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Dynamic Identification of a Lightly Damped Slender Structure Using Compressive Sensing

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ABSTRACT A great deal of signals coming from structural monitoring scenarios are sparse in the frequency domain, suggesting the application of Compressive Sensing (CS) techniques in order to reduce the quantity of transmitted information. CS can recover data vectors starting from a subset of the original vector entries, thus allowing to recover a previously sampled signal with much less samples than those suggested by the Nyquist-Shannon theorem, and much less than what is commonly used in dynamic identification of structures. A CS technique, specifically Basis Pursuit, is applied for the dynamic identification study of a 30-meter-high lightning rod, consisting of a steel monotubular pole, where possible issues had been raised concerning fatigue damage due to resonant response with the first and second modes of vibration. An experimental measurement campaign was carried out to estimate damping coefficients useful for structural verifications. The ambient response was collected using triaxial accelerometers positioned at the top and at an intermediate height, which transmit data via WiFi to a nearby workstation. Different sampling frequencies for the compressed records are utilized for the dynamic identification of the structure, comparing modal frequencies and damping ratios with values obtained from the original records to find the best trade-off between data reduction and accuracy of modal parameters. Despite the usual challenges inherent in identification problems, further complicated by the low damping levels of the structure under consideration, the comparisons demonstrate a very good approximation. Fundamental frequencies are accurately estimated, while the slight discrepancies in the damping coefficients are associated with the intrinsic uncertainties of this parameter. Regarding the structural aspect of the case study, the outcomes of the analysis indicate very low damping values, pointing to potential criticality, particularly in the second mode of vibration. Moreover, the solid approximation achieved with the CS technique marks a significant advancement in applying IoT solutions for structural monitoring, emphasizing a significant reduction in data flow without affecting data quality. This may lead to several benefits, including simpler installation and maintenance, lower costs, and decreased energy consumption.

INDEX TERMS Compressive Sensing, Modal Parameters, Operational Modal Analysis, Structural Monitoring, Vertical Slender Structures.

I. INTRODUCTION

STRUCTURAL Health Monitoring (SHM) of civil infrastructures is an activity of primary importance, allowing the improvement of people's safety, providing an increased knowledge about the lifespan and status of structures and the ability to ensure their correct functioning through maintenance. Many structures require constant monitoring, giving raise to a great quantity of transmitted data. The introduction of Compressive Sensing (CS) in this process brings various potential benefits. CS can recover data vectors (and, by proxy, signals) starting from a subset of the original vector entries, thus allowing to recover a previously sampled signal with less samples than those suggested by the Nyquist-Shannon theorem. Moreover, since most vectors are compressible, as is the case of frequency sparse signals, the number of used entries can potentially be a very small fraction of the original Nyquist samples. With a reduced data flow, such as the one coming from a CS application, IoT solutions become more viable, generating various advantages like less demanding installation and maintenance processes, reduced costs and energy consumption rates while at the same time enabling connection to more secluded areas, which may be difficult or outright impossible to reach with traditional approaches. Compressive Sensing has been extensively described in a great deal of books [1]–[6] and papers [7]–[15] from a theoretical standpoint. Even so, practical uses of CS are still limited. Some works describe possible applications of CS theory, such as the Non-Uniform Sampler [8], [12], [16], the Random Modulation Pre-Integrator [9], [12], [17], the Single-Pixel Camera [18] and other image/video processing examples, such as [19]. Among other works exploring possible practical applications of CS, some outline attempts at applying CS to SHM: [20], [21], [22] and [23] for example try to perform reconstruction starting from a slightly reduced sample pool arising from transmission errors and data loss that characterize wireless sensor networks. Although these papers don't explicitly tackle sub-Nyquist sampling, they examine a variety of data types, such as vibrational and ultrasonic sources. This broader perspective reinforces the notion that CS is effective across a range of scenarios, indicating its usefulness regardless of the specific type of data being analyzed. Applications appear promising in enhancing both the efficiency of data sampling and the effectiveness of transmission in Wireless Sensor Networks [24], [25], [26]. [27] uses a machine-learning approach for CS data reconstruction that is tested on simulated signals and on those from a long span bridge. [28] proposes discrete cosine transform for data compression applied to lab-scale building models for modal frequency and damping identification. [29] applies a rakeness-based CS based on data science paradigm to extract frequencies of a pinned-pinned steel beam. [30] implements a neural network model for SHM on a concrete bridge. [31] uses CS to avoid false mode identification.

Nevertheless, the applicability of CS techniques for structural dynamic identification purposes still needs further investigation. First, structural response records are not always sparse enough. Moreover, the accuracy of modal parameters (especially damping) that can be identified from reduced data records has not yet been thoroughly explored.

This paper presents the results of dynamic identification of a steel lightning rod based on in-field ambient response. The study extracts fundamental frequencies and damping ratios from the recovered compressed signals and evaluates modal damping coefficients using techniques of varying levels of accuracy. The results demonstrate the practicality of employing Compressive Sensing in structural monitoring applications. This approach enables a reduction in data transmission requirements when utilizing IoT solutions, effectively addressing challenges associated with common issues such as data loss and sampling/transmission challenges in wireless networks.

II. NOTES ON OMA TECHNIQUES

The class of Operational Modal Analysis (OMA) techniques provides the dynamic identification of structures from response records, under the assumption that excitation is random stationary [32]. Early applications were referred to as ambient vibration testing. In the last decades, OMA has been systematized with new effective output-only modal identification procedures that overcome the limitations in dealing with closely spaced modes, noise and excitation having a some spectral distribution. Nowadays, OMA is a widely applied tool with several successful applications in civil and mechanical engineering under immeasurable loadings such as environmental and operational loads.

Referring to the literature for overview on this topic (*e.g.*, [33]), this investigation employs three distinct procedures to identify modal frequencies and damping ratios: the Random Decrement Technique (RDT), the Frequency-Domain Decomposition (FDD), and the Data-Driven - Stochastic Subspace Identification (SSI-data). The former is a simple method for estimating the dynamic characteristics of structures by observing the stationary response, the second can be classified in the framework of OMA procedures, the latter belongs to advanced OMA.

RDT [34] expresses the random response in time t of a single degree of freedom system having modal frequency n_0 and damping ratio ξ_0 as the sum of a deterministic part due to initial conditions and a zero-mean random part due to random excitation [35], [36]. Averaging a large enough number of samples that are taken with the same starting level and slope, the random part is averaged out, while the deterministic part is transformed into a free decay system that is called Random Decrement Signature (RDS) [37]:

$$a(\tau) = \mathrm{G}\, e^{-\xi_0\omega_0\tau} \times \\ \times \left(\cos\sqrt{1-\xi_0^2}\,\omega_0\tau + \mathrm{H}\frac{\xi_0}{1-\xi_0^2}\sin\sqrt{1-\xi_0^2}\,\omega_0\tau\right) \quad (1)$$

where τ is the time delay while G, H, $\omega_0 = 2\pi n_0$ and ξ_0 are to be determined through least squares fitting. Although this technique is designed for dealing with Single Degree of Freedom (SDoF) systems only, it can be used to estimate fundamental frequencies and damping ratios of multi degree-of-freedom systems with well separated vibration modes by using suitable bandpass filtering. The RDS for multiple closely located vibration modes can be obtained by superimposition of different damped free oscillations as proposed in [38].

Relying on white noise excitation, at least in the neighborhood of the investigated frequencies, the FDD [39] estimates frequencies and damping ratios by line-fitting the first eigenvalue (or singular value) of the Power Spectral Density (PSD) matrix of the measured response in the frequency domain, n. In the neighborhood of each resonance frequency, the response is approximated by the square modulus of the frequency response function of a SDoF system:

$$\mathrm{SV}(\omega) = \frac{\mathrm{SV}_0}{\sqrt{\left(\omega_0^2-\omega^2\right)^2 + 4\xi_0^2\omega^2}} \quad (2)$$

where $\omega = 2\pi n$; ω_0, ξ_0 and SV_0 are to be determined through least squares fitting, also providing an estimate of the frequency and peak value that best approximate (in least-square terms) the experimental response [40].

Given the basic conditions of linearity, stationarity and response observability, SSI overcomes limitations due to non white noise excitation by interpreting the measured response as the output of the mechanical system under investigation excited by unknown forces that are the output of a so-called excitation system, loaded, in turns, by a Gaussian white noise. This way, the measured response is interpreted as the output of a combined system, made by the excitation system and by the structure under test in series. The measured response includes information about both. The discrimination between structural modes and properties of the excitation is possible since the structural system has a narrowband response and time invariant properties. SSI converts the second order differential equation of motion into two first order problems, defined by the so-called state equation, modelling the dynamic of the system, and the observation equation, supplying the observed dynamic response [41]. The following discrete-time stochastic state-space model holds:

$$\begin{aligned} \mathbf{j}_{\mathrm{k}+1} &= \mathbf{B}\mathbf{j}_{\mathrm{k}} + \mathbf{w}_{\mathrm{k}} \\ \mathbf{x}_{\mathrm{k}} &= \mathbf{D}\mathbf{j}_{\mathrm{k}} + \mathbf{v}_{\mathrm{k}} \end{aligned} \tag{3}$$

where $\mathbf{j}_k$, $\mathbf{x}_k$ are the state and the the observation vectors at time $t = \mathrm{k}\Delta t$; $\mathbf{B}$, $\mathbf{D}$ are the state and the observation matrices, $\mathbf{w}_k$ is the process noise, due to disturbances and model inaccuracies, and $\mathbf{v}_k$ is the measurement noise due to sensor inaccuracies. They are both unmeasurable, zero mean, white processes. Working in the time domain, the SSI-data algorithm carries out the identification of the state sequence before the estimation of the state-space matrices [41]. The separation of the physical poles from the spurious mathematical ones takes advantage of the so-called stabilization diagram. It shows the poles obtained for different model orders as a function of the corresponding frequency. By tracking the evolution of the poles, the physical modes can be identified from alignments of stable poles, since the spurious mathematical ones tend to be more scattered and typically do not stabilize.

III. COMPRESSIVE SENSING

Compressive Sensing is a sampling paradigm, introduced in [7], [12], [13], that allows recovery of vectors, regardless of the nature of their content, using a potentially very small subset of the original samples. This translates into the ability to recover a signal with less samples than those suggested by the Nyquist-Shannon theorem without introducing significant inaccuracies (in fact, the aforementioned theorem introduces a sufficient but not necessary condition for signal recovery). The ability to recover a signal using CS relies on two principles: the first related to the vector to be obtained, *i.e.*, Sparsity, the second related to the recovery modality, *i.e.*, Incoherence.

A. SPARSITY AND COMPRESSIBILITY

A vector $\mathbf{x}$ is said to be s-sparse

$$||\mathbf{x}||_0 := \mathrm{card}(\mathrm{supp}(\mathbf{x})) \leq \mathrm{s} \tag{4}$$

where $\mathrm{supp}(\mathbf{x})$ indicates the support of a vector, which is the index set of its nonzero entries (most likely a vector that is shorter than the original one); card(S) is the cardinality of a set, *i.e.*, the number of nonzero values in it. If the number of nonzero values in $\mathbf{x}$ is less than or equal to s that vector is s-sparse. Eq.4 is also known as the L_0-norm of a vector. Not many signals exactly satisfy the property of sparsity, as they may have negligible components which are nevertheless different from zero, that's why compressibility is considered instead. A vector which can be approximated as a sparse vector is compressible. If a vector is compressible then its s-term approximation $\mathbf{z}$ (an alternate version of the original vector with some nonzero entries put to zero in order to transform it into an s-sparse vector) is able to minimize a quantity called l_p error s-term approximation, defined as

$$\sigma_s(\mathbf{x})_p := \inf\{||\mathbf{x}-\mathbf{z}||_p,\ \mathbf{z} \text{ is } s\text{-sparse}\} \tag{5}$$

In other words, compression is successful if $\mathbf{z}$ is made of the highest valued nonzero entries in the original vector.

Many signals are compressible, as demonstrated by the large amount and efficiency of compressed digital formats that rely on this exact property. Sparsity and compressibility influence the ability to acquire signals in a nonadaptive way, something which improves the performance of the recovery algorithms employed in the process [1], [12]. To maximize the probability of properly retrieving the original vector another property is needed.

B. INCOHERENCE & MATRIX QUALITY MEASURES

To correctly perform Compressive Sensing recovery, the Sensing Matrix, also called Measurement Matrix, $\mathbf{A}$, size M×N, has to satisfy the property of Incoherence. The coherence of a matrix is defined as

$$\mu := \max_{1\leq i\neq j\leq N} |\langle a_i, a_j\rangle| \tag{6}$$

so it is equal to the maximum inner product between two different L_2-normalized columns of the matrix. It is also known as Mutual Incoherence Property (MIP) [42]. μ is equal to 0 when the columns of the matrix form an orthonormal system and is at most equal to 1. In general, the lower the coherence, the better.

Coherence is a simple way to assess the quality of a measurement matrix. The concept of Restricted Isometry Property (RIP), also known as Uniform Uncertainty Principle, is introduced as a more precise measure of the success rate of recovery algorithms, even though it's harder to compute [43]. Whereas Coherence considers pairs of columns, RIP considers all possibile s-tuples of columns, providing a more precise and descriptive result. The s-th restricted isometry constant δ_s of a matrix $\mathbf{A} \in \mathbb{C}$ is the smallest value of δ

such that the following expression holds true for all s-sparse vectors $\mathbf{x}$:

$$(1-\delta)||\mathbf{x}||_2^2 \leq ||\mathbf{A}x||_2^2 \leq (1+\delta)||\mathbf{x}||_2^2 \tag{7}$$

if the matrix has L_2-normalized columns, the relations between δ and μ are

$$\begin{aligned} \delta_1 &= 0 & s&=1 \\ \delta_2 &= \mu & s&=2 \\ \delta_s &\leq (s-1)\mu & s&\geq 2 \end{aligned} \tag{8}$$

CS theory regards random matrices as the most versatile to be adopted, irrespective of the sparse basis. Even so, using domain-specific matrices, like the DFT, DCT and DST matrices for the frequency domain and the DWT matrix for the wavelet domain, offers qualitatively better results.

C. THE RECOVERY PROBLEM

Performing CS recovery amounts to solving an optimization problem using L_1-Minimization algorithms:

$$\min ||\mathbf{x}||_1 \quad \text{subject to } \mathbf{Ax} = \mathbf{y} \tag{9}$$

$$\mathrm{M} \geq \mathrm{C\,s}\, ln(\mathrm{N/s}) \tag{10}$$

The involved quantities are: $\mathbf{y}$, the Measurement Vector in which the random measures obtained from the original data vector are stored, size M; the original vector $\mathbf{x}$, size N (with $N > M$) and the previously mentioned Measurement Matrix $\mathbf{A}$, a M×N matrix which models the measurement/recovery process (a simple way to obtain it is to select the rows of the transformation matrix corresponding to the indices of the values taken from $\mathbf{x}$ to create $\mathbf{y}$). Finally, in Eq.10, s is the degree of sparsity and C is a constant value greater than 0 ($C \approx 0.28$, according to [5]). It should be noted that the formula suggests a minimum required number of measurements lower than the one actually needed in practical uses. Still, it perfectly summarizes the quantities involved in the CS recovery process.

D. PRECISION AND EVALUATION PARAMETERS FOR CS

CS literature lists some parameters which can be used to evaluate obtained results [44], [45]. Among them, Recovery Error (RE), Root Mean Square Error (RMSE) and Compression Ratio (CR) are some of the most descriptive and intuitive metrics. The Recovery Error is defined as

$$\mathrm{RE} = \frac{||\mathbf{x}-\hat{\mathbf{x}}||}{||\mathbf{x}||} \tag{11}$$

and its meaning is very similar to that of RMSE, of which we propose a slightly modified version

$$\mathrm{nRMSE} = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\hat{x}_i)^2}{N}} * \frac{1}{\max|\mathbf{x}|} \tag{12}$$

where x is the original sample, $\hat{x}$ is the recovered value ($\mathbf{x}$ and $\hat{\mathbf{x}}$ being the vectors containing all samples in Eq.11), N is the length of both the original and of course the recovered vector and finally $\max|\mathbf{x}|$ denotes the largest component of the vector, irrespective of its sign. Using a normalized RMSE allows obtaining an evaluation metric which is not bound by signal dynamics, thus of more immediate understanding. Finally, Compression Ratio is a very simple parameter

$$\mathrm{CR} = \frac{M}{N} \tag{13}$$

it denotes the ratio between the number of compressive measures M and the original length N of the vector to be recovered, providing insight on the amount of actually needed data vs the originally gathered one.

IV. EXPERIMENTAL TESTS AND RESULTS

The effectiveness of compressive sensing is examined with a specific focus on its application for the dynamic monitoring of a 30 m high lightning rod. It is a steel tubular pole, clamped at the base, with a 16-sided polygonal section, outer diameter of 0.77 m at the base, 0.24 m at the top and a thickness t of 4 mm. It is free of any appendages or ancillaries along its height. This lightning rod was the object of an in-depth investigation after frequent wind-induced vibrations were observed, highlighting possible criticality for fatigue [46]. An experimental campaign was therefore conducted for its dynamic identification. The structure was equipped with two triaxial MEMS accelerometers positioned at the top of the pole, at 30 m above the ground and at an intermediate level of 16 m for appreciating the contribution of higher modes. Sensor measurements are performed at 500 Hz, with a 20-bit resolution and self-noise lower than $18 \times 10^{-6} g/\sqrt{\mathrm{Hz}}$, being g the gravity acceleration. A GNSS receiver embedded in the sensors allows to synchronize the signals with the absolute time. Each accelerometer acquires along the two horizontal directions denoted by u and v; the vertical component is not considered. Signals are sent via WiFi to a workstation near the pole, where they are stored on a laptop. Structural response was recorded under approximately stationary ambient conditions for a period of about two hours, during which measured wind speed was less than 2 m/s and the temperature nearly constant.

Figure 1 reports the main dimension of the structure, the u, v directions and the acquisition system at the top of the pole. The antenna shown in the figure is for transmitting the signal through WiFi while an integrated memory bank allows to manage a ring-buffer for long continuous recordings. Figure 2 provides an overview of the experimental process used in this work. Figure 3 shows the 2-hour long time record of ambient vibrations at the two levels 30 m and 16 m along the horizontal directions u and v, respectively. RMS values of acceleration components u, v are, respectively, 0.21 m/s^2 and 0.17 m/s^2 at the top of the pole. Figure 4 shows the PSD in the frequency range [0, 10] Hz for each response component at the two levels, highlighting the harmonic contributions of the first three modes of vibration.

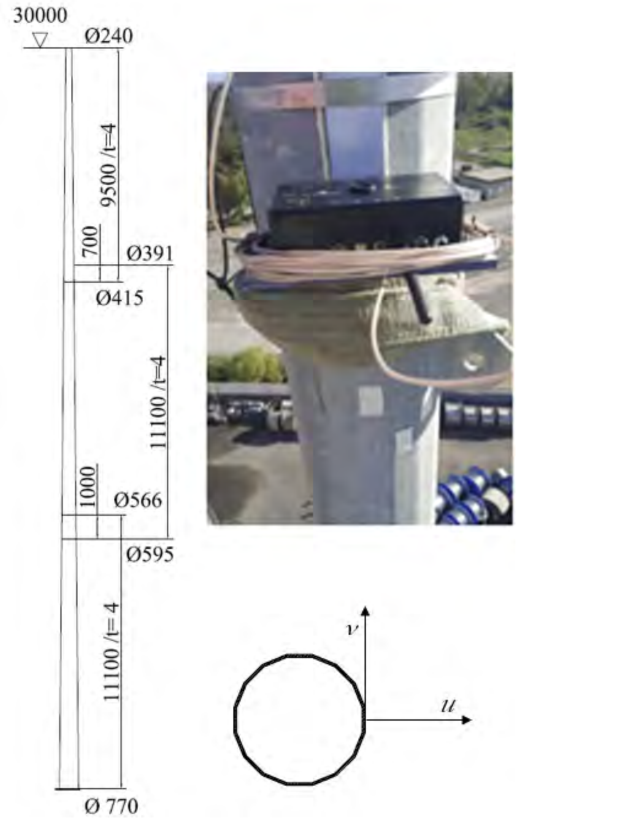

FIGURE 1: Lighting rod and recorded response components

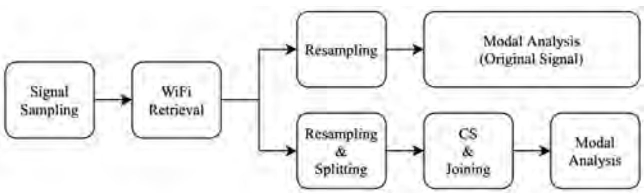

FIGURE 2: The employed methodology

A. MODAL IDENTIFICATION

This application specifically focuses on identifying modal frequencies and damping ratios, which are the key parameters for understanding dynamic behavior [47]. Fundamental frequencies of the first three vibration modes can be easily identified in Figure 4 by peak picking. They are, respectively: n_1 = 0.90 Hz, n_2 = 3.61 Hz, n_3 = 8.78 Hz. The estimate of modal damping is much more delicate due to the inherent randomness of the dissipation mechanism, data integrity and errors associated with the estimation technique. In the examined case, modal damping coefficients are expected to be very small, therefore estimates are particularly sensitive to the quality of the data.

By preconditioning the signals with a bandpass filter, RDT is applied to investigate each vibration mode individually. As an example, Figure 5a shows the RDS and the envelope function evaluated by a nonlinear fit of an exponential decay that simulates the equivalent viscous damping of the first mode. It is performed by using an iterative least squares estimation. First and second modal damping ratios derive: ξ_1 = 0.34%, ξ_2 = 0.019%. Figure 5b shows the first singular value of the PSD matrix and its local best fitting approximation (dashed green lines). Modal damping ratios derive: $\xi_1 = 0.31\%$, $\xi_2 = 0.022\%$. Both applied methods do not yield meaningful damping values for the third mode, likely due to its low modal contribution.

SSI-data is implemented using the commercial tool Macec® with MATLAB and the *Signal Processing Toolbox* of MATLAB. Figure 6 shows the stabilization poles. It supplies: ξ_1 = 0.29%, ξ_2 = 0.021%. The third modal damping ratio is also derived: ξ_3 = 0.06%.

The aerodynamic damping component, resulting from dissipation through the air, is negligible at low wind velocities. Consequently, the obtained values are considered merely representative of the structural damping.

The obtained results consistently indicate very low damping coefficients for the analyzed modes. The variations among the outcomes from the different procedures align perfectly with the inherent uncertainties characterizing the damping estimate. Values of the first modal damping of steel poles under ambient vibrations reported by the literature (*e.g.*, [48], [49] , [50], [51]) are generally in line with the present values, although they tend to be a little higher, potentially due to the usual presence of auxiliary elements. Available measurements on the second vibration mode are quite few. Values reported by [48] are much higher. This outcome highlights possible criticality for aeroelastic conditions related to higher modes and potential vortex induced vibrations in lock-in regime [52].

B. MODAL IDENTIFICATION BY CS RECOVERY

The PSD of recorded signals (Figure 4) benefits from sparsity in the frequency domain, highlighted by a clear harmonic content in the neighborhood of the fundamental frequencies and very small damping ratios. The terms Pseudofrequency and Pseudo-Hz (psHz) will be used to refer to the number of average random measurements per second which are taken during the initial phase of CS reconstruction. Measurement process is dealt with CS at different pseudofrequencies, from 6 Pseudo-Hz (*CR* = 1.2%) down to 2 Pseudo-Hz (*CR* = 0.4%), well below commonly used sampling frequencies for structural monitoring applications and below what is suggested by the Nyquist-Shannon theorem. The recovery process is performed on Matlab using the packages *CVX: Matlab Software for Disciplined Convex Programming* for solving convex programs [53], [54] and *ASP - A Matlab solver for sparse optimization* [55]–[57].

The employed CS technique for this research is Basis Pursuit, an optimization method which is both well-known in the related literature as well as the most applied one to explain and solve the CS problem (see, for example, [1]). Given that the attained results, as explained in the following, have proven to be satisfactory, only this method was used. Future works may also include comparisons between various CS

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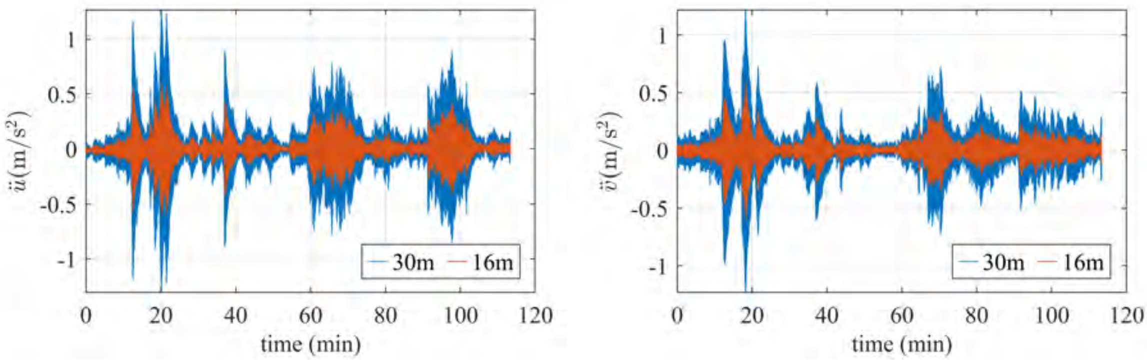


FIGURE 3: Time history of the structural acceleration records

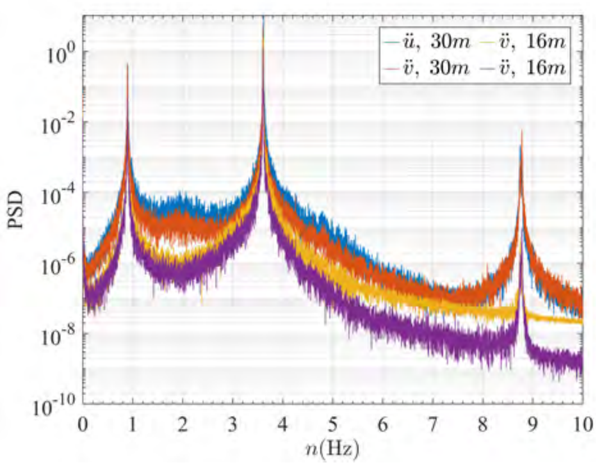


FIGURE 4: PSD of the structural acceleration records

TABLE 1: RMS and maximum values

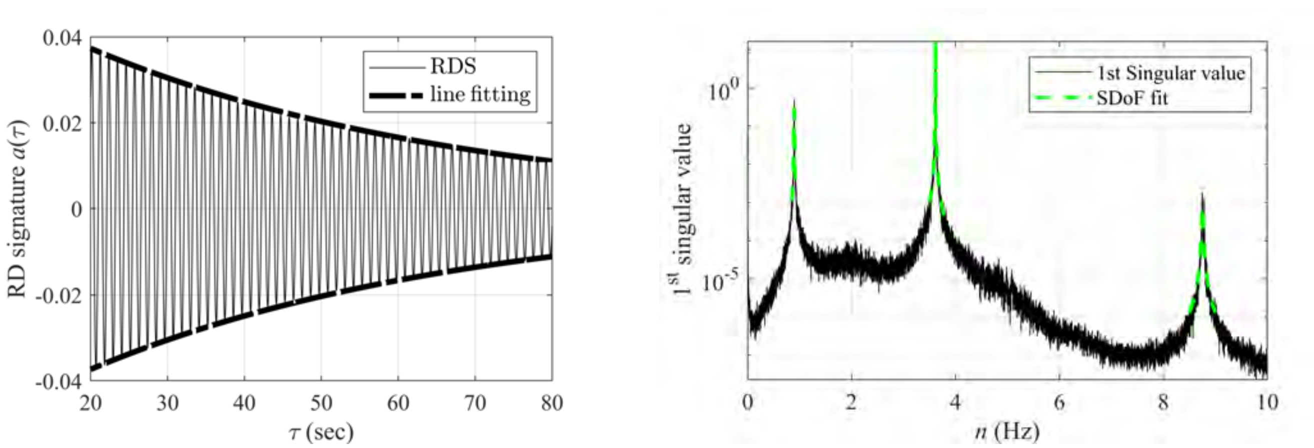
	500 Hz (Original)	6 psHz	4 psHz	2 psHz
$\ddot{u}_{rms}$ (m/s^2)	0.212	0.206	0.201	0.188
$\ddot{u}_{max}$ (m/s^2)	1.304	1.295	1.265	1.305
nRMSE (%)	-	2.53	3.52	5.40

TABLE 2: Fundamental frequencies

	500 Hz (Original)	6 psHz	4 psHz	2 psHz
n_1 (Hz)	0.90	0.90	0.90	0.90
n_2 (Hz)	3.61	3.62	3.61	3.62
n_3 (Hz)	8.78	8.88	-	-

techniques in relation to modal identification. The significant oversampling of the signal led to the necessity of resampling to a lower frequency, chosen at 20 Hz in this case, to better focus on the frequency components of the signal at hand. Such a course of action was needed because Compressive Sensing recovery techniques are more effective when the values used to construct the measurement vector vary widely [1], rather than being too similar. Initially, this wasn't the case, as the structure vibrates at 0.9 and 3.6 Hz, yet the data was being sampled at 500 Hz. Moreover, given the length of the original time records, signals were split into segments and CS was applied to each of them individually. Recovered segments were then joined back together for comparison. As an example, Figure 7 shows the recovered compressed signals in u, v at 2 Pseudo-Hz. Table 1 reports the RMS and maximum values for the original acceleration signal $\ddot{u}$ recorded at top and the recovered signals (6, 4 and 2 Pseudo-Hz), highlighting a very good approximation concerning maximum values; nRMSE error decreases from 5.4% to 2.5% passing from 2 to 6 Pseudo-Hz.

The comparison with the original record can be appreciated in Figure 8, showing the PSD of the recorded acceleration at top (u direction, blue line) together with the PSD of a selection of the recovered signals (6, 4 and 2 Pseudo-Hz). The lower Pseudo-Hz values exhibit noticeable background noise which is however small compared to the significant energy contents in the neighborhood of the structural frequencies n_1 and n_2, while it affects the contribution related to the third mode, around n_3. Fundamental frequencies are detected from the PSD functions and are listed in Table 2, showing satisfying agreement with the values obtained from the original signal for the first and second vibration mode. The identification of n_3 is only achieved at 6 Pseudo-Hz with 3% error. Estimates of damping ratios are reported in Tables 3, 4, 5, respectively obtained by RDT, FDD, SSI-data. Comparison with the results obtained from the original signals (sampled at 500 Hz, also reported in the tables) and the recovered ones highlights a very good approximation. The distinctions apparent in the tables are primarily attributed to variations in the error bounds commonly associated with structural damping. For this reason, there is no appreciable variation in the approximation by varying the CR, going from 6 to 2 Pseudo-Hz.



(a) Random Decrement signature of the top acceleration (1st mode)

(b) Singular value of PSD acceleration matrix

FIGURE 5: Damping estimate by RDT and FDD

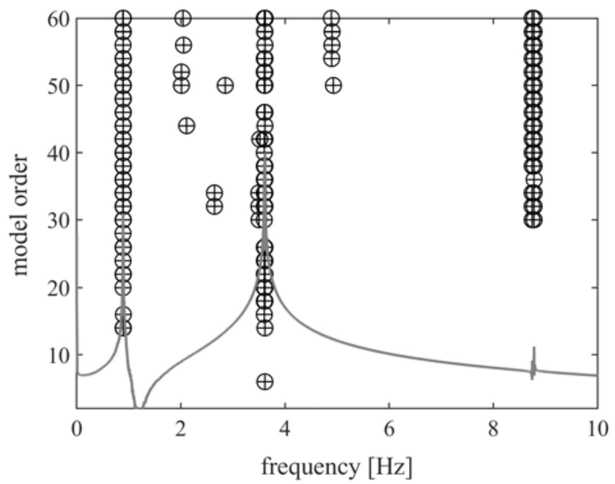


FIGURE 6: stabilization poles of the SSI algorithm

TABLE 3: Modal damping coefficients by RDT

	500 Hz (Original)	6 psHz	4 psHz	2 psHz
$\xi_1(\%)$	0.34	0.35	0.41	0.32
$\xi_2(\%)$	0.019	0.017	0.013	0.024

TABLE 4: Modal damping coefficients by FDD

	500 Hz (Original)	6 psHz	4 psHz	2 psHz
$\xi_1(\%)$	0.31	0.29	0.30	0.34
$\xi_2(\%)$	0.022	0.023	0.024	0.027

TABLE 5: Modal damping coefficients by SSI-data

	500 Hz (Original)	6 psHz	4 psHz	2 psHz
$\xi_1(\%)$	0.29	0.24	0.21	0.26
$\xi_2(\%)$	0.021	0.022	0.022	0.023

V. CONCLUSIONS & FURTHER DEVELOPMENTS

This paper has presented a CS based structural identification of a slender, lightly damped steel lightning rod. Using the results of a monitoring campaign, compressed signals of the structural acceleration due to ambient vibrations have been obtained at various Compression Ratios, from 1.2% to 0.4%, with sample rate from 6 Pseudo-Hz to 2 Pseudo-Hz. These values are less than what is suggested by the Nyquist-Shannon theorem and much less than what is commonly used in dynamic identification of structures. While fundamental frequencies can be easily obtained from the peak peaking of the PSD of the acceleration signals, modal damping ratios have been obtained according to three different modal identification techniques, *i.e.* the classical Random Decrement Technique, the Frequency Domain Decomposition, and the Stochastic Space Identification, which belongs to the field of advanced OMA procedures. The comparison of the results with the values obtained from the original records highlights the following outcomes:

- in the time domain, the recovered signals allow reproducing the original ones and the main statistic parameters with high accuracy. In the frequency domain, the PSDs exhibit some background noise which is however small compared to the significant energy contents in the neighborhood of the first and second modal frequencies, while it affects the contribution related to the third mode. This mode is however hardly identifiable and of little concern for structural applications;

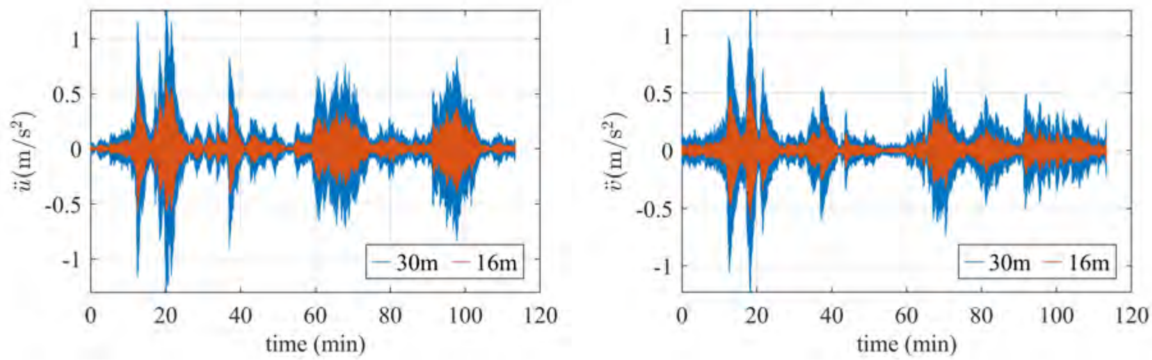


FIGURE 7: Structural acceleration records recovered at 2 Pseudo-Hz

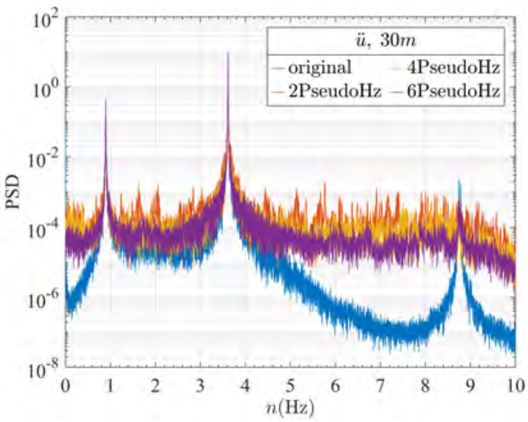


FIGURE 8: Random Decrement signature of the top acceleration (1st mode)

- fundamental frequencies of the first and second vibration modes are detected from the recovered signals without appreciable error; the third frequency is detected with good approximation at 6 Pseudo-Hz, already well below the Nyquist value;
- modal damping of the first and second vibration modes are detected from the recovered signals with very good approximation; errors committed are well below the usual uncertainties inherent to the estimate of this quantities. Therefore, despite the criticality in the estimation of this parameter, especially in the case of poorly damped structures, and despite its extreme sensitivity to the quality of the signal, this result shows that, in the field of CR values analysed, the applied procedures appear not sensitive to the sample reduction.

These findings represent a step forward the use of IoT solutions for structural monitoring applications relying on reduced data flow, generating various advantages such as less demanding installation and maintenance processes, reduced costs and decreased energy consumption.

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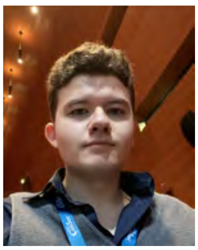

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