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A useful observable for estimating k_{eff} in fast subcritical systems

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Abstract. The neutron multiplication factor k_{eff} is a key quantity to characterize subcritical neutron multiplying devices and for understanting their physical behaviour, being related to the fundamental eigenvalue of Boltzmann transport equation. Both the maximum available power - and all quantities related to it, like, e.g. the effectiveness in burning nuclear wastes - as well as reactor kinetics and dynamics depend on k_{eff}. Nevertheless, k_{eff} is not directly measurable and its determination results from the solution of an inverse problem: minimizing model dependence of the solution for k_{eff} then becomes a critical issue, relevant both for practical and theoretical reasons.

1 Introduction

As it is well known, the neutron multiplication factor k_{eff} is both a fundamental quantity for the evaluation of the behavior of subcritical reactors (i.e. Accelerator Driven Systems, ADS) and a paramount indicator to monitor its subcriticality level. However, it is not directly an observable: its determination is essentially an inverse problem and various ad-hoc methods [1–7] have been developed for its experimental determination from flux measurements; they have been tested against experimental results from MUSE [8], YALINA [9], DELPHI [10] and VENUS-F [11] with different findings, mainly depending on the subcriticality level. Additionally, it is important to highlight that their applicability is debatable when the value of k_{eff} is below 1.

The key point to understand the relevance of the parameter k_{eff} is that in an ADS it is mandatory to avoid neutron flux divergence in all conditions: as already underlined [12], the concept of *margin to criticality* is questionable, so that "on-line" monitoring of k_{eff} is, at present, an unavoidable safety requirement for all the facilities with a non-negligible power [8]. Among others, also the EU FP7 project FREYA has investigated this issue [13].

To exemplify the reason of the difficulties in the determination of k_{eff} from total neutron flux measurements $\Phi_T = \int \Phi(E)\, dE$, as they can be performed in small volumes by a fission chamber, we show in Figure 1 the MCNP simuation of a typical flux behaviour as a function of k_{eff} in different in-core positions; see Section 2 for a detailed description of this model (LEADS) and for the convention we use to identify tally positions, all in the same z-plane. LEADS model can be considered a typical Generation IV [15, 16] configuration. Two relevant facts can be inferred

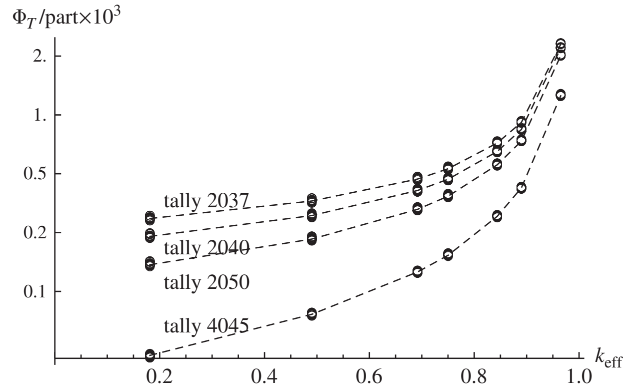

Figure 1. Logarithmic plot of the total neutron flux for 86 tallies at various distances from the source (see text).

from this plot: the first is that the supposed relation between a local quantity, the total neutron flux, and a global one, k_{eff}, is not functionally simple and moreover it is strongly position dependent; the second feature is not immediately evident from Figure 1 where actually a total of 86 different tallies are plotted together: however they are not randomly distributed in the core, but they have been chosen on purpose as the sets of tallies whose distances from the source differ from the one of 4 reference tallies (the one indicated in figure 1), by less than 5 mm. Then the second feature is that the flux position dependence effectively reduces with a very high confidence to a dependence

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only on the distance from the source: we think this to be a peculiar feature of a fast core, essentially because neutrons with longer mean free paths cannot distinguish fine details of the geometry.

What Figure 1 suggests is that any attempt to extract k_{eff} from total flux measurement [1–7] may be complicated and model dependent.

At least intuitively, being the fission neutron spectrum fast, one could expect that the "fast" component of the neutron flux could be more directly related to the ability of the core to multiply neutrons; however this is not sufficient because the fission rate is locally determined by the flux itself, so it becomes natural to normalize the fast flux component to the total one, by introducing a type of spectral index that we defined as [14]:

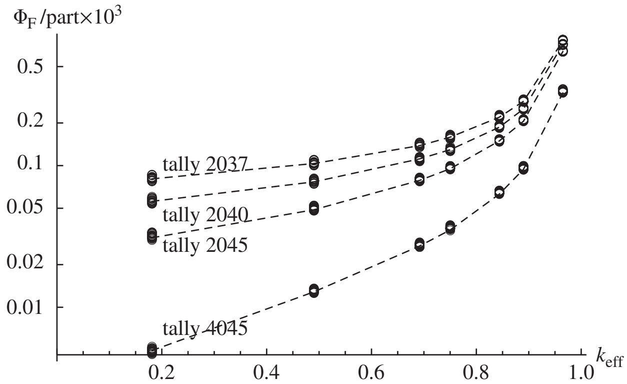

Figure 2. Logarithmic plot of the "fast" component of neutron flux for 86 tallies at various distances from the source (see text).

$$R(k_{eff}, \vec{x}; E_{th}) = \frac{\displaystyle\int_{E>E_{th}} \Phi(E)\, dE}{\displaystyle\int \Phi(E)\, dE} \qquad (1)$$

where Φ is the (local) neutron flux, E is the neutron energy and E_{th} is some specific energy threshold. In (1) we explicitly indicated the possible functional dependences of the ratio R; however we expect this definition to be effective to determine k_{eff} when measured in positions far from the source, where fast flux component is determined mostly by fission. In particular one expects a relation between R and k_{eff} simpler than the one displayed in Figure 1, because the fast component of the flux behaves itself in a very similar way, see Figure 2.

In order to define in some way the "fast" component of the spectrum, we distinguished in [14] between neutrons with kinetic energy below and above 0.5 MeV. The choice of this (in some sense arbitrary cutoff) is related to the threshold energy for fast fission in some actinides; consequently the "fast" neutrons so defined could be able to produce a signal in suitably coated fission chambers, while "slow" neutrons could not. Starting from this definition, an alternative observable, possibly related to k_{eff}, is given by the measurement of the relative fast flux component on fuel rods, in an ADS sufficiently far from the central target. If we suppose that the primary fast neutrons from the source are almost completely absorbed in the innermost part of the core, the fast component of the flux in the peripheral rods should be mostly due to those secondary fission neutrons, which are only partially slowed down by interactions with fuel and coolant [14].

If these arguments are correct we expect R, for peripheral rods and for energy threshold equal to E_{th}, to depend essentially on the fission rate i.e. it should be represented by a proper function of k_{eff} alone. If that were the case, such an observable would offer the advantage of being measurable in all operational conditions. Moreover, it should be relatively independent from the type of neutron source, being essentially a characteristic of the specific core, and it could be derived from two quantities measured with similar devices. In particular, it could be used for monitoring the reactivity level periodically when the accelerator is running in continuous mode, thereby providing a real-time, source- and current- independent cross check of the power-to-current indicator without turning off the beam.

However, the extraction of k_{eff} from the direct measurement of R could in principle be affected and complicated by the dependence on other parameters such as geometry (rod dimensions, fuel to coolant volume ratio, etc.), composition (fuel and coolant materials) and the detector radial position into the core. The main result from [14] was that for peripheral rods the dependence of R on many core details, but material composition, is very mild, leaving a very simple k_{eff} dependence, which turns out to be essentially linear over a wide range of k_{eff} values (at least for $k_{eff} \gtrsim 0.7$).

This result confirms our previous guess grounded on physical arguments. In order to investigate to some extent this residual parameter dependence, we performed simulations of $R(k_{eff})$ using MCNP6 on the previously mentioned LEADS model (see section 2).

2 Model

In order to have a (relatively) realistic model, we assumed the design of a research fast ADS (LEADS), based on solid lead, described in [17] and [18] and already used also for transmutation purposes [19]. The reactor model has the following features:

- inner core (1.6 x 1.6 x 90 cm^3 hexahedral type lattice containing the 0.37 cm radius fuel rods surrounded by a 0.05 cm thick AISI steel cladding and embedded in solid lead) with 50 cm radius and 90 cm height, surrounded by a 120 cm radius and 160 cm high lead reflector, in turn contained within a 5 cm thick AISI steel vessel;
- 2 cm radius central axial beam pipe hosting the Be target;
- beam pipe filled with helium gas for the purpose of target cooling;
- a mixture of ^{238}U and ^{235}U as fuel.
- helium gas core cooling

A sketch of the used (LEADS) model is reported in Fig. 3.

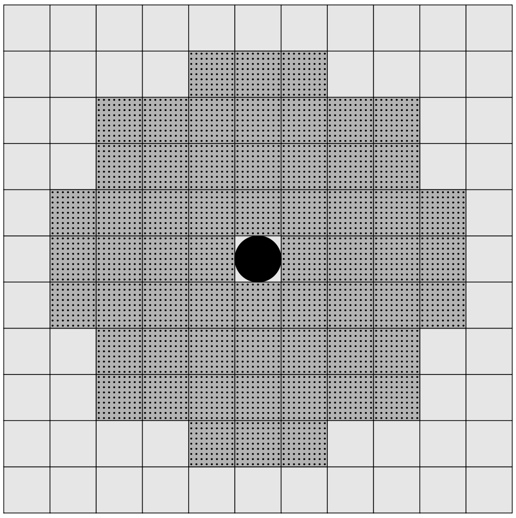

Figure 3. Core shape for LEADS model: in light grey the lead assemblies [14].

The present contribution aims to a more detailed investigation of the validity of the conclusion drawn in [14], in particular on the (residual) dependences of R on the choice of the cutoff energy E_{th} and on the position of the tally inside the core. To this purpose we evaluated the neutron flux in all the independent core positions for LEADS model: symmetry considerations restrict the number of different tallies to $81 \times 11 = 891$, see Figure 4. We identify each tally with four integer numbers, the first two indicating the x and y position of the fuels assembly (FA) with respect to the source in x=0, y=0, the second two indicating each individual tally inside a FA, from 01 to 81 from left to right and down to up (see fig. 4 for some examples of this convention). Flux has been averaged over a 2 cm high portion of the fuel pin at the mid plane in vertical direction.

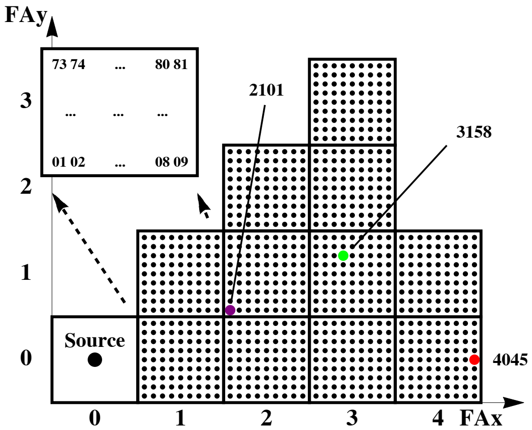

Figure 4. The convention used to identify tallies, In red tally 4045, the one used in [14], in green tally 3158 and in purple tally 2101. These tallies has been magnified with respect to real dimensions to make their identification easier.

3 Results

The most relevant conclusion drawn in [14] was that "*for $k_{eff} > 0.7$, the ratio calculated for LFR or GFR models turns out to show a quite similar trend well approximated by a straight line, while a different slope is observed for a sodium reactor. In our opinion, the results obtained show that such observable, after appropriate calibrations on the real systems under consideration, may be used for determining k_{eff} in all operational conditions, in particular for subcritical systems with different types of neutron source and different values of the beam current.*"

A first point to be investigated concerns the validity of a linear representation of the function $R\left(k_{eff}\right)$ at different positions in the core: we then considered all the tallies on the straight line from positions 1037 to 4045 and we performed two fits in the form $a + bk_{eff}$ and $c + dk_{eff} + ek^2_{eff}$, with $E_{th} = 0.5$ MeV, the same value assumed in [14]. Results for the coefficient of determination test [unfortunately usually indicated as R^2, here to not be confused with the function $R\left(k_{eff}\right)$ whose dependency of k_{eff} has been fitted] are shown in Figure 5: it is clear that for the

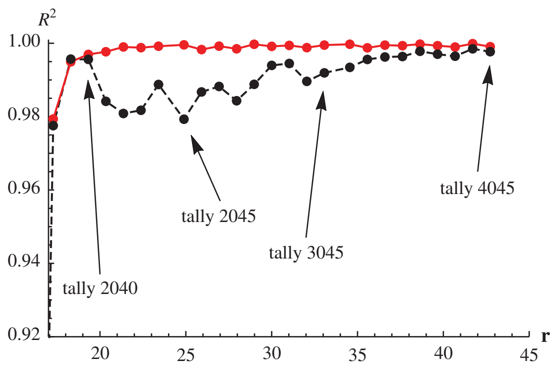

Figure 5. Coefficient of determination (R^2) tests for linear and parabolic fits to $R(k_{eff})$, for tallies from positions 1037 to 4045 as a function of their distance from the source. Upper (solid red) curve for the parabolic fit, lower (dashed black) curve for the linear fit. Tallies at lower distances have been omitted because they cannot be represented well by any of the two fits.

first few tallies near the reflector linear and parabolic fits to $R\left(k_{eff}\right)$ are equivalent, while as far as one considers tallies near to the source both become unable to represent the function. At intermediate distances parabolic fit is better than linear one. To better understand the meaning of the results in Figure 5 we show also the function $R\left(k_{eff}\right)$ for four different tallies 4045 (the original one used in [14]), 2045, 2040 and 2037, together with the corresponding linear (solid) and parabolic (dashed) fits. One immediately realizes that the increase in the coefficient of determination value observed for the linear fits in figure 5 around tally 2040, is accidentally due to a change of the sign in the second derivative of $R\left(k_{eff}\right)$ when moving towards the source.

We can observe another important feature in figure 6: the mean slope of $R\left(k_{eff}\right)$ decreases when moving towards the source: this confirms our original guess [14] that

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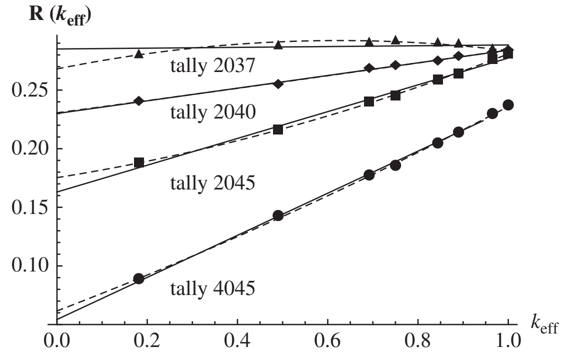

Figure 6. $R\left(k_{eff}\right)$ for 4 different tallies: 4045 (circles), 2045 (squares), 2040 (diamonds) and 2037 (triangles). Linear (solid) and parabolic (dashed) fits are also shown.

the best choice for the reference tally is at the border of the core (or near to it), because in this case the value of $R\left(k_{eff}\right)$ is more sensitive to k_{eff}.

3.1 E_{th} dependence

If we look at $R\left(k_{eff}\right)$ as an estimator for k_{eff}, sensitivity is larger when large variations in R are caused by small variations in k_{eff}, that is to say the optimal condition should be the maximum slope of $R\left(k_{eff}\right)$: in Figure 7 we show how the slope of $R\left(k_{eff}\right)$ depends on the choice of the energy cutoff parameter E_{th}. The maximum slope is attained

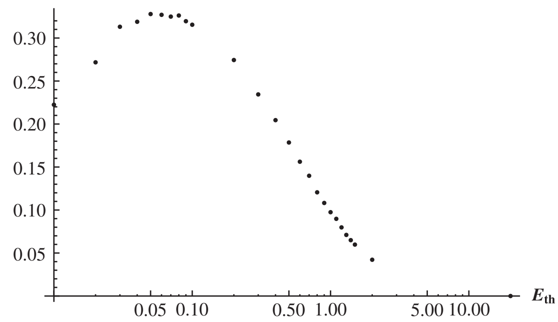

Figure 7. Variation of the slope of the linear fit to $R\left(k_{eff}\right)$ as a function of E_{th} for tally 4045.

for E_{th} in the range $0.03 - 0.1$ MeV, which is lower then the value used naively in [14]. This may be due to the fact that fission spectrum is somewhat softened by the medium. However it may not be obvious to practically implement such a low thresold in an experiment.

3.2 Position dependence

We now examine the position dependence of R: this is not so important from a practical point of view, in that to use $R\left(k_{eff}\right)$ as en estimator for k_{eff} requires anyway to perform a (local) calibration. However, as we shall see, position dependence of R reveals some non trivial features, which in our opinion deserve some attention.

The neutron flux depends on energy and position, then in principle $R\left(k_{eff}\right)$ is really $R\left(k_{eff}; \vec{x}, E_{th}\right)$; in Figure 8 we show the countour plots of R, for various values of k_{eff}, as a function of xy-coordinates of pins.

From Figure 8 we see that an approximate dependence on the distance from the source is effectively realized. This is only a qualitative observation: to make it more quantitative we collect together all the results from tallies with (approximately) the same distance from the source. This is naturally not exactly possible: the best we can do is to collect together results from all tallies whose distances from the source are the same within a given tolerance (tol), or, that is the same, all tallies with a distance from the source $d \in [d_{ref} - tol/2, d_{ref} + tol/2]$. We give an example of such selection procedure in Figure 9. For evident

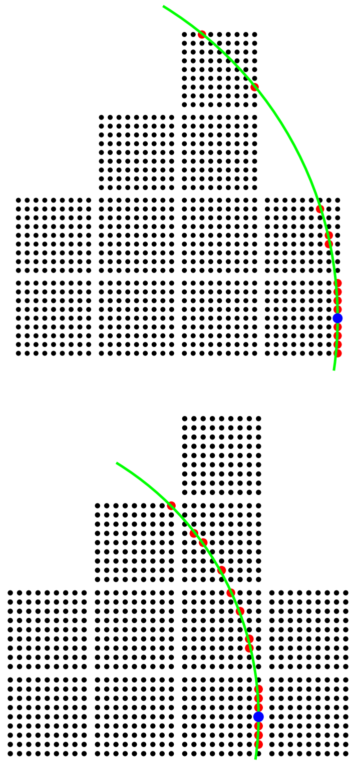

Figure 9. Two example of sets of tallies at the same distance from source with respect to a reference one (plotted in blue), with a tolerance d=2 mm

geometrical reasons the distribution of tallies selected by this procedure is more effective in testing different areas of the core at intermediate distances, as can be clearly seen in the figure. Then we show the same data as in Figure 6, but including all tallies within a tolerance of 2 mm. from the reference ones, together with the corresponding linear (solid) and parabolic (dashed) fits. Roughly speaking this means that $R\left(k_{eff}; \vec{x}, E_{th}\right)$ can be represented as $R\left(k_{eff}; \sqrt{x^2+y^2}, z, E_{th}\right)$; this (approximate) property holds better for a fast reactor, in our opinion, because of the longer mean free path of neutrons, which makes it more difficult to detect the specific details of geometry - including for instance pin diameter [14] - of the system

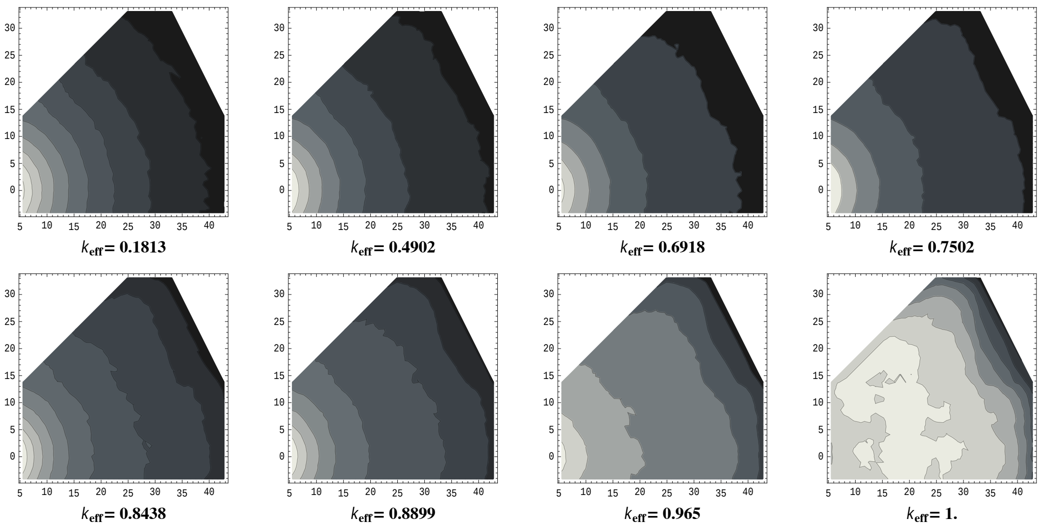

Figure 8. Countour plot of R as a function of tally position for various values of k_{eff} (E_{th} = 0.5).

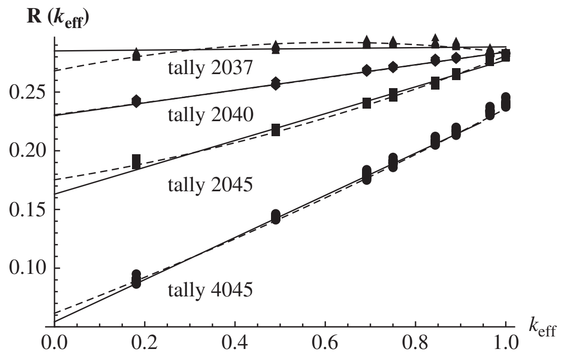

Figure 10. The same as Figure 6, but including all tallies at the "same distances" as reference ones, with a tolerance tol = 2 mm (see text).

as noted before in the introduction. We note also that the dispersions of values for different tallies at the same k_{eff} values are larger than for the case of the total neutron flux of Figure 1: there may be two reasons for this, the first being that since R is a ratio, absolute relative errors for numerator (the fast component of the flux) and the denominator (the total flux) sum up and the second being that the range of variability of R is reduced with respect to that of the total flux.

4 Conclusions

As a general conclusion we can confirm the results of [14]: R appears to be a promising and (relatively) robust estimator for k_{eff} in fast reactors, whose values can be determined by two measurements from the same or similar detector.

Moreover we observe that the maximum sensitivity of R to k_{eff} is obtained, for a fixed cutoff, at the boundary (but inside) the core. Therefore by appropriately choosing the energy threshold E_{th} as permitted by actual detectors, and by placing the detectors in that core region, the measurements can provide a good sensitivity to k_{eff} to be checked against calibrations.

We did not yet investigated how the efficiency of the detector for neutrons of different energies can influence the relation $R(k_{eff})$, a feature that can be studied by properly modifying the definition of R, and how this can interfere with the necessary calibration procedure.

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