Veering phenomena in cable-stayed bridge dynamics

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ABSTRACT: The analytic results of a simple model of a cable-stayed beam allows one to describe the dynamic characteristics of stayed-structures. The model is used either to evidence the capacity of a newly introduced localization factor to identify local modes involving mainly the cables, and to reveal a veering phenomenon occurring between local and global frequencies. The variation of a mechanical parameter produces a flipping between local/global (cable/beam) modal shapes through a veering region, where hybrid modes born as combination of the involved eigenfunctions. The occurrence of hybrid modes is also captured by the coincidence of the associated localization factors. These dynamical properties opportunely described by the introduced localization factor, have been recognized still valid in a refined finite element model of a cable-stayed bridge.

1 INTRODUCTION

Cable-stayed bridges actually represent a structurally efficient, economically competitive and aesthetically appealing solution to cover free central span in the medium-high length range (200-900 m). The recent technical evolutions and the introduction of high-performing materials have rapidly increased the lightness and flexibility of these structures and their substructural components, leading to significant nonlinear behaviour, due to the effects of large deflections and geometric stiffness (Abdel-Ghaffar & Nazmy 1991, Wang & Yang 1996).

Moreover, high amplitude localized oscillations can arise in the stay cables, whose possible source of vibrations are either direct rain-wind external excitation or cable-deck boundary interactions related to the occurrence of internal resonance conditions (1:1, 1:2, 2:1) between global and local frequencies (Gattulli & Lepidi 2003, Gattulli 2004). Even if the importance of local vibrations in the global dynamics of cable-stayed bridge has been recognized (Abdel-Ghaffar & Khalifa 1991, Tuladhar et al. 1995), cable transversal motion is usually neglected in common dynamic analysis, in which only their stiffening contribution to the global structural behaviour is taken into account by treating them as tendon elements with Ernst equivalent modulus (Ernst 1965, Wilson & Gravelle 1991, Eurocode 3, 2003).

Different authors have suggested to enhance the dynamical description through appropriate treatment of transversal cable motion in finite element models of stayed structures (Warnitchai et al. 1995, Ali & Abdel-Ghaffar 1995). In particular, Warnitchai and his co-authors proposed to investigate cable-deck interactions by means of a Ritz-type analytical model based on test functions representative of global and local modes. Their analytical model, also validated through an experimental set-up of a cable-stayed beam, describes global motions by cantilever eigenfunctions in the beam domain and quasi-static functions in the cable domain, while Irvine cable eigenfunctions account for local motions. Meanwhile, in their experiments they have observed a so-called modal distortion indicating that internal 1:1 resonance conditions yield a linear coupling between the beam and the cable, able to significantly modify the assumed cantilever modal shapes. Similar results on non-negligible effects produced by the cable transversal dynamics on the modal properties of real-scale structures have been obtained in finite element models by Au et al (2001) and Caetano et al (2000), in which the cables are modeled as a linkage of truss elements including transversal geometric stiffness. Thus, even if the evidence of such phenomenon is clear from experimental data and numerical results, no convincing attempts to analytically investigate its origin have been still performed.

The analytical models of several physical systems show that eigenvalues and eigenfunctions strongly depend on a set of mechanical parameters. This dependence is usually studied through the analysis of eigensolution loci curves versus a selected significant parameter. Two curves approaching each other may exhibit two different behaviours: an intersection of the two curves, for a critical parameter value, corresponding to a coalescence of two eigenfrequencies, is known as *crossing* condition; otherwise a sudden divergence of the two curves causes an avoided crossing, also known as *veering* phenomenon. In the latter case, the eigenfunctions associated with the eigenfrequencies involved in the veering interchange their shapes in a rapid but continuous way. The frequency veering has been often identified in assemblage of equal subsystems, as a consequence of regularity-breaking of structural periodicity, and it is widely known how this may lead to strong localization of vibrations (Perkins & Mote 1986, Pierre 1988).

In this paper a simple nonlinear model of a cable supported beam is used to replicate the cable-deck interaction in stayed structures (Gattulli et al. 2002). The analytical closed-form solution of the related linearized eigenproblem allows one to exactly evaluate the localization degree of the modal eigenfunctions in the cable domain. A wide parametric analysis then reveals that a veering phenomenon occurs in internal primary resonance conditions between the local and the global eigenfrequencies.

The modes born from the combination of these eigenfunctions are usually called hybrid modes (Triantafyllou & Triantafyllou 1986). The obtained results clearly show that the modal hybridization cannot anyway be described if Ritz-type functions are used instead of the exact system eigenfunctions, because they are not able to reproduce the eigenfrequency veering responsible of the phenomenon. A proper treatment of the cable transverse dynamics in finite element models is proved to efficiently match the modal behaviour of the cable-stayed beam in the veering region. The effects of mode hybridization in a real scale cable-stayed bridge with close global and local frequencies are finally described.

2 ANALYTICAL MODEL

A cable supported cantilever beam (see Fig.1) is analytically modeled to study the mechanical behaviour of cable-stayed bridges, since it is the simplest system still able to thoroughly describe the interactions between the cables and the deck.

Under the assumption of small sag D to length L_c ratio (i.e. $\frac{D}{L_c} \leq \frac{1}{10}$), the static equilibrium configuration can be approximated by the parabolic function $Y(X_c) = 4D\left[X_c/L_c - (X_c/L_c)^2\right]$ in the cable domain, while the beam static deflection is assumed to be negligible (Fig.1a). With respect to this reference

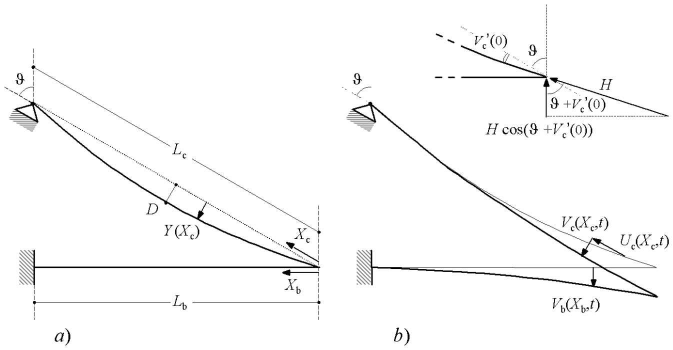

Figure 1. Configurations of the cable-stayed beam: (a) reference, (b) instantaneous.

configuration, and assuming an Euler-Bernoulli, axially rigid model of the beam, the planar actual configuration of the system is completely described by the cable displacement components $U_c(X_c)$, $V_c(X_c)$ and the beam transverse displacements $V_b(X_b)$ (Fig.1b). We denote ω_1 the first structural modal frequency, m_b and m_c the beam and cable mass per unit length, Ξ_b and Ξ_c the transverse damping coefficient per unit length, E_bI_b the beam flexural stiffness, E_cA_c the cable axial stiffness, and H the mean static tension in the cable.

The usual hypothesis that the longitudinal acceleration $\ddot{U}_c$ can be neglected in the prevalent transverse cable motion is adopted, and the following non-dimensional variables and parameters are introduced

$$y = \frac{Y}{D}; \;\; \tau = \omega_1 t; \tag{1}$$

$$x_i = \frac{X_i}{L_i}; \;\; v_i = \frac{V_i}{L_i}; \;\; \xi_i = \frac{\Xi_i}{m_i\omega_1}; \;\; (i = b, c) \tag{2}$$

$$\rho = \frac{m_c}{m_b}; \;\; \mu = \frac{E_cA_c}{H}; \;\; \nu = \frac{D}{L_c}; \;\; \chi = \frac{E_bI_b}{L_b^2 E_cA_c};$$

By using the classical extended Hamilton's Principle, and defining the non-dimensional constants

$$\beta_{b1}^4 = \omega_1^2 \frac{m_b L_b^4}{E_b I_b}; \;\;\; \beta_{c1}^2 = \omega_1^2 \frac{m_c L_c^2}{H} \tag{3}$$

the equations of motion governing the transverse vibrations are obtained

$$\begin{aligned} \beta_{b1}^4 \ddot{v}_b + \xi_b \beta_{b1}^4 \dot{v}_b + v_b'''' &= 0 \\ \beta_{c1}^2 \ddot{v}_c + \xi_c \beta_{c1}^2 \dot{v}_c - \left[v_c' + \mu\left(\nu y' + v_c'\right)\bar{e}\right]' &= 0 \end{aligned} \tag{4}$$

where the displacement components v_b and v_c, for any value of τ, satisfy the following geometric

$$\begin{aligned} &v_c(0) - v_b(0)\sin^2\vartheta = 0; \qquad v_c(1) = 0; \\ &v_b(1) = 0; \qquad\qquad v_b'(1) = 0 \end{aligned} \tag{5}$$

and mechanical boundary conditions

$$\mu\chi v_b'''(0) - v_c'(0)\sin\vartheta =$$

$$\mu\left[(\nu y'(0) + v_c'(0))\sin\vartheta - \cos\vartheta\right]\bar{e}; \qquad v_b''(0) = 0 \quad (6)$$

In Eqs.(4)-(6), primes and dots indicate differentiation with respect to the abscissae x_b and x_c and nondimensional time τ, and $\bar{e} = \bar{e}(\tau)$ represents the uniform dynamic elongation given by

$$\bar{e} = v_b(0)\sin\vartheta\cos\vartheta + \int_0^1 \left(\nu y' v_c' + \frac{1}{2}v_c'^2\right) dx_c \quad (7)$$

3 MODAL ANALYSIS

The equations governing the small amplitude structural oscillations can be obtained by linearizing Eqs.(4)-(6). The related eigenvalue problem has been solved in exact closed-form (Gattulli et al. 2002). It's then possible to apply simple continuation techniques directly to the characteristic equation in order to numerically follow the eigenfrequency loci $\beta_{bi}(\rho,\chi)$ in the bi-dimensional space of the significant mechanical parameters (ρ,χ). The natural frequencies of the system ω_i are directly related to the roots β_{bi} of the characteristic equation through Eq.(3) (see also Gattulli et al. 2002).

3.1 *Cable modal localization*

Wide numerical analyses on the eigensolution sensitivity to system parameter variations have been performed (Gattulli et. al 2002, Gattulli & Lepidi 2003). These analyses have suggested a shape-based distinction on the system eigenfunctions, already present in technical literature, because two different mode types can be basically recognized. The modal shape of some modes mainly involves the dynamic deflection of the beam, while the cable seems to be quasi-statically dragged by the beam tip (see Fig.2 - A1 and B1). Differently, other modes are extremely localized in the cable domain, while the beam participation to the modal shape appears to be negligible (see Fig.2 - B2 and C1). In the following these two types of modes are classified simply as *global modes* and *local modes*, respectively. Particularly, it is worth noting that, due to the strongly non-homogeneous mass distribution in cable-stayed beams, global modes, with beam-dominant modal shapes, present higher values of modal mass with respect to local modes, with cable-dominant modal shapes.

Complex systems made up of structural subsystems with strongly dissimilar dynamical properties (mass, stiffness, damping) have been proved to be affected by mode localization, if superposition between the natural frequency content of each subsystem exist (Natsiavas 1993). This mechanism of mode localization is obviously different by the usually studied source of localization due to the presence of a perturbation (disorder) in the order of regular or periodic structures (Pierre & Dowell 1987, Luongo 1992). Particularly, it is not the consequence of a casually introduced artificial irregularity, but it is an inherent structural property, systematically verified in almost the whole parameter space. Indeed, subsystems with different mechanical properties may exhibit nearly independent linear dynamics, or local oscillations autonomous from the main structure global motion.

In the studied system, till the elasto-dynamic parameters ensure that the global frequencies are well-separated from the local ones, the transversal motion of the cable may be effectively studied as independent from the beam motion, and vice versa. In this case it is then possible to describe with a sufficiently accurate approximation the global and local dynamics using Ritz-type functions (Warnitchai et al. 1995).

To properly describe the localization degree of each eigenfunction, an opportune nondimensional parameter has to be defined. To this aim, many studies on the localization effects introduced by regularity-breaking perturbations of periodical structures damping refer to the damping analogy (Luongo 1992). From a mechanical viewpoint, the confinement of inertial energy in the region of modal localization, and the absence of vibration diffusion in the remaining structure, has the same features of a strong structural damping, which reduces the propagation of the motion due to a point dynamic load. More recently, in the field of cable-stayed structures dynamics, a simple ratio between the maximum cable and deck displacements has been used to evaluate the localization degree of the modes obtained from finite element models of a real scale cable-stayed bridge (Caetano et. al 2000).

Differently, here, the following localization factor can be defined and exactly evaluated for the ith eigenfunction $\boldsymbol{\phi}_{\mathrm{i}}(\mathbf{x})$=$\{\phi_{bi}(x_b),\phi_{ci}(x_c)\}^{\mathrm{T}}$

$$\Lambda_{\mathrm{i}} = \frac{\int_0^1 m_c\,\phi_{ci}^2(x_c)\,dx_c}{\int_0^1 m_b\,\phi_{bi}^2(x_b)\,dx_b + \int_0^1 m_c\,\phi_{ci}^2(x_c)\,dx_c} \quad (8)$$

The above defined factor could be interpreted with the physical meaning of the ratio between the kinematic energy stored in the cable domain and the total kinematic energy of the system undergoing monofrequent (ω_i) free linear oscillations. Its definition is thus consistent with the need of paying major attention on the modal localization in the cable domain, since the flexibility and the low damping characteristics of the cables make them vulnerable to large amplitude oscillations, often amplified by a marked nonlinear behaviour. On the other hand, a high value the complementary factor $\Lambda_{\mathrm{i}}^{\mathrm{C}} = (1-\Lambda_{\mathrm{i}})$, means a strong kinetic

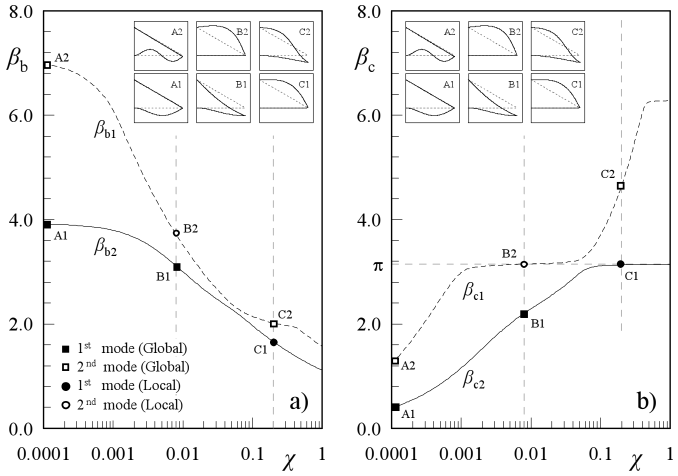

Figure 2. Eigenfrequency loci curves in the χ-space for $\{\rho, \mu, \vartheta, \nu\} = \{0.01, 486, 60^o, 0\}$

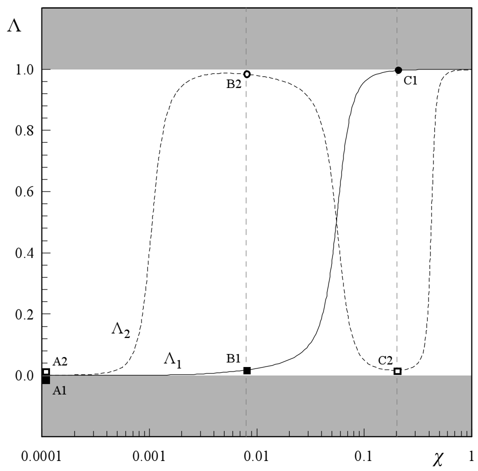

Figure 3. Localization factors in the χ-space for $\{\rho, \mu, \vartheta, \nu\} = \{0.01, 486, 60^o, 0\}$.

energy localization on the beam domain, with quasi-static participation of the cable. However, since here the beam ideally represents the main structure of a cable-stayed system, it is preferable from a technical viewpoint refer to the beam-dominant eigenfunctions as *global* modes.

Fig. 2(a) shows eigenfrequency loci of first and second beam eigenfrequencies β_{bi} in the χ-space. The corresponding loci of cable eigenfrequencies β_{ci} are shown in Fig. 2(b).

For vanishing χ-values, cable axial stiffness much greater than beam flexural stiffness, the cable-stayed beam behaves as a fixed-supported beam (Gattulli et al. 2002). Consequently, the two first eigenfunctions are both global and then exhibit low values of the related localization factors Λ_i (see the global-global sequence A1-A2 for $\chi = 0.0001$ in Fig.3).

Instead, larger χ-values lead to a monotonically decreasing of beam eigenfrequencies β_{bi}, while the cable eigenfrequencies β_{ci} monotonically increase till the higher one becomes close to the value $\beta_{c2} = \pi$.

The closeness to the first natural frequency of a taut string attracts the system to behave in a localized fashion in which the presence of the beam just minimally affects the dynamic behaviour of the cable. This system features are preserved for a wide χ-value range around $\chi = 0.01$ (see Fig.2(b)). Consequently, here, the localization factor reveals a strong localization of the second modal shape in the cable domain ($\Lambda_2 \simeq 1$ and see the modal shapes of the global-local sequence B1-B2 for $\chi = 0.008$).

Increasing further the χ-value yields to a frequency veering, which produces a sudden flipping between the localization factors Λ_i around $\chi = 0.05$ (Fig.3), and leads to an exchange of the modal shapes of the two modes (see Fig.2, the local-global sequence C1-C2 for $\chi = 0.2$). The introduced localization factor has a double property, indeed, its evaluation permits to evidence: the presence of localized modes in the cables when $\Lambda_i \simeq 1$, and the mode hybridization when $\Lambda_i \simeq \Lambda_j \simeq 0.5$. These remarks are easily extended to finite element modal analysis of a cable-stayed bridge, whose results can be properly used to clearly classify global, local and hybrid modes in these complex structures.

3.2 *Parameter influence on frequency veering*

The analytical model of the cable-stayed beam allows one to obtain the eigenfrequencies and the associated eigenfunctions depending on the $(\rho, \chi, \mu, \vartheta, \nu)$ parameter values. Parametrical investigations (Gattulli et al. 2002) lead to recognize beam - dominant modal shapes, namely global modes, and cable - dominant modal shapes, namely local modes.

However, as previously introduced, the occurrence of closeness between global and local frequency, produce a veering of the two approaching frequency loci curves, completely avoiding the possibility of a frequency coincidence. In the veering region the global and the local functions flips each other. The modal shape modification happens in a continuous way through the combination of the two approaching modes. When the frequencies reach their minimum distance, the participation of the global and the local modes is perfectly balanced, as shown by the equality of their localization factors. These modal shapes are named *hybrid*. The divergence in the eigenvalue loci β_b (or β_c) is generally dependent on the entire set of independent mechanical parameters affecting the characteristic equation. For the studied problem the two parameters (ρ, χ) have been considered for their mechanical significance and their strong influence on the modal properties. The eigenfrequency loci dependence on the selected parameters $\beta_b(\rho, \chi)$ has been numerically obtained through a continuation procedure applied to the characteristic equation, because standard perturbation schemes fail in describing the

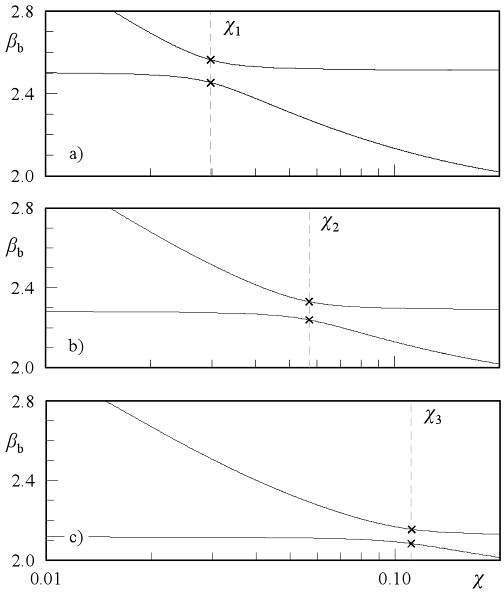

Figure 4. Frequency veering in the χ-space for different ρ-values

double-parameter loci dependence. Figure 4 shows the obtained veering of the two first frequencies in the χ-space for different mass ratios ρ. The depicted results show that the minimum distance between the involved frequencies, called *veering amplitude*, and its position in the parameters space, depend both on the mass ratio ρ and the stiffness ratio χ. In particular, decreasing the mass ratio ρ, lighter cable, the veering appears at higher value of χ, stiffer flexural beam, with smaller veering amplitude.

4 LOCALIZATION AND VEERING IN F.E. MODELS OF CABLE-STAYED BRIDGES

Finite element models of stayed systems require a complex geometrical description, since they present an high number of degrees of freedom. Therefore a simple cable modeling technique (Ernst 1965) is generally adopted: each cable is described through an hinged equivalent tendon which in term of global effects simulates the initial non straight configuration through an axial stiffness reduction.

The use of the Ernst equivalent modulus provides a reliable description of the global behaviour of cable stayed system, but it neglects the effort of cable transversal motion on their dynamics. Therefore more recently a second approach have been introduced in which the cable motion is described by the transverse nodal displacements dividing the cable in a selected number (n_c) of beam elements with low flexural and high geometric stiffness, the last depending on the pre-stressed axial tension (Abdel-Ghaffar & Khalifa 1991, Tuladhar et al. 1995, Caetano et al 2000).

According to the previous studies, the two presented approaches for modeling stayed structures will be named One Element Cable System (OECS) and

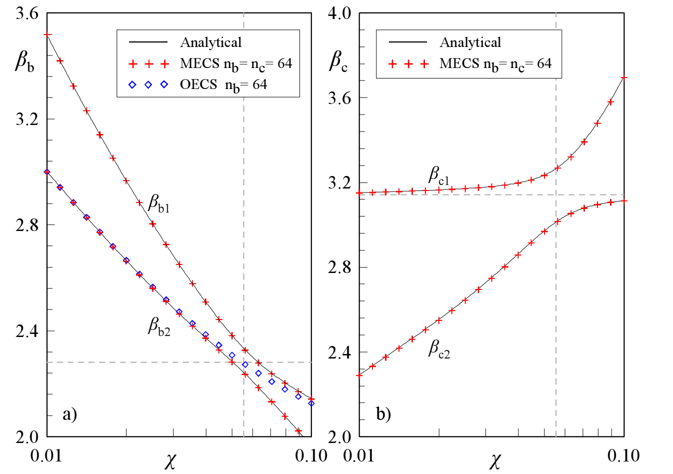

Figure 5. Comparison of veering curves obtained through analytical, OECS and MECS modeling approaches.

Multi Element Cable System (MECS).

In order to evaluate the different ability of the two approaches in the description of the veering phenomena, a simple selected stayed-beam has been used as numerical prototype. For this system the results obtained through the use of OECS and MECS models are compared with those obtainable from the exact solution of the previous presented analytical model, where the discretization error typical of the finite element method is absent.

Figure 5 reports the comparison between the first two frequencies varying the stiffness ratio χ for the OECS (n_c=1), MECS (n_c=64) and analytical models. The OECS approach is able to correctly predict the analytical results only away from the veering region, because modelization doesn't account for frequencies associated with local modes. Indeed in the veering region only a frequency associated with a global mode is present, and its locus seems to be a tangent to the curves representing the two analytical veering frequency curves (see Fig.5a). Differently, the MECS model, after a convergence analysis, allows the reproduction of the analytical frequencies loci in the entire χ domain.

Since this results seem to validate the MECS modelization approach, a MECS model of a cable-stayed bridge has been realized to verify the presence of cable localization and hybrid modes in real structures. To this aim the Bill Emerson Memorial Bridge

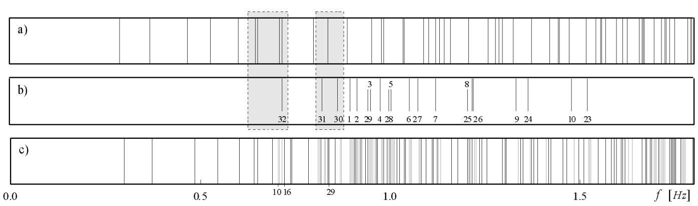

Figure 6. Spectral content of the Bill Emerson Memorial Bridge, frequencies of (a) OECS model, (b) stay cables Irvine model, (c) MECS model.

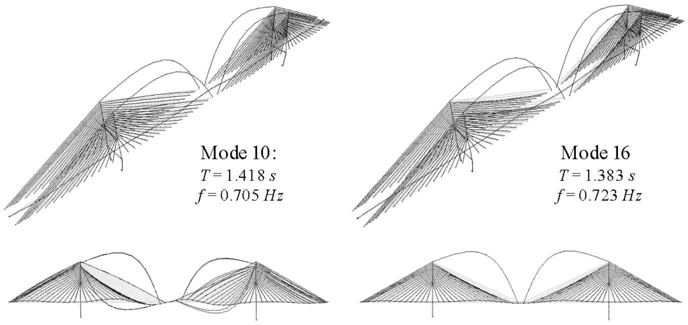

Figure 7. MECS model of the Bill Emerson Memorial Bridge: selected hybrid modes

has been selected, which has been the object of control benchmark studies. Figure 6 shows the spectral contents of both OECS and MECS models of the bridge, together with the cable frequencies with fixed support non interacting with the remaining bridge structure.The results show frequency bands in which OECS global mode frequency (Fig.6a) are close to frequencies of the isolated cables (Fig.6b). Around these frequencies the MECS model presents a large number of modes in which the global bridge dynamics strongly interacts with the cables local one (see e.g. Fig.7). It is reasonable to suppose that these can be classified as hybrid modes as it is confirmed by the evaluation of the their localization factors (for modes 10 and 16 the localization factors are: $\Lambda_{10} = 0.411$, $\Lambda_{16} = 0.647$).

5 CONCLUSION

Through a simple model of a cable-stayed beam a localization factor has been proposed to identify local modes involving mainly the cables, and to study a veering phenomenon occurring between local and global frequencies. The flipping between local/global modes in the veering region, caused by the variation of a mechanical parameter, has shown the presence of hybrid modes born from the combination of the involved eigenfunctions. The occurrence of hybrid modes is also captured by the coincidence of the associated localization factors. The MECS approach in modelling cable-stayed systems has been tested with respect to its ability to reproduce the veering phenomena and to describe hybrid modal shapes, where the commonly used OECS approach fails. The analysis reveals that MECS models are able to properly capture the interaction between local and global frequencies, responsible of the veering phenomena and of the modal hybridization. A studied case of a cable-stayed bridge has shown the presence of hybrid modes, efficiently detected by the introduced localization factor.

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