt relies upon conjectures and various approximations on which we need to meditate carefully (later sections are dedicated to comments on the delicate points). The aim at stake is to build effective field theories capable of describing the low-temperature behavior of condensed matter systems with particular attention to the scaling properties in T and corrections in B of the various thermodynamic and linear response quantities.¹ The attitude is in general phenomenological or bottom-up. Namely we choose gravity models that in a gauge/gravity spirit are conjectured to be dual to strongly coupled field theories. More precisely, we resort to a wide class of bulk models described by Einstein-Maxwell Lagrangians plus scalars; they feature the “axion-like” scalars accounting for momentum dissipation (and realizing a massive gravity through a Stueckelberg mechanism²) and a “dilaton-like” scalar encoding a non-trivial renormalization group flow.

We focus on infrared solutions only. This means that the various phenomenological functions of the model — specifically, the kinetic functions for the gauge field and momentum-dissipation scalars as well as the potential for the dilaton-like field — are approximated by means of a single exponential term which is the dominant contribution in the infrared. The input parameters into the game are therefore represented by the exponents of these exponential terms alongside the parameters coming from the dilaton black-brane solutions. These latter are in general of the hyperscaling-violating and Lifshitz type, characterized by three scaling exponents. Two of them, θ and z , appear directly in the metric and are associated to the “effective dimension” of the theory and the different scaling of space and time, respectively.³ The third one, ζ , parameterizes the IR anomalous dimension for the charge density operator and it affects directly the scaling of the conductivity⁴ (see [47, 71, 72, 75, 76]).

¹This idea is part of a long-standing program within the holographic community [22, 24, 80].

²See [60, 61] for more general holographic models making use of the Stueckelberg scalars and their connection with theories of massive gravity and Condensed Matter phenomenology. It is relevant to refer also to the wider non-holographic context where similar ideas have been proposed and explored, see for instance [65, 66].

³See also [20–27, 36–38, 62, 63] for previous analyses about transport properties of holographic systems with Lifshitz and hyperscaling violating geometries.

⁴One direct way of producing an anomalous dimension for the gauge field is provided by the unparticles models [73]. See also [74] for an alternative approach.

Such a class of models is at the core of a long-standing program to apply holography to condensed matter systems and constitutes a natural setup to describe IR criticality associated to a scaling fixed point. It provides a flexible and general framework for a phenomenological description of possible gauge/gravity models aiming to exhaustive classifications; notably, a wide subclass of such models features a vanishing residual entropy (further comments on this will prove to be in order). Moreover, some top-down glimpses have encouraged in convincing of the general relevance of this IR setup (see for instance [7]). Roughly, the philosophy consists in taking the minimal set of ingredients and allowing for the most general behavior which does not spoil completely the infrared geometry. All the actors on stage — the chemical potential, the magnetic field and the momentum dissipation device — are allowed to be either marginally relevant (roughly, weighting as much as the dilaton-like scalar in the IR flow) or irrelevant. Specifically, we start from an analytical background solution encoding the marginally relevant quantities and then deform it with perturbative irrelevant operators.⁵

We focus on bulk solutions that are nearly extremal, obtained perturbing slightly their extremal counterparts. Both the temperature and the irrelevant operators are therefore perturbations; focusing on the entropy we show that it is possible to consider both hierarchies and take either T or B^2 as a small parameter when compared to the other scales at play. This argument is based only on dimensional analysis and scaling properties and similar considerations hold for the transport quantities as well.

On the phenomenological side, it is important to recall that experimental evidence [2] shows that the characteristic cuprate scaling features, namely

$$\rho_{xx} \propto T \quad \text{and} \quad \cot \theta_H \propto T^2, \tag{1.1}$$

keep being present and robust also in the regime of strong magnetic field B . It is important to keep in mind however that an arbitrarily low- T regime requires some extra caution about the reliability of the models,⁶ and specifically the supergravity approximation; we discuss these subtleties and the difficulties of providing sharp bounds on T *a priori* of a clear string completion. At the outset it is important to underline that, as long as the charge density is a marginal or irrelevant operator (which are the two cases considered in detail in the present article), the cuprates scalings (1.1) can not be attained. We clarify later this point and comment on possible extensions to cases with a relevant charge density operator.

2 Results

In this paper we mainly consider the cases in which the magnetic field constitutes an irrelevant perturbation and comment on other possibilities. In the models at hand we find that, asking for the magnetic field to be irrelevant, automatically implies that the effective

⁵The method is clarified in the following sections and builds on the techniques described in [3]. The possibility of perturbing by means of relevant operators will be also commented later.

⁶Let us stress that also the experimental results have some low-temperature cut-off; the study of EMD geometries at exactly zero temperature could then be of no immediate phenomenological relevance or, at least, it does not constitute a primary worry.

dimension $d_\theta = d - \theta$ and the Lifshitz exponent z are negative. Even though this fact is strange when one thinks at standard effective quantum field theories, we do not find any “in principle” reason to exclude these solutions on the gravitational side.⁷ Consequently, we consider these black hole solutions as holographic duals of exotic strongly coupled quantum field theories keeping in mind that we do not know any realization of an effective standard quantum field theory where the effective dimension and the Lifshitz exponent are negative.⁸

One important result of the present analysis is that, also in the presence of a magnetic field, the picture outlined in [47, 48] remains valid: namely, as long as the charge density is marginal, the momentum-dissipation part of the conductivity always dominates the transport (in contrast with the basic requirement of [4])⁹ and the Hall angle and the resistivity have the following behavior:

$$\cot \theta_H \propto T^n, \quad \rho \propto T^n, \tag{2.2}$$

which evidently makes it impossible to fit the strange metals picture.

In the scaling geometries analyzed with the techniques adopted in this paper, the only way to achieve a “non-Fermi liquid like” behavior for the transport coefficients, namely different scalings for $\cot \theta_H$ and ρ , is to consider the charge density as an irrelevant perturbation. In such a case, the conductivity at zero net heat current dominates the momentum-dissipation piece leading to different scalings for the Hall angle and the resistivity, namely:

$$\cot \theta_H \propto T^n, \quad \rho \propto T^m. \tag{2.3}$$

However, also in this case, the irrelevance of the charge density operator implies that $m > n$, which excludes the cuprates scenario.¹⁰

In appendix C we extend the analysis to the Seebeck coefficient accounting for the thermopower. We do not consider the magneto-resistance because it is quadratic in B and this coincides with the perturbative order in B that we neglect.

⁷In this direction it would be interesting to perform an analysis along the lines of [62] in the presence of hyperscaling violation. One could also argue that $\theta < 0$ could be no worrisome on the basis that there exist Condensed Matter models featuring *dimensional crossover* [78].

⁸The same holds for a marginal B , see the dyonic solution of [11] which leads to $\theta = 4$.

⁹We refer to the inverse Matthiessen rule (at $B = 0$)

$$\sigma = \sigma_0 + \sigma_{\text{diss}}, \tag{2.1}$$

(see for instance [5]). The σ_0 term refers to the DC contribution at zero net heat current (which, at finite density, is in general different from the contribution at zero momentum current [31]); σ_0 adds up with a contribution σ_{diss} proportional to the charge density and it is directly connected to the momentum-dissipating dynamics. Note however that this splitting of the conductivity in two terms can be quite tricky in generic holographic effective theories. When one couples directly the charged sector with the momentum-dissipating sector, or in the case of a non-linear extension of the Maxwell action, the first term of the conductivity σ_0 can depend on the charge density and the momentum-dissipation rate as well (see e.g. [31, 32, 56, 57]). In order to identify properly the various time scales in the system one can resort to an analysis of the QNM spectrum because the DC transport features are in general not sufficient in providing such information [31].

¹⁰The scalings (2.2) and (2.3) can be predicted from the scalings worked out by [72], keeping in mind that $\Phi = \theta - d$ for EMD solutions with a marginal charge density in the IR, while $\Phi \neq \theta - d$ otherwise (as shown in [47, 71]).

Finally, concerning the case in which the magnetic field is a relevant perturbation, a detailed numerical analysis would be needed.¹¹ However, one can think in analogy with the analysis of [3] and consider that, as long as the momentum-dissipation operator is marginal or irrelevant, the nature of the IR fixed point found in [3] should not be modified. Then one can argue that, also in this case, the cuprate picture cannot be found. In fact, according to [3] the IR fixed point should be of the Reissner-Nordström type. This kind of black hole solutions do not presents the correct scalings for the Hall angle and the resistivity [29].

3 Model

The gravity model we consider features an Einstein-Maxwell action supplemented by “axion-like” scalars whose role is to incorporate momentum dissipation [6] and a “dilaton-like” field possessing a non-trivial potential and appearing in the kinetic functions of the other matter fields. All the fields are electromagnetically neutral. The explicit form of the action is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[R - 2(\nabla\phi)^2 - f(\phi)F_{\mu\nu}F^{\mu\nu} - V(\phi) - \frac{1}{2}Y(\phi) \sum_{I=1}^2 (\nabla\psi^I)^2 \right]. \quad (3.1)$$

The functions $f(\phi)$, $V(\phi)$ and $Y(\phi)$ are “phenomenological”, namely not derived in a top-down sense; the same being true for the field content. Moreover they are assumed to take the following form

$$f(\phi) = e^{2\alpha\phi} \quad V(\phi) = -|V_0| e^{2\delta\phi}, \quad Y(\phi) = e^{2\lambda\phi}. \quad (3.2)$$

This choice corresponds to assuming that a single exponential term dominates in the regime of interest (in the same spirit of the scaling solutions proposed, for instance, in [7]). It is important to underline that the sign of V in (3.2) is chosen to comply with the criterion of [8] when the exponential blows up.

The metric ansatz for the solutions is taken to be¹²

$$ds^2 = -a(r)^2 dt^2 + \frac{dr^2}{a(r)^2} + b(r)^2(dx_1^2 + dx_2^2), \quad (3.3)$$

with

$$a(r) = C_a r^\gamma, \quad b(r) = r^\beta, \quad (3.4)$$

where C_a is a numerical constant. For the bulk gauge field we consider

$$F = \frac{Q}{f(\phi)b(r)^2} dt \wedge dr + B dx_1 \wedge dx_2, \quad (3.5)$$

¹¹For instance an explicit computation is required to shed conclusive light to the interplay of the various scales and the radial region where the transition from the intermediate scaling geometry to the ultimate IR point occurs.

¹²We fix the normalization of the spatial subspace volume to one.

corresponding in general to a dyonic bulk solution charged both electrically and magnetically. The ansatz (3.5) solves the Maxwell equation coming from (3.1) automatically. Eventually for the scalar we consider the following ansatz

$$\psi_I = k x_I, \quad \phi = \kappa \log r. \quad (3.6)$$

namely a logarithmic profile for ϕ and a linearly sourced (radially constant) scalar for each spatial direction whose linear spatial dependence is the source for momentum dissipation [6]. In the dual perspective we have an electric charge density associated to a non-trivial Q and an “off-plane” magnetic field associated to B (implemented as in, e.g., [9]).

The variable r increases towards the boundary of the bulk. In the case of zero-temperature — corresponding to the absence of a horizon — there is no emblackening factor; therefore a naked singularity where ϕ diverges logarithmically appears in the deep IR. Its large value in the IR region constitute the rough motivation on which the hypothesis (3.2) relies; namely a single exponential term setting the behavior of the phenomenological functions. More comments are in order and we postpone them to the Discussion section. For now, let us just underline that (3.2) represents for us an IR hypothesis to build a low-energy effective holographic theory with scaling properties. In fact, the solutions (3.3), (3.5) and (3.6) are not UV complete and would need a matching to an asymptotic UV AdS solution. For this to happen, also the form of the full potential must allow for a minimum at some finite value for ϕ to be reached asymptotically at large r .

The metric ansatz (3.3) presents two parametric exponents γ and β ; as shown in appendix A, they encode θ and z as follows

$$\theta = \frac{2(1-\gamma)}{1-\beta-\gamma}, \quad z = \frac{1-2\gamma}{1-\beta-\gamma}. \quad (3.7)$$

We recall that θ and z are associated respectively to the hyperscaling violation and the different scaling among the time and space directions of the field theory. With (3.3) we are therefore dealing with a hyperscaling violating Lifshitz geometry.

Given the phenomenological functions (3.2), the ansatz (3.6) and the consequent structure of the equations of motions, a concomitant flip of α , δ , λ and κ leaves everything unchanged (a part from an unimportant sign flip in the dilaton profile). One can then restrict to

$$\delta > 0, \quad (3.8)$$

without losing in generality; we take this choice in line with the analysis of [3].

3.1 Equations of motion and effective potential

The equations of motion for the model (3.1) and the metric ansatz (3.3) read

$$\begin{aligned} (a^2 b^2)'' + 2b^2 V(\phi) + \frac{Y(\phi)}{2} \sum_I^2 (\partial_I \psi_I)^2 &= 0, \\ b'' + b \phi'^2 &= 0, \\ (a^2 b^2 \phi')' - \frac{1}{2} \partial_\phi V_{\text{eff}}(\phi) &= 0, \\ -\frac{1}{2} a^2{}' b^2{}' - a^2 b'^2 + a^2 b^2 \phi'^2 - V_{\text{eff}}(\phi) &= 0. \end{aligned} \quad (3.9)$$

The Maxwell equation is trivially satisfied by the ansatz (3.5). The effective potential $V_{\text{eff}}(\phi)$ is given by

$$V_{\text{eff}}(\phi) = \frac{Q^2}{b^2 f(\phi)} + \frac{B^2 f(\phi)}{b^2} + \frac{1}{2} b^2 V(\phi) + \frac{Y(\phi)}{4} \sum_I^2 (\partial_I \psi_I)^2 . \quad (3.10)$$

For more details about the complete set of equations and their expressions on the ansatz (3.2), (3.4), (3.6), we refer the reader to appendix B.

3.1.1 Scaling transformations

The system admits the following scaling transformations (the same as in [3])

$$r = \lambda \tilde{r} , \quad (3.11)$$

$$t = \lambda^{1-2\gamma} \tilde{t} , \quad (3.12)$$

$$x_I = \lambda^{1-\gamma-\beta} \tilde{x}_I , \quad (3.13)$$

upon which the metric acquires a conformal factor

$$ds^2 = \lambda^{2-2\gamma} \left\{ -C_a^2 \tilde{r}^{2\gamma} d\tilde{t}^2 + \frac{d\tilde{r}^2}{C_a^2 \tilde{r}^{2\gamma}} + \tilde{r}^{2\beta} (d\tilde{x}_1^2 + d\tilde{x}_2^2) \right\} ; \quad (3.14)$$

the dilaton is instead shifted

$$\phi(r) = \kappa \log(\tilde{r}) + \kappa \log(\lambda) = \phi(\tilde{r}) + \kappa \log(\lambda) . \quad (3.15)$$

This scaling behavior is a property of the IR solutions. Namely, the scaling maps IR solutions into IR solutions with rescaled parameters. To specify the scaling of the parameters one considers the various terms in the equations of motion (referring to the explicit form of $V_{\text{eff}}(\phi)$) and requires that they rescale in the same way. Later we will rely on this to fix the leading corrections to the thermodynamics due to irrelevant perturbations.

3.2 Warm-up

A finite temperature can be realized modifying the g_{tt} component of the metric introducing an emblackening factor,

$$a^2(r) = C_a^2 r^{2\gamma} \left[1 - \left(\frac{r_h}{r} \right)^{2\beta+2\gamma-1} \right] , \quad (3.16)$$

where r_h is the horizon radius of the black hole. Following [3], the specific form of the emblackening factor (3.16) is obtained by asking that the infrared equations of motion of the zero-temperature case are not modified by the introduction of a black hole horizon. The black hole temperature is

$$T = \left. \frac{(a^2)'}{4\pi} \right|_{r=r_h} = \frac{C_a^2}{4\pi} (2\beta + 2\gamma - 1) r_h^{2\gamma-1} . \quad (3.17)$$

Having assumed that the introduction of a small temperature does not affect the extremal geometry, the temperature must be small in comparison to the other scales of the system. We will be more precise on this point in discussing the thermodynamics of the solutions.

Finally, the entropy of the black hole is given as usual by the area law, namely

$$S \propto T^{\frac{2\beta}{2\gamma-1}} \propto T^{\frac{d_\theta}{z}}, \quad (3.18)$$

where d_θ is the effective dimension of the dual theory given by

$$d_\theta = 2 - \theta. \quad (3.19)$$

3.3 DC transport

The analytic computation of the DC transport coefficients relies on the “membrane paradigm” (in its generalized version to momentum-dissipating systems), namely the fact that the zero-frequency currents are radially conserved [17–19]. Notice that this plays a particularly important rôle in a circumstance — like the present one — where one does not dispose of a complete knowledge of the backgrounds.

We follow closely the computations of [28] to which we refer.¹³ The full set of DC transport coefficients reads

$$\sigma_{xx} = \frac{k^2 b^2 Y (B^2 f^2 + k^2 b^2 Y f + q^2)}{\Delta} \Big|_h, \quad \sigma_{xy} = \frac{B q (B^2 f^2 + 2 k^2 b^2 Y f + q^2)}{\Delta} \Big|_h \quad (3.20)$$

$$\alpha_{xx} = \frac{k^2 q S b^2 Y}{\Delta} \Big|_h, \quad \alpha_{xy} = \frac{B S (B^2 f^2 + k^2 b^2 Y f + q^2)}{\Delta} \Big|_h \quad (3.21)$$

$$\bar{\kappa}_{xx} = \frac{S^2 T (B^2 f + k^2 b^2 Y)}{\Delta} \Big|_h, \quad \bar{\kappa}_{xy} = \frac{B q S^2 T}{\Delta} \Big|_h \quad (3.22)$$

where the subscript h means that the quantities are evaluated at the horizon; we have defined

$$\Delta = (B^2 f + k^2 b^2 Y)^2 + B^2 q^2, \quad (3.23)$$

in analogy with [34] and we have also indicated the charge density with q and the entropy density with S ; these take the form

$$q = -f b^2 A'_t, \quad S = 4\pi b^2 \Big|_h, \quad (3.24)$$

accordingly to the metric (3.3). To show that q is the charge density one first notices that q is radially conserved because of the Maxwell equation and — when evaluated at a would-be $AAAdS$ boundary — the two quantities would coincide.

From (3.20) one can derive the resistivity and the Hall angle obtaining

$$\rho_{xx} = k^2 b^2 Y \frac{4 k^2 b^2 f Y + 16 B^2 f^2 + q^2}{16 f^2 (k^4 b^4 Y^2 + B^2 q^2) + 8 k^2 q^2 b^2 f Y + q^4} \Big|_h, \quad (3.25)$$

$$\tan \theta_H = \frac{B q}{k^2 b^2 Y} \frac{8 k^2 b^2 f Y + 16 B^2 f^2 + q^2}{4 k^2 b^2 f Y + 16 B^2 f^2 + q^2} \Big|_h. \quad (3.26)$$

¹³Similar analyses are present also elsewhere in the literature [29, 30, 33].

Inverting (3.17) we have

$$r_h = (\zeta T)^{\frac{1}{2\gamma-1}}, \quad (3.27)$$

where we have defined

$$\zeta = \frac{4\pi}{C_a^2} \frac{1}{2\beta + 2\gamma - 1}, \quad (3.28)$$

to avoid clutter.

4 Magnetic field as an irrelevant perturbation

In this section we focus on solutions where the magnetic field is an irrelevant perturbation that does not alter the nature of the IR fixed point described by the unperturbed gravitational solution. In particular, we are concerned with two types of solution: charged solutions where the electric charge density is marginal, and neutral solutions where both the charge density and the magnetic field are irrelevant perturbations.¹⁴

4.1 Charged solutions

In order to search for purely electric solutions, we set B to zero in (3.5). We require k , α and V_0 to be non-vanishing while δ to be positive as chosen in (3.8). Referring to the effective potential (3.10) on the ansatz, we impose that the exponents of the three remaining terms (the magnetic term vanishes as $B = 0$) are equal. This amounts to requiring that the operators associated to Q and to the scalars ψ_I are marginal. The remaining parameters are fixed by the equations of motion (3.9) to be

$$\begin{aligned} \kappa &= -\frac{2(\alpha + \delta)}{(\alpha + \delta)^2 + 4}, & \lambda &= \frac{\delta - \alpha}{2}, \\ \gamma = 1 + \delta\kappa &= 1 - \frac{2\delta(\alpha + \delta)}{(\alpha + \delta)^2 + 4}, & \beta = 1 - \gamma + \kappa\lambda &= \frac{(\alpha + \delta)^2}{(\alpha + \delta)^2 + 4}. \end{aligned} \quad (4.1)$$

along with

$$Q^2 = \frac{k^2(-\alpha^2 + \delta^2 - 4) - 2\delta|V_0|(\alpha + \delta) + 4|V_0|}{4[\alpha(\alpha + \delta) + 2]}, \quad (4.2)$$

$$C_a^2 = -\frac{[(\alpha + \delta)^2 + 4]^2(k^2 - 2|V_0|)}{4[\alpha(\alpha + \delta) + 2][(3\alpha - \delta)(\alpha + \delta) + 4]}. \quad (4.3)$$

The limit $k \rightarrow 0$ coincides with the solution found in [3].

Since β in (4.1) is manifestly positive, requiring a positive specific heat from (3.18) corresponds to

$$\gamma > \frac{1}{2}. \quad (4.4)$$

¹⁴These solutions have been already considered in the literature, see [22, 24, 47, 48].

We also have to demand that the squared parameters given by (4.2) and (4.3) are positive. We factorize $|V_0|$ and define $\xi = k^2/|V_0|$; with simple manipulations we get

$$\begin{aligned} 2 + \alpha(\alpha + \delta) &> 0, \\ 2 - \delta(\alpha + \delta) &> 0, \\ 4 + (3\alpha - \delta)(\alpha + \delta) &> 0, \\ 4 + (\alpha - 3\delta)(\alpha + \delta) &> 0, \end{aligned} \tag{4.5}$$

and

$$0 \leq \xi < 2 \frac{2 - \delta(\delta + \alpha)}{4 + \alpha^2 - \delta^2} < 1. \tag{4.6}$$

Considering the vanishing k limit we recover again the results described in [3] of which the present solution represents a finite k generalization. The validity domain (4.5) satisfies the null energy conditions. Relying on the map (3.7) and relations (4.1), one can express (4.5) in terms of the physical quantities z and θ ,

$$\begin{aligned} z < 0 \wedge \theta > 2 + z &\quad \text{or} \\ 1 < z \leq 3 \wedge \theta < 2z - 2 &\quad \text{or} \\ z > 3 \wedge \theta < 1 + z &\quad \end{aligned} \tag{4.7}$$

and

$$0 \leq \xi < \frac{2(z - 1)}{2z - \theta}. \tag{4.8}$$

It is important to stress that in these new variables one has to additionally require $\kappa^2 > 0$, after which the null energy conditions (see [11] for a derivation)

$$(z - 1)(z + 2 - \theta) \geq 0, \tag{4.9}$$

$$(\theta - 2)(\theta + 2 - 2z) \geq 0, \tag{4.10}$$

are automatically satisfied.

4.1.1 Perturbing with a magnetic field

We perturb the charged solutions (4.1) by means of a small magnetic field B and focus on the case where it is dual to an irrelevant operator. First, we compare the various terms in the effective potential $V_{\text{eff}}(\phi)$ in (3.10) and ask for the term in B to be small compared to all the other terms, namely

$$\frac{B^2}{Q^2} \ll e^{-4\alpha\phi}, \quad \frac{B^2}{k^2} \ll \frac{b^2}{2} e^{2(\lambda - \alpha)\phi}, \quad \frac{B^2}{|V_0|} \ll \frac{b^4}{2} e^{-2\phi(\alpha - \delta)}. \tag{4.11}$$

Considering the ansatz (3.3), we obtain

$$\frac{B^2}{Q^2} \ll r^{-4\alpha\kappa}, \quad \frac{B^2}{k^2} \ll \frac{r^{-4\alpha\kappa}}{2}, \quad \frac{B^2}{|V_0|} \ll \frac{r^{-4\alpha\kappa}}{2}; \tag{4.12}$$

these encode the fact that B is a perturbation. The purely charged solution (4.1) can be considered as a reliable solution of the Einstein equations in a range of the radial coordinates

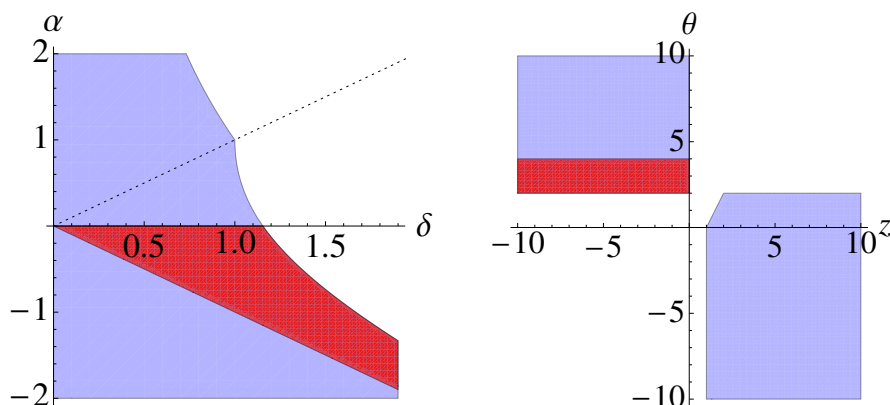


Figure 1. The colored areas correspond to the domain of validity of the charged solution depicted in the (δ, α) -plane (*left*) and in the (z, θ) -plane (*right*). The red areas represent the restriction to the region where B corresponds to an irrelevant operator. The dotted line corresponds to $\alpha = \delta$.

where the conditions (4.12) are satisfied. The appearance of the same exponent in all three conditions (4.12) is a direct consequence of the marginality of Q , k and V .

We focus on cases where the magnetic field is an irrelevant perturbation and (4.12) are satisfied all the way down to the IR, namely we consider

$$\alpha(\alpha + \delta) < 0, \quad (4.13)$$

which corresponds to $\alpha\kappa > 0$ upon using (4.1). Translating this condition in terms of θ and z , one obtains

$$z \leq 2 \quad \wedge \quad 2 < \theta < 4 \quad \text{or} \quad (4.14)$$

$$2 < z < 3 \quad \wedge \quad 2z - 2 < \theta < 4. \quad (4.15)$$

This, once intersected with the regime of validity of the charged solution (4.7), yields

$$z < 0 \quad \wedge \quad 2 < \theta < 4; \quad (4.16)$$

to be considered together with (4.8). The validity domain of the charged solution where B corresponds to a dual irrelevant operator is shown in red in figure 1.

4.1.2 Thermodynamics

To understand the interplay of the different scales in the charged solution (4.1), it is important to discuss the thermodynamic corrections due to the introduction of the B perturbation. To this end we rely on the scaling symmetry (3.11), (3.12), (3.13) of the IR solution. Since μ and k are UV quantities, they do not rescale. Notice that for k the statement is particularly non-trivial because k appears also as a parameter in the IR. We come back to this shortly.

From (3.18) and dimensional analysis we have that the entropy density S has to scale as follows:

$$S \propto \bar{\mu}^2 \left(\frac{T}{\bar{\mu}} \right)^{\frac{2\beta}{2\gamma-1}}. \quad (4.17)$$

The UV scale $\bar{\mu}$ is defined by

$$\bar{\mu} = \mu h\left(\frac{k}{\mu}\right), \quad (4.18)$$

where μ is the chemical potential, k the momentum dissipation scale and h is a dimensionless function of the dimensionless ratio k/μ which is analytical in k and satisfies

$$h(k/\mu) \rightarrow 1 \quad \text{for} \quad k \rightarrow 0; \quad (4.19)$$

this last condition is imposed asking that we reduce to the analysis of [3] when we switch off the momentum dissipation.

Since B enters always quadratically in the equations of motion, the first correction to the entropy has to be quadratic in B . Such leading correction term must be dimensionless and scaling invariant. We have first to determine the scaling of B , which is derived asking that all the terms in $V_{\text{eff}}(\phi)$ scale as the other terms of the equations of motion where $V_{\text{eff}}(\phi)$ appears. From this we have that upon rescaling

$$B \rightarrow B r^{2\beta+\kappa(\delta-\alpha)} = B r^{-2\alpha\kappa}, \quad (4.20)$$

where we have used relations (4.1) among the exponents. Eventually, the correction to the entropy density due to the introduction of B are given by

$$S \propto \bar{\mu}^2 \left(\frac{T}{\bar{\mu}}\right)^{\frac{2\beta}{2\gamma-1}} \left[1 + s_1 \left(\frac{B}{\bar{\mu}^2}\right)^2 \left(\frac{T}{\bar{\mu}}\right)^{\frac{4\alpha\kappa}{2\gamma-1}}\right], \quad (4.21)$$

where s_1 is a constant parameter whose determination would require the analysis of the backreaction of B on the geometry.

Finally, the condition for the entropy correction to be small is given by:

$$\left(\frac{T}{\bar{\mu}}\right)^{\frac{4\alpha\kappa}{2\gamma-1}} \frac{B^2}{\bar{\mu}^4} \ll 1. \quad (4.22)$$

It is important to note that, due to the irrelevance of the magnetic perturbation, the exponent $\frac{4\alpha\kappa}{2\gamma-1}$ is always positive. As long as the condition (4.22) is satisfied, one can consider either a regime where T is small compared to the magnetic field B or the other way around.

4.1.3 DC transport on the charged solutions

In this section we discuss the temperature scalings of the longitudinal electric resistivity and Hall angle on the charged solutions (4.1). Inserting the phenomenological functions (3.2) into expressions (3.26) and working up to linear order in B , we get

$$\rho_{xx} = \frac{k^2 (\zeta T)^{2\frac{\beta+\kappa\lambda}{2\gamma-1}}}{q^2 + 4k^2 (\zeta T)^{2\frac{\beta+\kappa(\alpha+\lambda)}{2\gamma-1}}} + \mathcal{O}(B^2), \quad (4.23)$$

and

$$\tan \theta_H = \frac{Bq}{k^2} (\zeta T)^{-2\frac{\beta+\kappa\lambda}{2\gamma-1}} \frac{q^2 + 8k^2 (\zeta T)^{2\frac{\beta+\kappa(\alpha+\lambda)}{2\gamma-1}}}{q^2 + 4k^2 (\zeta T)^{2\frac{\beta+\kappa(\alpha+\lambda)}{2\gamma-1}}} + \mathcal{O}(B^2). \quad (4.24)$$

Using the relations coming from the charged background solution (4.1) to rewrite the exponents in (4.23) and (4.24),

$$2 \frac{\beta + \kappa(\alpha + \lambda)}{2\gamma - 1} = 0, \quad 2 \frac{\beta + \kappa\lambda}{2\gamma - 1} = \frac{4\alpha(\alpha + \delta)}{4 + (\alpha - 3\delta)(\alpha + \delta)}, \quad (4.25)$$

we obtain

$$\rho_{xx} = \frac{k^2}{q^2 + 4k^2} (\zeta T)^{\frac{4\alpha(\alpha + \delta)}{4 + (\alpha - 3\delta)(\alpha + \delta)}} + \mathcal{O}(B^2), \quad (4.26)$$

and

$$\tan \theta_H = \frac{Bq}{k^2} \frac{q^2 + 8k^2}{q^2 + 4k^2} (\zeta T)^{-\frac{4\alpha(\alpha + \delta)}{4 + (\alpha - 3\delta)(\alpha + \delta)}} + \mathcal{O}(B^2). \quad (4.27)$$

It is crucial to note that the scalings of the transport coefficients arising from the class of charged solutions perturbed with an irrelevant magnetic field are of the *Fermi liquid* type, namely

$$\rho_{xx} \propto T^n, \quad \tan \theta_H \propto T^{-n}. \quad (4.28)$$

Within this class of solutions there is no room for a *non-Fermi liquid* behavior and specifically for a holographic accommodation of the cuprate scalings,

$$\rho_{xx} \propto T, \quad \tan \theta_H \propto T^{-2}. \quad (4.29)$$

It is interesting to discuss whether the solutions in the charged class provide a metallic or an insulating behavior. To this purpose, we refer to the following definitions

$$metal: \frac{d\rho_{xx}}{dT} > 0, \quad insulator: \frac{d\rho_{xx}}{dT} < 0. \quad (4.30)$$

From (4.26) we have that the resistivity in this class scales like $\rho_{xx} \propto T^{m_1}$ with:

$$m_1 = \frac{4\alpha(\alpha + \delta)}{4 + (\alpha - 3\delta)(\alpha + \delta)} < 0.$$

Therefore this class of solutions corresponds to insulating systems¹⁵ and the hunt for the *strange metal* phenomenology has to consider other setups.

4.2 Neutral solutions

The analysis of the transport properties of the charged solutions described in subsection 4.1.3 shows that the temperature scaling of ρ (4.26) and $\cot \theta_H$ (4.27) is the same. This is due to the fact that the electric conductivity is dominated by the momentum-dissipation physics which determines the scale of $\cot \theta_H$ as well.

In order to have different temperature scalings for ρ and $\cot \theta_H$ we consider a geometry in which both the charge density and the magnetic field are perturbative (and irrelevant). The momentum-dissipation term in the longitudinal conductivity is controlled by the square of the parameter Q which is now perturbative.

¹⁵This statement holds only if one insists on an irrelevant magnetic field. Allowing for the magnetic field to be marginal or relevant might change the picture.

It is important to stress that, even though the solution at hand allows for the possibility of considering both B and Q of the same order of magnitude, we restrict only to cases where $Q \gg B$. This choice is in line with generic experimental circumstances.

We solve Einstein equations (B) with both B and Q set to zero obtaining

$$\begin{aligned} \kappa &= \frac{\lambda - \delta}{(\delta - \lambda)^2 + 1}, & \gamma &= 1 + \kappa\delta = 1 + \frac{\delta(\lambda - \delta)}{(\delta - \lambda)^2 + 1}, \\ \beta &= 1 - \gamma + \kappa\lambda = \frac{(\delta - \lambda)^2}{(\delta - \lambda)^2 + 1}, \end{aligned} \tag{4.31}$$

and

$$C_a^2 = \frac{|V_0| ((\delta - \lambda)^2 + 1)^2}{2[1 - \lambda(\delta - \lambda)] (\delta^2 - 4\delta\lambda + 3\lambda^2 + 1)}, \tag{4.32}$$

$$k^2 = |V_0| \frac{\delta(\lambda - \delta) + 1}{\lambda^2 - \delta\lambda + 1}. \tag{4.33}$$

The domain of validity of these neutral solutions is obtained by demanding the positivity of the squared quantities C_a^2 and k^2 along with the positivity of the specific heat (3.18); this leads to

$$\begin{aligned} 1 - \lambda(\delta - \lambda) &> 0, \\ 1 + \delta(\lambda - \delta) &> 0, \\ 1 + (\lambda + \delta)(\lambda - \delta) &> 0, \\ 1 + (\delta - 3\lambda)(\delta - \lambda) &> 0. \end{aligned} \tag{4.34}$$

We still refer to the choice $\delta > 0$ commented around Equation (3.8).

The domain of validity of the neutral solutions (4.34) can be expressed in terms of θ and z and, as in the charged case, one has to consider explicitly the null energy conditions (4.9); eventually one obtains for the neutral solutions the same validity domain on the (z, θ) -plane as for the charged solutions (4.1), namely the colored region (both blue and red areas) of the right plot in figure 1.

4.2.1 Perturbing with a magnetic field and a charge density

The irrelevance of the magnetic field and and the charge density means that the corresponding terms in $V_{\text{eff}}(\phi)$ (3.10) become smaller and smaller towards the IR faster than the pure dilatonic term. In addition, Q and B can be considered to introduce small perturbations of the neutral solutions (4.31) in the radial interval where the following conditions are satisfied:

$$\frac{B^2}{k^2} \ll \frac{1}{2} r^{2\beta+2(\lambda-\alpha)\kappa}, \quad \frac{B^2}{|V_0|} \ll \frac{1}{2} r^{4\beta-2\kappa(\alpha-\delta)}. \tag{4.35}$$

and

$$\frac{Q^2}{k^2} \ll \frac{1}{2} r^{2\beta+2(\lambda+\alpha)\kappa}, \quad \frac{Q^2}{|V_0|} \ll \frac{1}{2} e^{4\beta+2\kappa(\alpha+\delta)}. \tag{4.36}$$

Again, the case where both the electric and the magnetic perturbations are irrelevant corresponds to having the conditions (4.35) and (4.36) valid all the way down to the IR.¹⁶ Explicitly, the irrelevance requirements correspond to

$$\begin{aligned} (\lambda - \delta)(2\lambda - \delta + \alpha) &< 0, \\ (\lambda - \delta)(2\lambda - \delta - \alpha) &< 0. \end{aligned} \tag{4.37}$$

4.2.2 Thermodynamics

The analysis of the Q and B corrections to the thermodynamics (specifically, to the entropy density) instructs us about the relations among the different physical scales of the system and about the regime of validity of the perturbative approximations.

The analysis is analogous to that performed already for the charged solutions. At zero order in Q and B , the temperature scaling of the entropy density is obtained combining the ansatz for the unperturbed solution (3.3) with the relation connecting the temperature and the horizon radius (3.17); one obtains

$$S \propto \bar{\mu}^2 \left(\frac{T}{\bar{\mu}} \right)^{\frac{2\beta}{2\gamma-1}}. \tag{4.38}$$

The appropriate insertions of $\bar{\mu}$ (see (4.18) and comments around it) follow from dimensional considerations.

Relying upon the scaling symmetry (3.1.1) and enforcing that all the terms in $V_{\text{eff}}(\phi)$ scale as the other terms of the equations of motion where $V_{\text{eff}}(\phi)$ appears, we obtain that both Q and B have to scale as follows:

$$B \rightarrow B r^{2\beta+\kappa(\delta-\alpha)} = B r^{-\kappa(\alpha+\delta-2\lambda)}, \tag{4.39}$$

$$Q \rightarrow Q r^{2\beta+\kappa(\delta+\alpha)} = Q r^{\kappa(\alpha-\delta+2\lambda)}. \tag{4.40}$$

The corrections to the entropy have to be quadratic in Q and B (as these quantities enter quadratically in the bulk equations of motion); relying on dimensional analysis and the scalings (4.39) and (4.40), we find the following form for the leading corrections to the entropy density (4.38):

$$S \propto \bar{\mu}^2 \left(\frac{T}{\bar{\mu}} \right)^{\frac{2\beta}{2\gamma-1}} \left[1 + s_1 \left(\frac{B}{\bar{\mu}^2} \right)^2 \left(\frac{T}{\bar{\mu}} \right)^{2\kappa \frac{\alpha+\delta-2\lambda}{2\gamma-1}} + s_2 \left(\frac{Q}{\bar{\mu}^2} \right)^2 \left(\frac{T}{\bar{\mu}} \right)^{-2\kappa \frac{\alpha-\delta+2\lambda}{2\gamma-1}} \right], \tag{4.41}$$

where s_1 and s_2 are two numerical parameters which could be fixed considering the back-reaction of Q and B on the background. Finally, in order for both the correction terms in (4.41) to be small perturbations, the various scales into play are constrained by the following relations:

$$\left(\frac{B}{\bar{\mu}^2} \right)^2 \left(\frac{T}{\bar{\mu}} \right)^{2\kappa \frac{\alpha+\delta-2\lambda}{2\gamma-1}} \ll 1, \quad \left(\frac{Q}{\bar{\mu}^2} \right)^2 \left(\frac{T}{\bar{\mu}} \right)^{-2\kappa \frac{\alpha-\delta+2\lambda}{2\gamma-1}} \ll 1. \tag{4.42}$$

¹⁶In (4.36) and (4.35) one obtains the same r exponent when comparing with both k^2 and $|V_0|$ (one can appreciate this explicitly upon using relations (4.31)). This fact descends directly from the marginality of both the momentum-dissipating scalars and the dilaton.

Similarly to what obtained in (4.22), due to the irrelevance of the perturbations, the exponents of $T/\bar{\mu}$ appearing in both the previous relations are positive; analogous comments made there hold here as well.

4.2.3 DC transport on the neutral solutions

We now analyze the transport properties of the neutral solution (4.31). The electric resistivity and the Hall angle (3.26) on this solution take the form

$$\rho_{xx} = \frac{1}{4}(\zeta T)^{\frac{2\alpha(\lambda-\delta)}{\delta^2-\lambda^2-1}}, \quad \tan \theta_H = \frac{2Bq}{k^2}(\zeta T)^{\frac{2(2\lambda-\delta)(\lambda-\delta)}{\delta^2-\lambda^2-1}}. \quad (4.43)$$

We recall that, since both the charge density and the magnetic field are perturbations, we are working up to linear order in q and B . In particular, let us emphasize that the electric DC conductivity is no longer dominated by the momentum-dissipating term; this feature leaves room for a possible *non-Fermi liquid* behavior. As already anticipated, in this scenario the scalings of the resistivity ρ_{xx} and the Hall angle $\cot \theta_H$ are *a priori* different. Nevertheless, demanding the two scalings to fit the cuprate behavior, namely

$$\frac{2\alpha(\lambda-\delta)}{\delta^2-\lambda^2-1} = 1, \quad \frac{2(2\lambda-\delta)(\lambda-\delta)}{\delta^2-\lambda^2-1} = -2, \quad (4.44)$$

one finds no solution that lie inside the domain of the neutral backgrounds (4.34), (4.37). In conclusion, despite the possibility of having a *non-Fermi liquid* behavior in the class of neutral solutions, it is anyhow not possible to accommodate the cuprate phenomenology.

The scaling of the electric resistivity within this class goes like $\rho_{xx} \propto T^{m_2}$ where:

$$m_2 = \frac{2\alpha(\lambda-\delta)}{\delta^2-\lambda^2-1}.$$

Being the denominator negative definite on the regime of definition of the solutions, the requirements of having a metal or an insulator translate into:

$$\text{metal: } \alpha(\delta-\lambda) < 0, \quad \text{insulator: } \alpha(\delta-\lambda) > 0.$$

Both the conditions have non null overlap with the regime of validity (4.34) and the conditions (4.37) for the irrelevance of the perturbations; within this class both metallic and insulating behaviors are then allowed.

5 Discussion

5.1 Other backgrounds

The analysis so far has delved into two backgrounds where the momentum-dissipating scalars were always dual to marginal deformations of the quantum field theory. Moreover, the magnetic field has always been considered as an irrelevant perturbation. Some comments on other setups and their physical features is in order.

First, one can consider cases where the momentum-dissipating scalars are perturbations dual to irrelevant operators. The unperturbed background can be either charged or neutral.

The analysis proceeds similarly to the cases presented explicitly above however, being now the momentum-dissipating device perturbative, the leading order of the corrections to the thermodynamics is k^2 (to be seen as the counter-parts of the B^2 or Q^2 corrections discussed above). The computation of the transport coefficients yields a longitudinal resistivity which goes as k^2 at leading order. This is however the order at which the momentum-dissipating scalars would back-react on the background. Before performing a deeper perturbative analysis, it is therefore unclear whether this result concerning the resistivity could be relied upon.

Another possibility is to consider a magnetically charged background (with marginal momentum-dissipating scalar); this being in spirit the electro-magnetic dual of the charged solution analyzed above. The analysis of such a setup does not present technical obstructions, nevertheless is arguably at odds with typical experimental circumstances as it would feature a magnetic field dominating over all the other scales along the entire portion of renormalization flow described by the holographic background.¹⁷ These cases, which are physically relevant in general, appear not to be interesting in view of describing the cuprate phenomenology, they do not accommodate the right scalings in T of the resistivity and Hall angle.

5.2 UV

All the solutions presented in this paper are based on the hypothesis that the field ϕ is marginally relevant and that its absolute value becomes large at the horizon. To the aim of having an IR effective picture, we then assumed that the complete potential for ϕ is dominated by one single term (3.2); therefore (3.1) is to be regarded as in IR effective theory, namely a deep bulk model.

The proper definition of the field/operator map requires in general a precise knowledge of the UV behavior of the fields and a full-fledged holographic renormalization procedure. Our solutions and even our bulk Lagrangian (3.1) are however not physically trustworthy in the UV.¹⁸ To amend for this, we could in principle resort to a UV completion where the full potential for ϕ features minima at finite field values and admits $AAdS$ solutions. In other words, one could repeat a program along the lines of [3, 15] extended to comprehend the momentum-dissipating scalars. Instead of investigating this possibility of completing or matching the IR solution (and theory) with some UV counterpart, we assume AdS asymptotics and argue about the insensitivity of our results to the UV detail.¹⁹

A fundamental property of the DC currents that has been considered here consists in being radially constant in line with a generalized version of the “membrane paradigm” [17–19]. Such crucial feature should be respected in a would-be UV completion. Suppose that we completed the theory in the UV by means of a bigger model admitting $AAdS$ solutions. Given the dimensionality of interest, it is known that both the electric current and the heat current are finite. They do not need to be renormalized and, importantly,

¹⁷More precisely, a marginal B could be comparable with the momentum-dissipating sector but dominating by construction over a perturbative charge density and temperature.

¹⁸A systematic program of Lifshitz holographic renormalization is described in [12–14].

¹⁹Relevant comments on similar issues are contained in the recent paper [16].

there are no finite counter-terms leading to ambiguities in their definition [35]. The same is true for the associated conductivities too. So, on the basis of an assumed completion featuring a UV conformal fixed point (namely AdS asymptotics) and the membrane paradigm, we can argue that the current operators that we define and their associated conductivities are the same that we would define in a complete theory with full knowledge of the solutions.

5.3 IR

The $T = 0$ temperature solutions have a mild (namely, logarithmic) naked singularity and one could question its acceptability. A definite answer to this would probably require a precise string completion of the bulk model. This is of course beyond the aim of the present paper, so we resort to the standard “bottom-up” criteria described in [8]: the boundedness of the potential from *above*, and the possibility of obtaining the extremal solution as a zero-temperature limit of black-brane solutions (what usually goes under the name of cloaking the singularity with a horizon).²⁰ As we have already noted, our solutions pass the former criterion because we take $V \sim -|V_0|e^{2\delta\phi}$ in (3.2). The second criterion is passed as well; we obtained the nearly-extremal solutions finding a suitable one-parameter deformation of the $T = 0$ solutions. Therefore, by construction, we have that the extremal solutions are obtained as the $T = 0$ limit of the one-parameter family itself.

The acceptability (in string theory) of the singularity does not instruct us about how actually to resolve it. In other words, the approximation regarding the description by means of a classical and two-derivative gravity could breakdown close to the singularity. Namely the IR problems could be of two kinds: large curvatures and quantum corrections. These in turn could imply a temperature threshold below which the model is not reliable. On this issues we borrow the comments on how to tame the singularity described in [39] to which we refer. In fact, the presence of additional scalars does not change the qualitative bottom-up picture. The moral is that the large N limit helps in controlling both issues even though conclusive answers could only be provided by precise string insight. Indeed the quantities involved in possible lower bounds for T , such as the four-dimensional Plank scale, are derived quantities. Namely, their definition relates to the original string model and involves non-trivially the higher dimensional fields and dynamics thereof.

It is interesting to consider a direct approach to the problem of IR subtleties as in [15] where additional terms in the gauge kinetic function (the $f(\phi)$ in (3.1)) are explicitly considered in the cases where the bulk gauge coupling becomes large. Such corrections drive the geometry away from the hyperscaling-violating-Lifshitz ansatz and furnish an IR completion to that. We do not enter into the detail of such IR completion which generically features AdS factors and therefore residual entropies.

All in all, the gravity description we adopt is likely to be safe from possible lower bounds for T provided by either high curvatures and by a large bulk gauge coupling thanks to the large N limit and the possibility to tune the UV value of ϕ [15, 39]. Any statement involving a strict $T \rightarrow 0$ limit however should be taken with extreme care. Notably, comments about the residual entropy fall in this class.

²⁰The well-posedness of the Sturm-Liouville problem for the fluctuations is proposed as a further acceptability criterion for the background in [22, 36]; our solutions meet this criterion too.

6 Future perspectives

One important question that remains open is the individuation of a holographic model capable of reproducing the temperature behavior of the longitudinal resistivity and the Hall angle characterizing the cuprate physics. A possible continuation of the present analysis could be pursued considering also the cases where the charge density is a relevant operator in the IR; this could potentially overcome the obstructions we pointed out and allow one to accommodate the cuprate scalings. To the same purpose it could be inspiring to go back to the original proposal of [52], where the insertion of an effective spin-spin interaction led to a description characterized by two different scattering rates for longitudinal and transverse modes. It is interesting to ask ourselves how to realize a similar proposal within the holographic context and, more specifically, in generalizations of the present setup. One could either try to actually embed some devices describing the spin dynamics²¹ or (at least as a starting point) to just consider features which lead to an extra scale and a decoupling of the longitudinal and transverse physics.

Additionally, it could be interesting to think about Chern-Simons terms and their impact on IR scaling geometries. Another way to expand the range of results presented here would be to consider two U(1)'s, one carrying the electric charge and the other the magnetic field (along the lines of [77]). This could relax some of the constraints encountered in the present analysis.

Finally, it would be interesting to use the solutions here analyzed to address the investigation about bounds in momentum-dissipation holography, both for diffusion²² [10, 41–45, 53] and entropy production by a strain [46]. In particular to systematically test the proposal of [46] to relate the irrelevance of the momentum dissipation mechanism to a constant η/S bounding value at zero temperature and eventually studying the temperature fall-off scaling of the η/S ratio (see for example [79]).

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²¹In this sense it is interesting to see whether the setups described in [67–70] could provide a useful starting point.

²²We refer also to the line addressing disorder-driven metal-insulator transitions [54–56, 58, 59].

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A Relation among γ , β and θ , z

Consider the generic form for a hyperscaling-violating Lifshitz geometry (see for instance [47, 63])

$$ds^2 = \bar{r}^\theta \left[-\mathcal{U}(\bar{r}) \frac{dt^2}{\bar{r}^{2z}} + \frac{d\bar{r}^2}{\mathcal{U}(\bar{r})\bar{r}^2} + \frac{dx_1^2 + dx_2^2}{\bar{r}^2} \right]. \quad (\text{A.1})$$

We consider the following radial change of coordinate

$$r = \bar{r}^\nu, \quad dr = \nu \bar{r}^{\nu-1} d\bar{r}. \quad (\text{A.2})$$

We demand

$$a[r(\bar{r})]^2 = \mathcal{U}(\bar{r})\bar{r}^{\theta-2z}, \quad \frac{dr^2}{a[r(\bar{r})]^2} = \bar{r}^\theta \frac{d\bar{r}^2}{\mathcal{U}(\bar{r})\bar{r}^2}, \quad b[r(\bar{r})]^2 = \bar{r}^{\theta-2}, \quad (\text{A.3})$$

where the functions a and b are those appearing in the ansatz (3.3). We obtain

$$\bar{r}^{2\gamma\nu} \sim \bar{r}^{\theta-2z}, \quad \bar{r}^{2(\nu-1)} \sim \bar{r}^{\theta+2(\gamma\nu-1)}, \quad \bar{r}^{2\beta\nu} \sim \bar{r}^{\theta-2}, \quad (\text{A.4})$$

from which we eventually get

$$\nu = \frac{1}{1-\beta-\gamma}, \quad \theta = \frac{2(1-\gamma)}{1-\beta-\gamma}, \quad z = \frac{1-2\gamma}{1-\beta-\gamma}. \quad (\text{A.5})$$

This is in agreement with similar computations performed in [3]. Note however that they referred to the hyperscaling metric as given in [64]; this latter is mapped to (A.1) by means of a further change of radial coordinate $r \rightarrow r^{(2-\theta)/2}$.

B Equations of motion on the ansatz

The equations of motion (3.9) on the ansatz (3.2), (3.3), (3.4), (3.5), (3.6), after some simple manipulations, take the form of

$$\begin{aligned} 2C_a^2(\beta+\gamma)(2\beta+2\gamma-1) - 2|V_0|r^{-2\gamma+2\delta\kappa+2} + k^2 r^{2-2\beta-2\gamma+2\kappa\lambda} &= 0, \\ C_a^2\kappa(2\beta+2\gamma-1) - \alpha(B^2 r^{2\alpha\kappa} - Q^2 r^{-2\alpha\kappa}) r^{-2(2\beta+\gamma-1)} \\ &+ \frac{1}{2}\delta|V_0|r^{-2\gamma+2\delta\kappa+2} - \frac{1}{2}\lambda k^2 r^{2-2\beta-2\gamma+2\kappa\lambda} = 0, \\ C_a^2(\beta^2+2\beta\gamma-\kappa^2) + (Q^2 r^{-2\alpha\kappa} + B^2 r^{2\alpha\kappa}) r^{-2(2\beta+\gamma-1)} \\ &- \frac{1}{2}|V_0|r^{-2\gamma+2\delta\kappa+2} + \frac{1}{2}k^2 r^{2-2\beta-2\gamma+2\kappa\lambda} = 0, \\ &(\beta-1)\beta + \kappa^2 = 0. \end{aligned} \quad (\text{B.1})$$

C Wider exploration

In this appendix we examine both the solutions that have not been examined in the main text and repeat the analysis of the two already illustrated as a way of checking the results and bridging the approaches of [3] and [47]. We work directly adopting the θ, z parametrization, namely the solutions are obtained by solving the equations of motion for the action 3, using the ansatz:

$$\begin{aligned} ds^2 &= -\tilde{r}^{\theta-2z} dt^2 + L^2 \tilde{r}^{\theta-2} d\tilde{r}^2 + \tilde{r}^{\theta-2} (dx^2 + dy^2), \\ f(\phi) &= e^{2\alpha\phi} & V(\phi) &= -|V_0| e^{2\delta\phi}, & Y(\phi) &= e^{2\lambda\phi}, \\ A &= \tilde{Q} \tilde{r}^{\zeta-z} dt - B y dx, & \phi &= \tilde{\kappa} \ln(\tilde{r}), & \psi^I &= k x^I, \end{aligned} \tag{C.1}$$

where $\zeta = 2(1 - \alpha\tilde{\kappa})$ is the IR anomalous dimension of the charge density and is obtained by solving the Maxwell equation.

The NEC conditions (4.9) (which include $\tilde{\kappa} > 0$) and the consistency condition coming from the positivity of the specific heat constitute general requirements valid for all the classes here presented; they fix the allowed domain to coincide with the colored region in the right panel of figure 1 (either color). Throughout all the cases listed here we have

$$\tilde{\kappa}^2 = (\theta - 2)(\theta + 2 - 2z). \tag{C.2}$$

We exclude from the exploration all the solutions where the momentum-dissipation scalar correspond to irrelevant operators. As discussed in subsection 5.1, in such cases the transport coefficient gets corrected at the quadratic in k , this being the same order that we neglect in the background.

C.1 No charge density, marginally relevant magnetic field

Consider $Q = 0$ and non-trivial B , namely the “electro-magnetic” dual configuration of the charged solution in the main text. The marginal relevance of B , the momentum-dissipation scalars and the dilaton leads to

$$2\tilde{\kappa}\alpha = \theta - 4, \quad \tilde{\kappa}\lambda + 1 = 0, \quad 2\tilde{\kappa}\delta + \theta = 0. \tag{C.3}$$

Solving the equation we obtain

$$B^2 = \frac{k^2(\theta - 2z) + 2|V_0|(z - 1)}{4(\theta - z - 1)}, \quad L^2 = \frac{2(\theta - z - 1)(\theta - z - 2)}{2|V_0| - k^2}. \tag{C.4}$$

The validity domain is given by

$$|V_0| > \frac{k^2(2z - \theta)}{2(z - 1)} \tag{C.5}$$

intersected with the following set of alternative conditions

$$2z > \theta + 2 \quad \wedge \quad 0 < \theta < 2 \quad \text{or} \tag{C.6}$$

$$\theta > 2 \wedge z < 0 \quad \text{or} \tag{C.7}$$

$$z > 1 \wedge \theta \leq 0. \tag{C.8}$$

C.2 Marginally relevant charge, magnetic field and momentum-dissipation scalars

The system encoding the marginality conditions for all the operators reads

$$2\delta\tilde{\kappa} + \theta = 2\tilde{\kappa}\lambda + 2 \tag{C.9}$$

$$\tilde{\kappa}(\gamma + \delta) + 4 = 2\theta \tag{C.10}$$

$$2\delta\tilde{\kappa} + \theta = 0 \tag{C.11}$$

$$\tilde{\kappa}(\alpha + \delta) + \theta = 2 . \tag{C.12}$$

This system however is impossible.

C.3 Irrelevant charge density, marginally relevant magnetic field

The marginal relevance of B , the momentum-dissipation scalars and the dilaton leads to (C.3). Solving the EOM one gets

$$\begin{aligned} \theta = 4 + 2\alpha\tilde{\kappa}, \quad B^2 &= \frac{k^2(\theta - 2z) + 2|V_0|(z - 1)}{4(\theta - z - 1)}, \\ L^2 &= \frac{2(\theta - z - 1)(\theta - z - 2)}{2|V_0| - k^2} \end{aligned} \tag{C.13}$$

The validity domain (including the condition enforcing the irrelevance of the charge density) are given by

$$|V_0| > \frac{k^2(2z - \theta)}{2(z - 1)}, \tag{C.14}$$

intersected with

$$2 < \theta < 4 \quad \wedge \quad z < 0 \quad \text{or} \quad \theta > 4 \wedge z < 0 . \tag{C.15}$$

C.4 Marginally relevant charge density, irrelevant magnetic field

This corresponds to the charged solutions already considered in the main text. Marginal relevance for the charge density, the momentum-dissipation scalar and the dilaton yields

$$\tilde{\kappa}\lambda + 1 = 0, \quad 2\tilde{\kappa}\delta + \theta = 0, \quad 2\tilde{\kappa}\alpha = 4 - \theta . \tag{C.16}$$

The irrelevance of the magnetic field corresponds to

$$\theta < 4 . \tag{C.17}$$

Solving the EOM's we further get

$$\tilde{Q}^2 = \frac{k^2(2z - \theta) - 2|V_0|(z - 1)}{2(2|V_0| - k^2)(\theta - z - 2)}, \quad L^2 = \frac{2(\theta - z - 1)(\theta - z - 2)}{2|V_0| - k^2}; \tag{C.18}$$

the quantities on the r.h.s.'s must therefore be positive. All in all, the validity domain of the solution is

$$2 < \theta < 4 \quad \wedge \quad z < 0 \quad \text{and} \quad |V_0| > \frac{k^2(2z - \theta)}{2(z - 1)} . \tag{C.19}$$

We agree with (4.8) and (4.16) where $\xi = k^2/|V_0|$.

C.5 Irrelevant charge density and magnetic field

This corresponds to the neutral solution already considered in the main text. Asking for marginal of the momentum-dissipation scalars and the dilaton leads to

$$\tilde{\kappa}\lambda + 1 = 0, \quad 2\tilde{\kappa}\delta + \theta = 0. \quad (\text{C.20})$$

Irrelevance of the charge density and the magnetic field requires

$$\theta - 4 + 2\alpha\tilde{\kappa} < 0, \quad \theta - 4 - 2\alpha\tilde{\kappa} < 0. \quad (\text{C.21})$$

which corresponds to (4.37). Solving the EOM's we get

$$|V_0| = \frac{k^2(2z - \theta)}{2(z - 1)}, \quad L^2 = \frac{2(z - 1)(-\theta + z + 2)}{k^2}. \quad (\text{C.22})$$

Eventually, the validity domain is

$$2 < \theta < 4 \quad \text{and} \quad z < 0, \quad (\text{C.23})$$

to be considered along with (C.21).

D Temperature scaling of the Seebeck coefficient

The definition of the Seebeck coefficient s is

$$s = \frac{\alpha_{xx}}{\sigma_{xx}}. \quad (\text{D.1})$$

From the transport coefficients (3.21), (3.25) we can extract the generic form of the Seebeck:

$$s = \frac{qS}{b^2 f k^2 Y + B^2 f^2 + q^2} \Big|_h. \quad (\text{D.2})$$

Considering the T scalings of the various quantities appearing in (D.2), we obtain

$$s = \frac{q(\zeta T)^{\frac{2\beta}{2\gamma-1}}}{B^2 (\zeta T)^{\frac{4\alpha\kappa}{2\gamma-1}} + k^2 (\zeta T)^{\frac{2\kappa(\alpha+\lambda)+2\beta}{2\gamma-1}} + q^2}. \quad (\text{D.3})$$

D.1 Charged solutions

Expanding up to linear order in B we get

$$s = \frac{q(\zeta T)^{\frac{2\beta}{2\gamma-1}}}{k^2 (\zeta T)^{\frac{2\kappa(\alpha+\lambda)+\beta}{2\gamma-1}} + q^2} + \mathcal{O}(B^2). \quad (\text{D.4})$$

Considering relations (4.1) and the domain of definition of the charged solutions (4.5), at low T we have

$$s = \frac{q}{k^2 + q^2} (\zeta T)^{\frac{2\beta}{2\gamma-1}} + \mathcal{O}(B^2). \quad (\text{D.5})$$

D.2 Neutral solutions

Expanding up to linear order in both B and q we obtain

$$s = \frac{q}{k^2} (\zeta T)^{-\frac{2\kappa(\alpha+\lambda)}{2\gamma-1}} + \mathcal{O}(B^2, q^2) . \quad (\text{D.6})$$

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