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On analyzing the vulnerabilities of a railway network with Petri nets

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Abstract

Petri nets are used in this paper to estimate the indirect consequences of accidents in a railway network, which belongs to the class of the so-called transportation Critical Infrastructures (CIs), that is, those assets consisting of systems, resources and/or processes whose total or partial destruction, or even temporarily unavailability, has the effect of significantly weakening the functioning of the system. In the proposed methodology, a timed Petri ne<t represents the railway network and the trains travelling over the rail lines; such a net also includes some places and some stochastically-timed transitions that are used to model the occurrence of unexpected events (accidents, disruptions, and so on) that make some resources of the network (tracks, blocks, crossovers, overhead line, electric power supply, etc.) temporarily unavailable. The overall Petri net is a live and bounded Generalized Stochastic Petri Net (GSPN) that can be analyzed by exploiting the steady-state probabilities of a continuous-time Markov chain (CTMC) that can be derived from the reachability graph of the GSPN. The final target of such an analysis is to determine and rank the levels of criticality of transportation facilities and assess the vulnerability of the whole railway network.

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Keywords: Railway networks; Critical infrastructures; Petri nets; Vulnerability assessment; Performance analysis

1. Introduction

In the last two decades, the interest of institutions, managers, enterprises, and researchers has been focused on improving the safety and security levels of Critical Infrastructures (CIs). In particular, as regards transportation systems, the interest about this topic has grown due not only to the potentially very high number of people killed or injured when an incident or accident occurs (direct losses), but also to the high costs and the large amount of time

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needed in many cases to restore the infrastructure (indirect losses). Moreover, the main characteristic that makes CI safety and security a particularly important problem to tackle with is the mutual interdependence of their components whose operation depends on the proper functioning of all the others; this is especially true in railway networks where the occurrence of unexpected accidental events (hereafter referred to as safety or failure events) makes some resources of the network (tracks, blocks, crossovers, overhead line, electric power supply, etc.) temporarily unavailable, with consequences that are in some cases amplified by the chain-effect phenomenon resulting from the interdependency of resources. A methodology to assess the consequences of failures in a railway network is presented in this paper; it is based on Petri Nets (PNs), which have been proven to be a valuable and powerful modelling tool to represent and analyze the behavior of discrete event systems, as they are able to capture the precedence relations and interactions among the concurrent and asynchronous events typical of such systems.

While for specific literature relevant to security and safety of critical infrastructures the reader may refer to Lewis (2006) and Macaulay (2008), the problem of assessing the criticality of infrastructures has been faced in Arulselvan et al. (2009), where an approach based on the graph theory is proposed to detect the most critical nodes in large networks. Although the graph theory appears to be a suitable tool for such analyses, Petri Nets may represent a better modelling formalism, being capable to model in a unique framework different kinds of dynamics and to easily model concurrency and synchronism of different events, often in a modular way. In this framework, an interesting application of PNs to infrastructure interdependence analysis is described in Gursesli and Desrochers (2003) where different kinds of critical infrastructures are modelled in a unique framework and some considerations about interdependence of CIs are provided by applying the analysis of PN structural properties (Murata, 1989). In addition, the "intrinsic modularity" of PNs is useful whenever a large network of different kinds of infrastructures has to be considered, such as, for instance, an electric power distribution network and a railway transportation network. For what concerns the modelling capabilities of PNs in the field of transportation engineering, they have been put into evidence by the relevant vast literature. For the cases of highway networks or urban transportation networks, readers can refer, for instance, to Tolba et al. (2005) and Di Febbraro et al. (2016). Instead, for the case of railway systems here considered, it can be mentioned the works of Ren and Zhou (1995), Fanti et al. (2006), Hagalisetto et al. (2007), Giua and Seatzu (2008), Ricci (2009), and Liu et al. (2016). Recently, in Giglio and Sacco (2016) a very detailed Petri Net representation of a railway traffic system has been proposed, with the aim of providing a comprehensive model to be used for analysis, optimization, and control purposes. In this paper, that model is taken into account to analyze the CIs characterizing the railway network and assess the level of criticality of failure events.

The level of criticality of a safety or failure event which affects one or more transportation facility is here considered as the product of a value representing the impact of the occurrence of the event on the system's operation with the probability of being in a state which is affected by the event. This is consistent with the classic definition of risk. In general, the impact is assessed through the computation of a weighted sum of the estimations of some losses (both direct and indirect) caused by the occurrence of the event. In the proposed Petri net-based approach, the impact of a certain failure event will be determined directly by means of the reachability graph of the generalized stochastic Petri net (GSPN) representing the railway network, whereas the probability of occurrence of the event will be calculated through the continuous-time Markov chain (CTMC) that can be derived from the GSPN taking into account the tangible states of the reachability graph, the firing rates of exponentially-timed transitions, and the stochastic switches associated with immediate transitions.

2. The railway network and the adopted Petri net model

The proposed Petri net-based model of a railway network consists of two interconnected PNs that represent, respectively, the physical part of the network (stations, blocks, crossovers, tracks, and so on, as well as the rolling stock moving on the network) and the logical part of it (train schedule over the available lines). A sketch of a railway network with 7 stations and 8 railways is reported in Fig. 1(a), whereas in Fig. 1(b) the overall PN model is illustrated. The two nets have been described in detail in Giglio and Sacco (2016), to which the reader can refer for more details on the modelling approach. The extension presented in this paper is aimed at representing the occurrence of accidents, disruptions, and other unexpected events that make some resources of the network unavailable; this is done by including in the Timed Petri Net (TPN) modelling the physical part of the system some places and some stochastically-timed transitions whose firings model such failure events; the resulting net is the "extended" TPN depicted in Fig. 1(b) which is actually a Generalized Stochastic Petri Net. Since the proposed methodology to determine the levels of criticality of transportation facilities requires that the GSPN is live and bounded, a Petri net supervisor is employed to model real railway operational constraints so that the reach of some states is prevented (in particular, deadlock states); nevertheless, due to the lack of space, such a net is not described in this paper. Besides, also the logical part of the overall PN model, including the PN actuator, is not considered here as it is not strictly pertinent to the research presented in this paper.

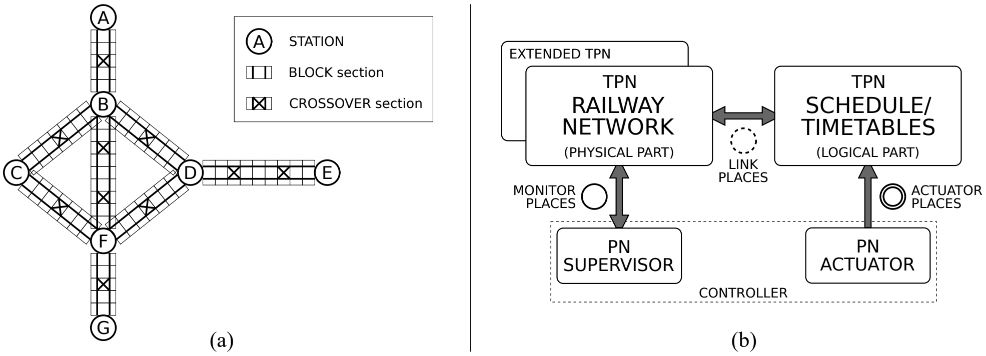

Fig. 1. (a) A railway network with 7 stations and 8 railways. (b) Schematization of the overall Petri net model.

2.1. The extended TPN of the physical part

The representation of failure events in the railway network is briefly illustrated in the following by means of the PN reported in Fig. 2, which models a block section within a railway.

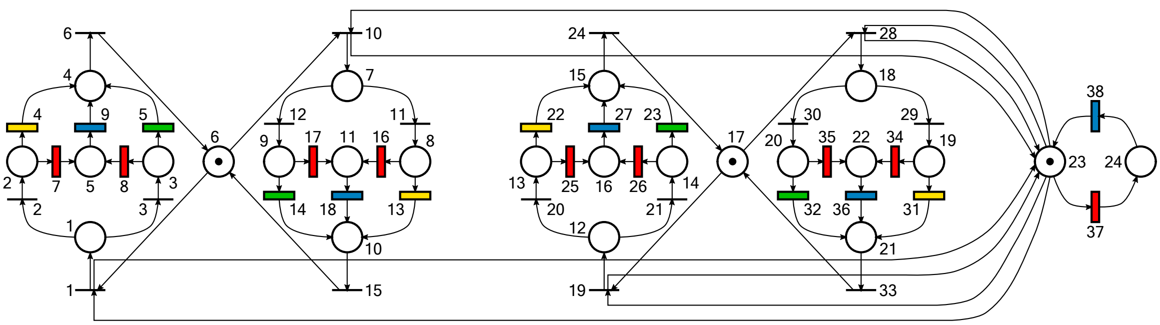

Fig. 2. Petri net model of a block section with failure events (red transitions).

In such a net, places from p_1 to p_{11} (resp., from p_{12} to p_{22}) and transitions from t_1 to t_{18} (resp., from t_{19} to t_{36}) model the left (resp., right) track of the double-track railway section, whereas places p_{23}-p_{24} and transitions t_{37}-t_{38} model the physical availability of the block section. With regards to the left track, a northbound train is represented by a token which enters p_1 and reaches p_4 by passing through either t_2-p_2-t_4 (in case of reduced speed) or t_3-p_3-t_5 (nominal speed). Transitions t_7 and t_8 model a train failure (for instance due to an engine failure) which occurs when the train is travelling the block section, and t_9 represents the repair process which makes the train able to restart its travel; t_7, t_8, and t_9 are timed transitions whose firing time is distributed exponentially in order to properly model the randomness of the (memoryless) fault events. Analogous considerations can be made for a southbound train (the token representing the train flows from p_7 to p_{10} and for the part of the net relevant to the right track. An accident involving the whole block section (such as, for instance, a mudslide) is modelled through the firing of t_{37} which moves the token in p_{23} to p_{24}; in such a marking, the absence of a token in p_{23} prevents the firing of immediate transitions t_1, t_{10}, t_{19}, and t_{28} which means that trains cannot enter the disrupted block section until it is repaired (the repair process is represented through the firing of t_{38} which put back the token to p_{23}).

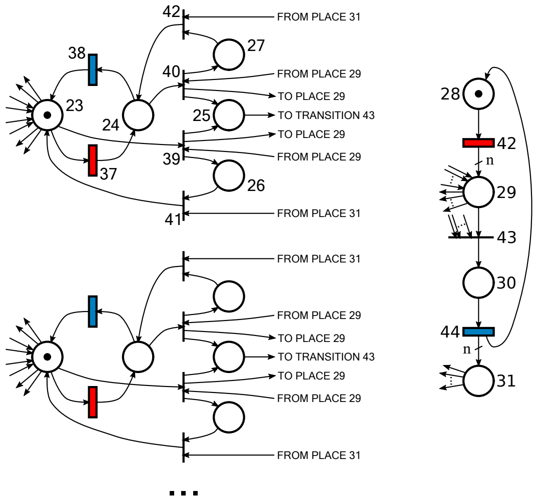

Fig. 3. Representation of a failure event affecting the overhead line with Petri nets.

As discussed in the introduction, in a critical infrastructure some resources depend on the proper availability of some other resources; for example, in a railway network, the block and crossover sections constituting a railway line are available only if the overhead line provides power to the railway line; in other words, a failure event affecting the overhead line prevents the execution of any travel activity on the involved block and crossover sections. In the proposed model, this is represented with the Petri net structure illustrated in Fig. 3. In such a part of the whole "extended TPN", the considered failure event is modelled through the firing of t_{42} which put n tokens to p_{29} (n is the number of blocks and crossovers connected to the overhead line); due to the arcs between the portion of the net in the left part of Fig. 3 (which actually represents the physical availability of block sections powered by the overhead line) and p_{29}, t_{43}, and p_{31} in the right part, the tokens in p_{29} and the firings of immediate transition t_{43} remove all tokens from the left part, making all the block sections no longer available until the line is restored (this happens when t_{44} fires and the token is put back to p_{28}). In a similar way, it is possible to model the dependency of the overhead lines from an electric power supply which provides power to a subset of adjacent overhead lines; in this case, an electric power failure impedes any connected overhead line to properly work.

3. On determining the levels of criticality of transportation facilities

The level of criticality of a certain transportation resource or facility is determined from the Petri net model described in the previous section; in particular, the reachability graph is used to assess the impact of a certain failure event, and the continuous-time Markov chain that can be derived from the reachability graph is used to determine the probability of occurrence of the event. The impact and the probability of occurrence of a certain event lead to the determination of the level of the relevant criticality. In this connection, let a failure event e_h, $h = 1, 2, \dots$, be described by the markings of places that characterize the states after the occurrences of the event; as an example, with reference to the part of the net represented in Fig. 3, the event "overhead line failure" is characterized by the absence of a token in p_{28} and by the presence of one or more tokens in p_{29} or one token in p_{30}; thus it can be defined as $e_1 = \{m(p_{28}) = 0 \wedge (m(p_{29}) > 1 \vee m(p_{30}) = 1)\}$. Giving the reachability graph RG and being M_i the generic marking of RG, the set of markings of the graph in which the failure event e_h is active is defined as

$$NR(e_h) = \{M_i \in RG \mid M_i \cap e_h \neq \emptyset\} \tag{1}$$

Then, the impact of the failure event e_h is

$$I(e_h) = \frac{\text{card}\{NR(e_h)\}}{\text{card}\{RG\}} \tag{2}$$

being $\text{card}\{NR(e_h)\}$ the number of markings of the reachability graph in which the event is active and $\text{card}\{RG\}$ the number of markings of the reachability graph. Moreover, giving the continuous-time Markov chain MC (it is determined taking into account the tangible states of the reachability graph, the firing rates of exponentially-timed transitions, and the stochastic switches associated with immediate transitions) and being M_i the generic node of it, the probability of occurrence of the failure event e_h is

$$\Pr\{e_h\} = \sum_{M_i \in MC | M_i \cap e_h \neq \emptyset} \Pr\{M_i\} \tag{3}$$

Finally, the level of criticality of the failure event e_h is given by

$$V(e_h) = \Pr\{e_h\} \cdot I(e_h) \tag{4}$$

and the vulnerability of the whole railway network is assessed at

$$V_{\text{SYS}} = \sum_{e_h \in \mathcal{E}} V(e_h) \tag{5}$$

being $\mathcal{E} = \{e_h, h = 1, 2, \dots\}$ the set including all the failure events affecting the system.

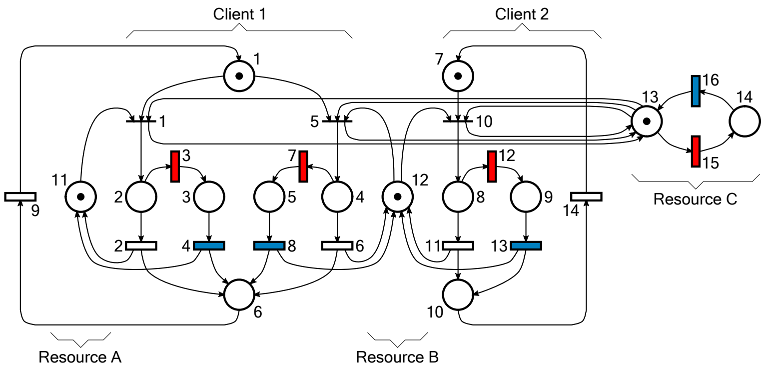

Fig. 4. Petri net modelling the execution of activities for two kinds of clients by means of two partially-shared "local" resources A and B, and with the availability of a "global" resource C.

In order to better explain the proposed methodology, a toy example is taken into account. Let consider the Petri net illustrated in Fig. 4, which models a simple system consisting of three resources A, B, and C which are used by two kinds of clients, namely clients 1 and clients 2, to carry out some activities (whose duration is exponentially-distributed). Places p_{11}, p_{12}, and p_{13} model the state (availability or non-availability) of resources A, B, and C, respectively. Clients 1 can use either resource A (modelled by the sequence of places and transitions p_1-t_1-p_2-t_2-p_6) or resource B (sequence p_1-t_5-p_4-t_6-p_6) to perform its activity, whereas clients 2 are allowed to use resource B only (sequence p_7-t_{10}-p_8-t_{11}-p_{10}). All activities need the availability of resource C to start; thus, resource C can be considered as a primary "global" resource with respect to the secondary "local" resources A and B. Each activity is renewed after its completion; in this connection, transitions t_9 and t_{14} represent the renewal processes of activities of client 1 and client 2, respectively. It can be noted that such an example is analogous to the case of a block section with two parallel tracks, in which trains (clients) 1 can use the two of them, while trains (clients) 2 can use only one.

A disruption may affect a client when it is using resource A or B; in this case, the involved client keeps the resource occupied until it is repaired; once repaired, the client leaves the system and the resource is again available. In the net of Fig. 4, disruptions are modelled with exponentially-timed transitions t_3 (for clients of type 1 when using resource A), t_7 (again for clients of type 1 but when using resource B), and t_{12} (for clients of type 2). The repair activities, which are obviously mandatory when a client suffers a disruption, are respectively modelled with exponentially-timed transitions t_4, t_8, and t_{13}. The primary resource C is also failure-prone; it can suddenly break (due to unexpected events) thus preventing the start of any activity on both resource A and B; in the Petri net, the failure of resource C and its consequent repair process are modelled with exponentially-timed transitions t_{15} and t_{16}, respectively; besides, the firing of t_{15} (which moves the token in p_{13} to p_{14}) prevents the firing of immediate transitions t_1, t_5, and t_{10} that represent the starts of activities.

It is evident that 4 kinds of failure events affect the system, which are defined as follows:

- $e_1 = \{m(p_3) = 1\}$ (disruption of client 1 when using resource A);
- $e_2 = \{m(p_5) = 1\}$ (disruption of client 1 when using resource B);
- $e_3 = \{m(p_9) = 1\}$ (disruption of client 2 when using resource B);
- $e_4 = \{m(p_{14}) = 1\}$ (break down of resource C).

As a matter of fact, such an example is a simplification of the Petri net model of a block section in a railway and therefore can be taken as representative of the considered problem.

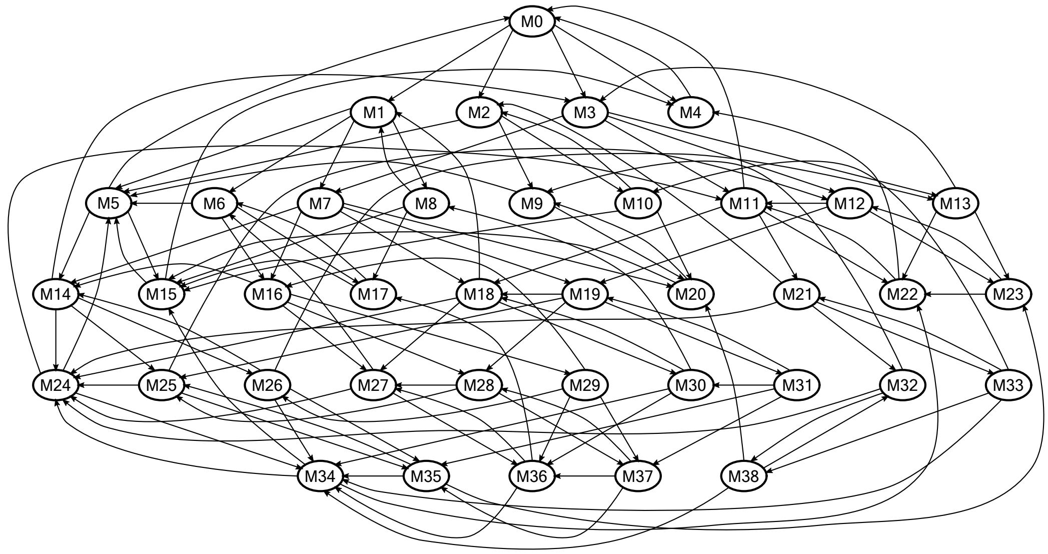

Fig. 5. Reachability graph of the Petri net illustrated in Fig. 4.

The reachability graph of the Petri net is illustrated in Fig. 5 and the list of reachable markings is reported in Table 1. From the table it results that the sets $NR(e_h)$, with $h = 1, 2, \ldots$, are:

- $NR(e_1) = \{M_6, M_{16}, M_{17}, M_{27}, M_{28}, M_{29}, M_{36}, M_{37}\}$;
- $NR(e_2) = \{M_9, M_{20}, M_{32}, M_{38}\}$;
- $NR(e_3) = \{M_{12}, M_{19}, M_{23}, M_{25}, M_{28}, M_{31}, M_{35}, M_{37}\}$;
- $NR(e_4) = \{M_4, M_8, M_{10}, M_{13}, M_{15}, M_{17}, M_{20}, M_{22}, M_{23}, M_{26}, M_{29}, M_{30}, M_{31}, M_{33}, M_{34}, M_{35}, M_{36}, M_{37}, M_{38}\}$;

which means that the impacts of the failure events are:

- $I(e_1) = 0.2051$;
- $I(e_2) = 0.1026$;
- $I(e_3) = 0.2051$;
- $I(e_4) = 0.4872$.

In order to compute the steady-state probabilities of being, in a generic time instant, in a certain tangible state of the graph, the CTMC associated with the considered GSPN (which is a live and bounded net) is determined. In this connection, the firing rates and the stochastic switches reported in Tables 2 and 3 are taken into consideration.

Table 1. Markings of the reachability graph of the Petri net illustrated in Fig. 4 (the notation $[j\rangle Mk$ used in the "Next Markings" column means that the next marking Mk is reached by firing transition t_j).

Label	Marking	Next Markings	Type
M0	(1,0,0,0,0,0,1,0,0,0,1,1,1,0)	[1⟩M1 ; [5⟩M2 ; [10⟩M3 ; [15⟩M4	vanishing
M1	(0,1,0,0,0,0,1,0,0,0,0,1,1,0)	[2⟩M5 ; [3⟩M6 ; [10⟩M7 ; [15⟩M8	vanishing
M2	(0,0,0,1,0,0,1,0,0,0,1,0,1,0)	[6⟩M5 ; [7⟩M9 ; [15⟩M10	tangible
M3	(1,0,0,0,0,0,0,1,0,0,1,0,1,0)	[1⟩M7 ; [11⟩M11 ; [12⟩M12 ; [15⟩M13	vanishing
M4	(1,0,0,0,0,0,1,0,0,0,1,1,0,1)	[16⟩M0	tangible
M5	(0,0,0,0,0,1,1,0,0,0,1,1,1,0)	[9⟩M0 ; [10⟩M14 ; [15⟩M15	vanishing
M6	(0,0,1,0,0,0,1,0,0,0,0,1,1,0)	[4⟩M5 ; [10⟩M16 ; [15⟩M17	vanishing
M7	(0,1,0,0,0,0,0,1,0,0,0,0,1,0)	[2⟩M14 ; [3⟩M16 ; [11⟩M18 ; [12⟩M19 ; [15⟩M20	tangible
M8	(0,1,0,0,0,0,1,0,0,0,0,1,0,1)	[2⟩M15 ; [3⟩M17 ; [16⟩M1	tangible
M9	(0,0,0,0,1,0,1,0,0,0,1,0,1,0)	[8⟩M5 ; [15⟩M20	tangible
M10	(0,0,0,1,0,0,1,0,0,0,1,0,0,1)	[6⟩M15 ; [7⟩M20 ; [16⟩M2	tangible
M11	(1,0,0,0,0,0,0,0,0,1,1,1,1,0)	[1⟩M18 ; [5⟩M21 ; [14⟩M0 ; [15⟩M22	vanishing
M12	(1,0,0,0,0,0,0,0,1,0,1,0,1,0)	[1⟩M19 ; [13⟩M11 ; [15⟩M23	vanishing
M13	(1,0,0,0,0,0,0,1,0,0,1,0,0,1)	[11⟩M22 ; [12⟩M23 ; [16⟩M3	tangible
M14	(0,0,0,0,0,1,0,1,0,0,1,0,1,0)	[9⟩M3 ; [11⟩M24 ; [12⟩M25 ; [15⟩M26	tangible
M15	(0,0,0,0,0,1,1,0,0,0,1,1,0,1)	[9⟩M4 ; [16⟩M5	tangible
M16	(0,0,1,0,0,0,0,1,0,0,0,0,1,0)	[4⟩M14 ; [11⟩M27 ; [12⟩M28 ; [15⟩M29	tangible
M17	(0,0,1,0,0,0,1,0,0,0,0,1,0,1)	[4⟩M15 ; [16⟩M6	tangible
M18	(0,1,0,0,0,0,0,0,0,1,0,1,1,0)	[2⟩M24 ; [3⟩M27 ; [14⟩M1 ; [15⟩M30	tangible
M19	(0,1,0,0,0,0,0,0,1,0,0,0,1,0)	[2⟩M25 ; [3⟩M28 ; [13⟩M18 ; [15⟩M31	tangible
M20	(0,0,0,0,1,0,1,0,0,0,1,0,0,1)	[8⟩M15 ; [16⟩M9	tangible
M21	(0,0,0,1,0,0,0,0,0,1,1,0,1,0)	[6⟩M24 ; [7⟩M32 ; [14⟩M2 ; [15⟩M33	tangible
M22	(1,0,0,0,0,0,0,0,0,1,1,1,0,1)	[14⟩M4 ; [16⟩M11	tangible
M23	(1,0,0,0,0,0,0,0,1,0,1,0,0,1)	[13⟩M22 ; [16⟩M12	tangible
M24	(0,0,0,0,0,1,0,0,0,1,1,1,1,0)	[9⟩M11 ; [14⟩M5 ; [15⟩M34	tangible
M25	(0,0,0,0,0,1,0,0,1,0,1,0,1,0)	[9⟩M12 ; [13⟩M24 ; [15⟩M35	tangible
M26	(0,0,0,0,0,1,0,1,0,0,1,0,0,1)	[9⟩M13 ; [11⟩M34 ; [12⟩M35 ; [16⟩M14	tangible
M27	(0,0,1,0,0,0,0,0,0,1,0,1,1,0)	[4⟩M24 ; [14⟩M6 ; [15⟩M36	tangible
M28	(0,0,1,0,0,0,0,0,1,0,0,0,1,0)	[4⟩M25 ; [13⟩M27 ; [15⟩M37	tangible
M29	(0,0,1,0,0,0,0,1,0,0,0,0,0,1)	[4⟩M26 ; [11⟩M36 ; [12⟩M37 ; [16⟩M16	tangible
M30	(0,1,0,0,0,0,0,0,0,1,0,1,0,1)	[2⟩M34 ; [3⟩M36 ; [14⟩M8 ; [16⟩M18	tangible
M31	(0,1,0,0,0,0,0,0,1,0,0,0,0,1)	[2⟩M35 ; [3⟩M37 ; [13⟩M30 ; [16⟩M19	tangible
M32	(0,0,0,0,1,0,0,0,0,1,1,0,1,0)	[8⟩M24 ; [14⟩M9 ; [15⟩M38	tangible
M33	(0,0,0,1,0,0,0,0,0,1,1,0,0,1)	[6⟩M34 ; [7⟩M38 ; [14⟩M10 ; [16⟩M21	tangible
M34	(0,0,0,0,0,1,0,0,0,1,1,1,0,1)	[9⟩M22 ; [14⟩M15 ; [16⟩M24	tangible
M35	(0,0,0,0,0,1,0,0,1,0,1,0,0,1)	[9⟩M23 ; [13⟩M34 ; [16⟩M25	tangible
M36	(0,0,1,0,0,0,0,0,0,1,0,1,0,1)	[4⟩M34 ; [14⟩M17 ; [16⟩M27	tangible
M37	(0,0,1,0,0,0,0,0,1,0,0,0,0,1)	[4⟩M35 ; [13⟩M36 ; [16⟩M28	tangible
M38	(0,0,0,0,1,0,0,0,0,1,1,0,0,1)	[8⟩M34 ; [14⟩M20 ; [16⟩M32	tangible

Table 2. Firing rates in the GSPN illustrated in Fig. 4.

Transition	Firing Rate	Note
t_2	$\mu_2 = 4\ \text{hour}^{-1}$	service rate
t_3	$f_3 = 1\ \text{week}^{-1}$	failure rate (disruption)
t_4	$r_4 = 0.5\ \text{hour}^{-1}$	repair rate (disruption)
t_6	$\mu_6 = 6\ \text{hour}^{-1}$	service rate
t_7	$f_7 = 1\ \text{week}^{-1}$	failure rate (disruption)
t_8	$r_8 = 0.5\ \text{hour}^{-1}$	repair rate (disruption)
t_9	$\lambda_9 = 2\ \text{hour}^{-1}$	renewal process rate
t_{11}	$\mu_{11} = 2\ \text{hour}^{-1}$	service rate
t_{12}	$f_{12} = 2\ \text{week}^{-1}$	failure rate (disruption)
t_{13}	$r_{13} = 0.5\ \text{hour}^{-1}$	repair rate (disruption)
t_{14}	$\lambda_{14} = 1\ \text{hour}^{-1}$	renewal process rate
t_{15}	$f_{15} = 1\ \text{month}^{-1}$	failure rate (breakdown)
t_{16}	$r_{16} = 2\ \text{day}^{-1}$	repair rate (breakdown)

Table 3. Stochastic switches in the GSPN illustrated in Fig. 4.

Marking	Enabled Immediate Transitions	Probabilities
M0	$t_1 - t_5 - t_{10}$	0.2 – 0.3 – 0.5
M1	t_{10}	1
M3	t_1	1
M5	t_{10}	1
M6	t_{10}	1
M11	$t_1 - t_5$	0.4 – 0.6
M12	t_1	1

After having built the matrix Q and determined the steady-state probabilities of being in the tangible markings of the CTMC (by solving the system of linear equalities $\pi Q = 0$ with $\sum_i \pi_i = 1$), the probabilities of occurrence of the failure events turn out to be the following ones:

- $\Pr\{e_1\} = 2.4501 \cdot 10^{-3}$;
- $\Pr\{e_2\} = 1.3378 \cdot 10^{-3}$;
- $\Pr\{e_3\} = 7.6165 \cdot 10^{-3}$;
- $\Pr\{e_4\} = 1.6393 \cdot 10^{-2}$;

and, consequently, the levels of criticality of the failure events are:

- $V(e_1) = 2.4501 \cdot 10^{-3} \cdot 0.2051 = 5.0259 \cdot 10^{-4}$;
- $V(e_2) = 1.3378 \cdot 10^{-3} \cdot 0.1026 = 1.3721 \cdot 10^{-4}$;
- $V(e_3) = 7.6165 \cdot 10^{-3} \cdot 0.2051 = 1.5624 \cdot 10^{-3}$;
- $V(e_4) = 1.6393 \cdot 10^{-2} \cdot 0.4872 = 7.9865 \cdot 10^{-3}$;

and the vulnerability of the whole railway network is assessed at $V_{\mathrm{SYS}} = 1.0189 \cdot 10^{-2}$. In conclusion, the most critical failure event among those considered in this toy example is the breakdown of the "global" resource C.

4. Conclusions

In this paper, a methodology to assess, from a quantitative point of view, the accident events that affect a Critical Infrastructure has been presented with specific reference to the case of railway networks. The proposed methodology is applied to the detailed Petri net representation of the system and exploits the properties of the Generalized Stochastic Petri Net, that can be analyzed by means of the Markov chain theory. As future research direction, in order to limit some computational troubles in the determination of the reachability graph of the net, an efficient building procedure can be defined by exploiting the modularity of the adopted Petri net representation.

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