

Analytic dc thermoelectric conductivities in holography with massive gravitons

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We provide an analytical derivation of the thermoelectric transport coefficients of the simplest momentum-dissipating model in gauge/gravity where the lack of momentum conservation is realized by means of explicit graviton mass in the bulk. We rely on the procedure recently described by Donos and Gauntlett for holographic models where momentum dissipation is realized through nontrivial scalars. The analytical approach confirms and supports the results found previously by means of numerical computations and the associated holographic renormalization procedure. Importantly, it also provides a precise identification of the range of validity of the hydrodynamic approximation.

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I. INTRODUCTION AND CONVENTIONS

The purpose of the present paper is to provide an analytic derivation of the dc thermoelectric transport coefficients for holographic models with massive gravitons in the bulk. These formulas were recently proposed on the basis of numerical computations in [1]. Holographic models featuring momentum dissipation due to specific scalar field setups have been treated analytically in [2]. As suggested by the authors of [2], the same analytical approach can also be exported to holographic models where the momentum dissipation is simply realized by means of explicit mass terms for the bulk gravitons. In line with this hint, we apply the method of [2] to provide an analytic derivation of the formulas emerged from the numerical analysis of the thermoelectric transport of the holographic massive gravity model proposed in [3].

The approach described in [2] to compute the dc thermoelectric response is based on the analysis of quantities which do not evolve from the IR to the UV. In other words, it relies on a “membrane paradigm” [4] for massive gravity bulk models. The analytic results corroborate our previous numerical results about the behavior of the dc transport coefficients which were inspired by field theoretical expectation for systems featuring elastic scattering due to impurities [5].

In order to fix the conventions, let us recall the main properties of the massive gravity model which we have

analyzed in [1] (previously studied in [3,6,7]).¹ The gravitational action of the model is

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa_4^2} \left(R + \frac{6}{L^2} + \beta([\mathcal{K}]^2 - [\mathcal{K}^2]) \right) - \frac{1}{4q^2} F_{\mu\nu} F^{\mu\nu} \right] + \frac{1}{2\kappa_4^2} \int_{z=z_{UV}} d^3x \sqrt{-g_b} 2K + \frac{1}{2\kappa_4^2} \int_{z=z_{UV}} d^3x \sqrt{-g_b} \frac{4}{L}, \quad (1.1)$$

where β is an arbitrary parameter having the dimension of a mass squared² and, as usual, L represents the curvature radius of the asymptotically anti-de Sitter bulk solutions. The matrices \mathcal{K}_ν^μ and $(\mathcal{K}^2)_\nu^\mu$ are defined in the following way:

$$(\mathcal{K}^2)_\nu^\mu \equiv g^{\mu\rho} f_{\rho\nu}, \quad \mathcal{K} \equiv (\sqrt{\mathcal{K}^2})_\nu^\mu, \quad \text{with} \\ f_{\mu\nu} = \text{diag}(0, 0, 1, 1). \quad (1.2)$$

The small square brackets in (1.1) represent the trace operation and $f_{\mu\nu}$ is a nondynamical fiducial metric which

¹For the details of the computations and the physical meaning of this model we refer to [1,3,6,7].

²We consider just the terms which are quadratic in \mathcal{K} . Linear terms or higher-power terms in \mathcal{K} could be considered as well, see [3]. The motivation for retaining just the quadratic terms in \mathcal{K} is twofold: the proven absence of ghosts and the fact that the bulk gauge field has the same UV falloffs as in the standard massless gravity case.

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explicitly breaks the bulk diffeomorphisms along the spatial x, y directions.³ In (1.1) we have also added to the bulk action two boundary terms. The first one is the usual Gibbons-Hawking term, expressed in terms of the induced metric $(g_b)_{\mu\nu}$ and the trace K of the extrinsic

curvature $K_{\mu\nu}$ on the manifold $z = z_{UV}$,⁴ which is necessary to have a well-defined bulk variational problem; the second one is a counterterm necessary in order to make the on-shell action finite. This model admits black-brane solutions corresponding to the following radial ansatz:

$$\begin{aligned} ds^2 &= \frac{L^2}{z^2} \left[-f(z)dt^2 + dx^2 + dy^2 + \frac{1}{f(z)}dz^2 \right], \quad A = \phi(z)dt, \\ \phi(z) &= \mu - q^2\rho z = \mu \left(1 - \frac{z}{z_h} \right), \quad \rho \equiv \frac{\mu}{q^2 z_h}, \\ f(z) &= 1 - \left(1 + \frac{z_h^2 \gamma^2 \mu^2}{2L^2} \right) \left(\frac{z}{z_h} \right)^3 + \frac{z_h^2 \gamma^2 \mu^2}{2L^2} \left(\frac{z}{z_h} \right)^4 + \beta z_h^2 \left(\frac{z^2}{z_h^2} - \frac{z^3}{z_h^3} \right), \end{aligned} \quad (1.3)$$

where $\gamma = \kappa_4/q$ and z_h is the horizon radius defined by $f(z_h) = 0$. In (1.3) μ is the chemical potential associated to the charge density ρ . The other thermodynamical quantities, namely the temperature T , the energy density \mathcal{E} , the pressure P and the entropy density \mathcal{S} are (see [1])

$$T = -\frac{f'(z_h)}{4\pi} = -\frac{\gamma^2 \mu^2 z_h}{8\pi L^2} + \frac{3}{4\pi z_h} + \frac{\beta z_h}{4\pi}, \quad (1.4)$$

$$\begin{aligned} \mathcal{S} &= \frac{2\pi L^2}{\kappa_4^2 z_h^2}, \quad \mathcal{E} = \frac{L^2}{z_h^3 \kappa_4^2} + \frac{\mu^2}{2q^2 z_h} + \frac{L^2 \beta}{z_h \kappa_4^2}, \\ P &= \frac{L^2}{2\kappa_4^2 z_h^3} + \frac{\mu^2}{4q^2 z_h} - \frac{\beta L^2}{2\kappa_4^2 z_h}. \end{aligned} \quad (1.5)$$

We underline that holographic massive gravity models as well as other specific setups (e.g. [8,9]) realize momentum dissipation while preserving the homogeneity of the bulk differential problem. This leads to the noteworthy technical advantage of dealing with ordinary differential equations instead of partial differential equations.

II. DC TRANSPORT COEFFICIENTS COMPUTATION

A. The electric conductivity and the Seebeck coefficient

Due to the isotropy of the system we are allowed to consider just perturbations in the x direction without loss of generality. Then the static electric conductivity σ_{dc} and the Peltier coefficient Π_{dc} are defined in terms of the electric field E_x , the charge density current J_x and the heat flow Q_x in the following way:

³For more details look at [3] and references therein.

⁴Actually z_{UV} represents a UV cutoff that is eventually sent to zero in the final step of the holographic renormalization procedure.

$$\sigma_{dc} \equiv \left. \frac{J_x}{E_x} \right|_{\nabla_x T=0}, \quad \Pi_{dc} \equiv \left. \frac{Q_x}{E_x} \right|_{\nabla_x T=0}. \quad (2.1)$$

The definitions (2.1) imply that, in order to compute σ_{dc} and s_{dc} , we must consider nonzero electric field and vanishing thermal gradient.

Inspired by [2], we turn on the following fluctuating components:

$$a_x(t, z) = -Et + \tilde{a}_x(z), \quad (2.2)$$

$$h_{tx}(t, z) = \tilde{h}_{tx}(z), \quad (2.3)$$

$$h_{zx}(t, z) = \tilde{h}_{zx}(z), \quad (2.4)$$

where the temporal dependence of the 4-potential a_μ corresponds to a constant electric field E along x and with \tilde{h} we denote fluctuations of the metric components.

The set of coupled linearized equations of motion for the fluctuating fields are

$$\tilde{h}_{tx}''(z) + \frac{2}{z} \tilde{h}_{tx}'(z) + 2 \left(\frac{\beta}{f(z)} - \frac{1}{z^2} \right) \tilde{h}_{tx}(z) - \frac{2\gamma^2 \mu}{z_h} \tilde{a}_x'(z) = 0, \quad (2.5)$$

$$\frac{E z^2 \mu \gamma^2}{z_h f(z)} + \left(\frac{z^4 \mu^2 \gamma^2}{2z_h^2 L^2} - z f'(z) + 3f(z) - 3 \right) \tilde{h}_{zx}(z) = 0, \quad (2.6)$$

$$\tilde{a}_x''(z) + \frac{f'(z)}{f(z)} \tilde{a}_x'(z) - \frac{z^2 \mu}{L^2 z_h f(z)} \tilde{h}_{tx}'(z) - \frac{2z \mu}{L^2 z_h f(z)} \tilde{h}_{tx}(z) = 0. \quad (2.7)$$

Note that Eq. (2.6) for \tilde{h}_{zx} can be solved algebraically and, recalling the explicit expression of the emblackening factor $f(z)$ given in (1.3), the solution can be expressed as follows:

$$\tilde{h}_{zx}(z) = -\frac{E\gamma^2\mu}{z_h\beta f(z)}. \quad (2.8)$$

In order to completely determine the solution of the remaining two equations (2.5) and (2.7), we have to provide suitable boundary conditions for the fluctuation fields $h_{tx}(t, z)$ and $a_x(t, z)$ at the conformal boundary $z = 0$ and at the horizon $z = z_h$. At the horizon we require the regularity of the fluctuations; this requirement can be easily fulfilled by switching to the Eddington-Finkelstein time coordinate,

$$v = t - \frac{1}{4\pi T} \log\left(\frac{z_h - z}{L}\right), \quad (2.9)$$

leaving untouched all the other coordinates. From the IR regularity requirement for all the metric components in the new coordinate system we derive the behavior of \tilde{h}_{tx} at the horizon, namely,

$$\tilde{h}_{tx}(z) = -\frac{E\gamma^2\mu}{z_h\beta} + \mathcal{O}(z - z_h). \quad (2.10)$$

An analogous regularity requirement at the horizon applied to the gauge field yields

$$\tilde{a}_x(z) = \frac{E}{4\pi T} \log\left(\frac{z_h - z}{L}\right) + \mathcal{O}(z - z_h). \quad (2.11)$$

Considering the conformal boundary located at $z = 0$, we have to furnish boundary conditions in such a way that the dual system has an external electric field and vanishing thermal gradient. Given the ansatz (2.2), the small- z leading behavior of the fluctuating field $a(t, z)$ corresponds to a constant electric field E . According to the standard holographic dictionary, the coefficient of the subleading falloff in z of the field $a(t, z)$ corresponds to the charge density current J_x ; namely $\tilde{a}_x \sim J_x z$ at small z . Regarding the metric fluctuation \tilde{h}_{tx} , from Eq. (2.5) it is easy to see that it has two independent behaviors in the near-boundary region, i.e. z and z^{-2} . The holographic dictionary prescribes that imposing that there are no sources associated to thermal gradients corresponds to setting to zero the coefficient of the leading z^{-2} term. All in all, the set of boundary conditions that we have just illustrated determines the solution of the differential equations (2.5) and (2.7) completely.

In order to compute the dc transport coefficients it is fundamental to note that there are two linear combinations of the fluctuations which are independent of the coordinate z and are respectively related to the charge density current and the heat current. The first conserved current is

$$\bar{J}^\mu = -\frac{\sqrt{-g}}{q^2} F^{z\mu}, \quad (2.12)$$

with $\mu = t, x, y$. Given the ansatz (2.2), the only nonzero component of the Maxwell equation $\sqrt{-g} \nabla_M F^{MN} = \partial_z (\sqrt{-g} F^{zx}) = 0$ states that \bar{J}^x is independent of the radial coordinate z and it assumes the following explicit form in terms of the fluctuating fields:

$$\bar{J}^x = -\frac{\sqrt{-g}}{q^2} F^{zx} = \frac{z^2\mu}{L^2 q^2 z_h} \tilde{h}_{tx}(z) - \frac{f(z)}{q^2} \tilde{a}'_x(z). \quad (2.13)$$

Relying on the “radial conservation” we can compute \bar{J}^x both at the horizon $z = z_h$ and at the boundary $z = 0$ knowing that the two results must correspond. Computing it at $z = 0$ and recalling the above-mentioned UV behaviors, it is possible to see that this quantity is actually the charge density current J^x of the dual field theory. Then, evaluating (2.13) at $z = z_h$ we find

$$J^x = \left(\frac{1}{q^2} - \frac{\gamma^2\mu^2}{L^2 q^2 \beta} \right) E. \quad (2.14)$$

The electric conductivity σ_{dc} is now easily computed by means of (2.1),

$$\sigma_{dc} = \frac{J^x}{E} = \frac{1}{q^2} - \frac{\gamma^2\mu^2}{L^2 q^2 \beta}, \quad (2.15)$$

which corresponds exactly with the analytical expression found in [7].

The second conserved quantity, which is related to the heat current, is subtler to identify. Indeed, as noted in [2], it is associated to the existence of the Killing vector $k = \partial_t$ and it assumes the following form:

$$\begin{aligned} \bar{Q} &= \frac{\sqrt{-g}}{\kappa_4^2} \nabla^z k^x - \phi J^x \\ &= \left(\frac{\phi \tilde{a}'_x}{q^2} + \frac{\tilde{h}'_{tx}}{2\kappa_4^2} + \frac{\tilde{h}_{tx}}{\kappa_4^2 z} \right) f + \left(-\frac{f'}{2\kappa_4^2} - \frac{\mu z^2 \phi}{L^2 q^2 z_h} \right) \tilde{h}_{tx}, \end{aligned} \quad (2.16)$$

where in the second passage we gave an explicit expression of \bar{Q} in terms of the fluctuation fields. Evaluating \bar{Q} on the equations of motion (2.5) and (2.7) and keeping into account the particular form of the background quantities f and ϕ (1.3), one has that $\partial_z \bar{Q} = 0$. Namely \bar{Q} is radially conserved.

Having obtained a radially conserved quantity, in order to repeat the same steps done previously for the electric conductivity, we have to prove that \bar{Q} coincides, if evaluated on the boundary, with the heat current in the x direction $Q^x = T^{tx} - \mu J^x$ of the dual field theory. Actually, in this case, the proof of this statement is straightforward. In fact, J^x is constant along the radial direction and $\phi(0) = \mu$, and consequently the term ϕJ^x reduces to μJ^x when evaluated at the conformal boundary. Moreover, it is not

difficult to see that $\frac{\sqrt{-g}}{\kappa_4^2} \nabla^z k^x$ coincides with the linearized tx component of the stress-energy tensor of the dual field theory⁵ evaluated at the conformal boundary $z = 0$, namely,

$$\begin{aligned} T^{tx} &= \frac{L^5}{\kappa_4^2 z^5} \left(-K^{tx} + K g_b^{tx} + \frac{2}{L} g_b^{tx} \right) \\ &= \frac{\tilde{h}'_{tx}}{2\kappa_4^2 \sqrt{f}} - \frac{\tilde{h}_{tx}}{\kappa_4^2 z \sqrt{f}} + \frac{2\tilde{h}_{tx}}{\kappa_4^2 z f}. \end{aligned} \quad (2.17)$$

Finally, having associated the quantity \bar{Q} with the heat flow, the Peltier coefficient is straightforwardly obtained evaluating \bar{Q} at the horizon $z = z_h$ and relying on the IR behavior of the fluctuating fields. We obtain

$$\Pi_{dc} = \frac{\bar{Q}}{E} = -\frac{2\pi\mu}{\beta q^2 z_h} \left(-\frac{\gamma^2 \mu^2 z_h}{8\pi L^2} + \frac{3}{4\pi z_h} + \frac{\beta z_h}{4\pi} \right). \quad (2.18)$$

The expressions for the dc electrical conductivity (2.15) and the Peltier coefficient (2.18) obtained here with an analytical computation along the lines described by [2] coincide with those found in [1] using numerical methods. Note that the term inside the parenthesis of Eq. (2.18) coincides exactly with the temperature (1.4). Consequently, by Onsager reciprocity (see later), one has $s_{dc} = \frac{\Pi_{dc}}{T}$, finding exactly formula (1.4) of [1], which was satisfied by numerical data.

B. Thermal conductivity and Onsager reciprocity

The thermal conductivity $\bar{\kappa}_{dc}$ is defined as

$$\bar{\kappa}_{dc} \equiv \left. \frac{Q_x}{-\nabla_x T} \right|_{E_x=0}. \quad (2.19)$$

Symmetrically to what we have done in the previous section, to compute this quantity we must consider a thermal gradient at vanishing electric field. To this end, we rely on the elegant method described in [2] (to which we refer the reader for further details); we consider the following set of fluctuations:

$$a_x(t, z) = \alpha_2 \phi(z) t + \tilde{a}_x(z), \quad (2.20)$$

$$h_{tx}(t, z) = -\alpha_2 \frac{L^2}{z^2} f(z) t + \tilde{h}_{tx}(z), \quad (2.21)$$

$$h_{zx}(t, z) = \tilde{h}_{zx}(z). \quad (2.22)$$

Note that, following the standard holographic prescription (see for instance [11–13]), the coefficient α_2 corresponds to the thermal gradient $-\nabla_x T/T$. It actually represents the thermal source dual to the boundary value of the h_{tx} bulk field [see (2.21)].

Considering the ansatz (2.20), (2.21) and (2.7), one finds that the equations of motion for \tilde{h}_{tx} and \tilde{a}_x are the same as those given in Sec. II A, namely (2.5) and (2.7). The equation for \tilde{h}_{zx} looks slightly different but is still algebraically solvable; its solution being

$$\tilde{h}_{zx}(z) = -\frac{L^2 \alpha_2 \{2L^2 q^2 [3z(\beta z_h^2 + 1) - 2\beta z_h^3] - \kappa_4^2 \mu^2 z z_h^2\}}{2\beta z(z - z_h) \{\kappa_4^2 \mu^2 z^3 z_h - 2L^2 q^2 [z^2(\beta z_h^2 + 1) + z z_h + z_h^2]\}}. \quad (2.23)$$

As regards the boundary conditions, the regularity of the fluctuations at the horizon, in this case, implies that

$$\tilde{h}_{tx}(z) = -\frac{\alpha_2 (2L^2 q^2 (\beta z_h^2 + 3) - \kappa_4^2 \mu^2 z_h^2)}{4\beta z_h^3 q^2} + \frac{\alpha_2 L^2 f(z)}{2\pi T z^2} \log\left(\frac{z_h - z}{L}\right) + \mathcal{O}(z - z_h), \quad (2.24)$$

and

$$\tilde{a}_x = \mathcal{O}(z - z_h). \quad (2.25)$$

Moreover, at the boundary $z = 0$ we require that \tilde{h}_{tx} is proportional to z and, as before, we have $\tilde{a}_x = J_x z + \dots$, where the ellipsis indicates higher power of z .

Importantly, since the equations of motion for \tilde{h}_{tx} and \tilde{a}_x are the same as in the case studied in the previous section, the quantities \bar{J}^x (2.13) and \bar{Q} (2.16) are still independent of z .

⁵To have an explicit expression of the stress-energy tensor we refer for instance to [10].

Enforcing the near-boundary conditions discussed previously, we have that also in this case \bar{J}^x corresponds to the x component of the dual charge density current. However, the relation between \bar{Q} and the heat current Q^x is in this case more subtle than before. Indeed the tx component of the holographic stress-energy tensor evaluated on the ansatz (2.20), (2.21) and (2.22) for the fluctuations assumes the following form:

$$\begin{aligned} T^{tx} &= t \left(-\frac{\alpha_2 L^2 f'}{2\kappa_4^2 z^2 \sqrt{f}} + \frac{2\alpha_2 L^2 \sqrt{f}}{\kappa_4^2 z^3} - \frac{2\alpha_2 L^2}{\kappa_4^2 z^3} \right) \\ &\quad + \frac{\tilde{h}'_{tx}}{\kappa_4^2 \sqrt{f}} - \frac{\tilde{h}_{tx}}{\kappa_4^2 z \sqrt{f}} + \frac{2\tilde{h}_{tx}}{\kappa_4^2 z f}. \end{aligned} \quad (2.26)$$

Therefore the quantity $\frac{\sqrt{-g}}{\kappa_4^2} \nabla^z k^x$ (contained in \bar{Q}) computed at $z = 0$ equals just the time-independent part of T^{tx} [i.e. the last three terms in (2.26)]. Nevertheless, as discussed in [2], in order to compute the dc response, only the time-independent part of T^{tx} is relevant. To the purpose of computing the dc thermal conductivity we can then assume that \bar{Q} corresponds to the heat current, and evaluating it at the horizon, and considering in this case the horizon conditions (2.24) and (2.25), we find

$$\bar{\kappa}_{dc} = \frac{1}{T \alpha_2} \bar{Q} = \frac{\pi [\gamma^2 \mu^2 z_h^2 - 2L^2(\beta z_h^2 + 3)]}{2\beta \kappa_4^2 z_h^3}, \quad (2.27)$$

which agrees with our previous numerical result reported in [1].

Eventually, relying on the boundary conditions (2.24) and (2.25) we evaluate the charge density current \bar{J}^x (2.13) at $z = z_h$. We can then prove that the Onsager reciprocity relation holds, namely,

$$s_{dc} = \left. \frac{J_x}{-\nabla_x T} \right|_{E_x=0} = -\frac{2\pi\mu}{\beta q^2 z_h} = \frac{\Pi_{dc}}{T}, \quad (2.28)$$

where the last identity can be obtained by using (2.18) and dividing it by the temperature (1.4).

Said otherwise, the conductivity matrix is symmetrical and the Peltier coefficient Π_{dc} is equal to the Seebeck coefficient multiplied by T .

III. DISCUSSION AND CONCLUSIONS

In the present paper we have computed analytically the dc thermoelectric transport coefficients for holographic models featuring momentum dissipation realized by explicit graviton mass in the bulk. We followed precisely the method illustrated in [2] and applied it to the holographic model first introduced in [3]. Introducing the explicit form of the dissipation rate for the model

$$\tau^{-1} = -\frac{S\beta}{2\pi(\mathcal{E} + P)}, \quad (3.1)$$

which was found in [6] by studying the pole of the propagators in the hydrodynamic regime, and using the explicit expression of the thermodynamical quantities (1.4) and (1.5), the transport coefficients assume the following beautiful form:

$$\begin{aligned} \sigma_{dc} &= \frac{1}{q^2} + \frac{\rho^2}{\mathcal{E} + P} \tau, & s_{dc} &= \frac{S\rho}{\mathcal{E} + P} \tau, \\ \bar{\kappa}_{dc} &= \frac{S^2 T}{\mathcal{E} + P} \tau. \end{aligned} \quad (3.2)$$

Such explicit expressions coincide with those found in [1] relying on a numerical analysis.

The analytical results outlined in this paper put a firm ground to the statements made in [1], which were mainly obtained on purely phenomenological observation. First of all, we are now able to establish precise limits on the validity of the hydrodynamic regime. In fact in [1] we have found that for certain values of the thermodynamic quantities and of the parameters of the model the thermoelectric transport coefficients computed in massive gravity are compatible with the hydrodynamic prediction made in [14], namely,

$$\begin{aligned} \sigma_{dc} &= \frac{1}{q^2} + \frac{\rho^2}{\mathcal{E} + P} \tau, & s_{dc} &= -\frac{1}{q^2 T} + \frac{S\rho}{\mathcal{E} + P} \tau, \\ \bar{\kappa}_{dc} &= \frac{1}{q^2 T} + \frac{S^2 T}{\mathcal{E} + P} \tau. \end{aligned} \quad (3.3)$$

By the comparison of the previous formulas with the exact results (3.2) it is easy to see that, in order for the hydrodynamic approximation to be valid, the following inequality must hold:

$$\tau^{-1} \ll \min \left\{ \frac{S^2 q^2}{\mathcal{E} + P} \frac{T^2}{\mu^2}, \frac{S\rho q^2}{\mathcal{E} + P} \frac{T}{\mu} \right\}. \quad (3.4)$$

Since both of the terms $\frac{S\rho q^2}{\mathcal{E} + P}$ and $\frac{S^2 q^2}{\mathcal{E} + P}$ are approximatively constant at sufficiently low T , one can see that the first of the two bounds to the dissipation rate τ^{-1} in (3.4) is the lowest bound near $T/\mu \sim 0$; in contrast the second bound is the most stringent at higher values of the ratio T/μ .

It is also interesting to note that the present analysis allows us to pinpoint a technical aspect of [1]. In fact, the analysis described in [1] highlighted the importance of a careful holographic renormalization procedure to the aim of obtaining a physically meaningful thermoelectric response. Specifically, the reduced amount of symmetry due to the mass term for the bulk graviton allows for a wider set of possible boundary terms. Requiring a well-behaved high-frequency response and the absence of unphysical dissipationless heat transport fixes the coefficients of the finite counterterms. This latter feature corresponds to the physical need of absorbing a dc delta function in the real part of the thermal conductivity signalled by a pole in the corresponding imaginary part. To be more precise, in order to have a well-behaved imaginary part of the spectral thermal conductivity in the limit $\omega \rightarrow 0$ we found it necessary to add to the action (1.1) the finite counterterm

$$S_{c.t.}^{(fin)} = -\frac{1}{4} \mathcal{E} \int_{z=z_{UV}} d^3 x \frac{z}{L} \sqrt{-g_{tt}} g^{tt} g^{xx} h_{tx} h_{tx}. \quad (3.5)$$

Note that the coefficients in front of this counterterm are unusually state dependent and the reason of the presence of this term in the action requires a more specific analysis, which we postpone to future studies. However, adding

these finite counterterms to the action (1.1) would have resulted in the presence of additional terms in the tx component of the holographic stress-energy tensor T^{tx} and such additional terms, in accordance with the ansatz (2.2)–(2.4), result in a linear time-dependent term (which is of no importance in computing the dc response).

Finally, as regards the future perspective, they are manifold. All the extensions of the simplest massive gravity holographic model could be addressed from the membrane paradigm standpoint developed in [7] and [2], the hope

being to extend the analytical reach in the context of holographic systems featuring momentum dissipation.

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