



The flow of sound pathways through the music network: introduction and analysis of music connections

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Abstract— In this paper, the concept related to the music connections is presented. It concerns a detailed analysis of the creative processes related to music composition. An innovative geometrical-based approach aimed to trace, analyze and understand the melodic and harmonic music processes has been introduced. In particular, the connections graph representing the geometry of the harmonic space which includes all the potential music paradigms and sonorities is presented. This diagram allows to map and analyze the human composition processes by observing its geometric superposed trajectories. An interactive software application tool (via hardware connected to the instrument of each musician) has been made in order to determine and handle the melodic and harmonic connections. The present application can be employed to view and analyze the music transient passages along with a song or improvisation through geometrical figures. The present application provides a real-time view of the flow of sound pathways through the music network which is constantly expanding. Its shape and size depend on the musician's subjective knowledge.

I. INTRODUCTION: THE AIM OF THE PRESENT STUDY AND THE MAIN SOFTWARE FUNCTIONS

The music concept introduced in the present paper is not related to a specific type of instrument but has a more general aim. It involves chords, scales, and more in general any generic group of notes. Scales and chords can be considered as the melodic and harmonic subgroups in which the set of 12 notes is organized. In some terms, scales and chords represent the result of an iterative study process through "trials and errors". Scales and chords can be finally employed to reduce the number of attempts, to minimize the number of "trials and errors". The algorithm implemented in the calculation program of the present study is able to map harmonic and melodic movements involving groups of notes arbitrarily played in real-time (or typed on a pc-keyboard) by the musician. From this

process, geometric graphs are automatically drawn (a network of interconnected nodes) in which each node represents, in general, a specific musical scale or a specific chord. These maps can be employed for performing the melodic and harmonic analyses (any sort of music investigation) of any kind of tune. With these maps, it is possible to draw melodic and harmonic movements on graphs instead of on pentagrams. The graphs' shape is arbitrary since it is given by the arbitrary choice related to nodes placement. The interconnections between nodes are not arbitrary. The interconnections are computed by the program according to the algorithm described in the present study. The interconnections determine the graphs' shape, depending on where the competent nodes are chosen to be placed. These graphs are not intended to replace the pentagram but are intended as an auxiliary device to see and analyze the music transient passages through the music geometrical network. These graphs can

be employed to recognize recurring patterns in music, and also to represent and better explain the reasons behind specific melodic and harmonic choices. These graphs can be also intended as an auxiliary device for the visual detection and prediction of innovative unexplored harmonic and melodic solutions/ways in the human composition process. More generally, this system is not dependent on the instrument played by the musician, representing in synthesis a geometric vision (divisible into different sub-networks) of all possible interconnections between different groups of frequencies (these groups are arbitrarily considered by the individual musician, so they depend on the subjective degree of knowledge of the musician himself who, as he learns, will enrich his graphic-musical network).

Music treatises connected to math and art are those by the Yusef Lateef [1] theory (which is extrapolated by the Coltrane notebook), Arnold Schoenberg [2, 3], Nicolas Slonimsky [4], and numerous other studies [5, 6, 7, 8]. The innovative aspect introduced and analyzed in the present paper is represented by the possibility of building a network in which all the music scales (and chords) are mutually interconnected. A specific algorithm has been designed to trace the connections among scales (and chords, more in general generic groups of notes). This algorithm has been implemented in an interactive software application tool via hardware connected to the instrument of each musician. The present method considers each chord as generated by specific degrees of different scales as well as an autonomous entity (a generic group of notes) and completely unrelated to one (or more) specific scale. The main chords constituted by three notes (triads) can be interconnected by following a parsimonious voice-leading approach (i.e. two triads that have two common tones are mutually connected, while the third tone changes by one generic scale step). According to the present method, the parsimonious voice-leading approach can be adopted for connecting triads, quadriads (chords of four notes) and more in general all the chord and scale types (more in general any group of one up to twelve notes). One of the advantages carried out by the method introduced in the present study is related to the fact that connections can include whatever music scale and chord type (more in general any generic group of notes). The same paradigmatic logic to connect the triads (or whatever chord type) has been used to connect the scales (more in general any group of one up to twelve notes). Thus each scale has been considered as a container of chords (or better, each scale is like a big chord that can be subdivided into its singular components that are the chords). Chords and scales are in turn represented in a condensed/synthetic manner through the use of different symbols and colors.

Chords and scales constituted by the same or a different number of notes can be reciprocally interconnected by following the specific logic that has been presented in the next sections through the main examples. In this way, the interconnected chord and scale patterns confer an original representation of the 2D music network (that could also be considered as a 3D music solid network). The network's shape depends on the number of chords and scales considered by the musician.

The methodology implemented in the in-house built software by the Author of the present study is aimed to calculate what concerns chords and scales and their mutual interconnections (starting from scale harmonizations on several levels). The methodology reported in the present study can demonstrate how there will be almost no distinction between scales and arpeggios (chords) or a single note. Generic groups of notes are considered and treated in the same manner by the software, regardless of whether the groups of notes are specifically chords or scales.

The software can provide bold insights since the use of the computer has the advantage of detecting the mutual relationships between generic groups of notes in an automated way. At the same time, the automated process makes simpler the task of the musician in discovering new relationships. The software allows performing a space-time discretization of musical pieces calibrated on the subjective preparation and knowledge of the user musician, amateur or professional. As the user learns, he becomes able to receive information of increasing detail level.

Some of the answers provided by the software are summarized in the following:

- the software allows the user to choose the environment of scales and chords to be considered and handled by the process. This feature allows the user to deal with familiar scales and chord types, according to the knowledge and goals that the musician-user wants to reach.

- the software provides the list of all the scales mutually similar at the melodic level (i.e. based on the number of notes in common between the compared scales). Any type of scale (which in turn generates chords) can be investigated.

- the software provides the list of chords that can be generated by the chosen scale starting from each note of the scale itself (scale harmonization starting from each degree of the scale).

- the software provides a list of all the scales harmonically connected to the scale chosen by the user. By

harmonically connected, it means that each scale can be connected to others through the chords it generates on each of its degrees and that at the same time are in common with other scales (for instance the *C major/ionian* scale is harmonically connected to the *G major/ionian* scale because both generate and therefore share the *Am*, *Am7*, *Amadd4*, *Amadd9*, *C6*, *Cadd9*, *Cmaj*, *Cmaj7*, *Em*, *Em7*, *Emadd4*, *G6*, *Gadd4*, *Gadd9*, *Gmaj* chords (pivot chords); many other examples of this kind exist).

- the software provides the list of all the chords melodically similar in absolute terms (i.e. regardless of the scale, in a way completely unrelated to a reference scale and the related and potential harmonic areas) to any type of chord chosen by the user, having a certain fundamental (namely the reference first degree of the chord and denoted by number 1; the fundamental can be the bass note of the chord or not). Even if the scales did not exist and only notes and generic groups of notes (i.e. chords) were assumed to exist, the *Em7* chord (here reported as an example) is in any case melodically similar to the *G6* chord based on the number of notes in common between these two chords. Many other relationships of this type exist in the comparison between chords (or any generic groups of notes) at the melodic level, considered as autonomous entities and completely unrelated to one (or more) specific scale.

- many interesting relationships between chords exist also at the harmonic level: the software provides the list of chords that can be generated by the chosen chord starting from each note of the chord itself (chord harmonization starting from each "degree" of the chord).

- the software provides a list of all the chords harmonically connected to the chord chosen by the user. By harmonically connected, it means that each chord can be connected to others through the chords it generates on each of its degrees and that at the same time are in common with other chords (for example the *Cmaj7* chord is harmonically connected to the *G6* chord because both generate and therefore share the *Em* chord on their third and sixth-degree respectively; many other examples of this kind exist).

- The software provides the list of all the chords which do not allow a harmonic connection among the scales chosen by the user. These chords are not in common between scales since they are specific and proper of a specific scale besides of course the *12 notes chromatic scale*.
- For each scale and for each chord chosen by the user the software provides the list of the scales and chords included in that scale and that chord. Furthermore, for each scale and for each chord

the software provides the list of the scales and chords in which that scale or chord is included. This because each scale and each chord can be included by other bigger scales and chords if all the notes of the smaller scale and smaller chord are in common with the bigger ones.

- the software provides the list of the scales and chords which include a single or a generic group of single notes chosen/played by the user.

- the software provides the list of the scales and chords which include a single or a generic group of chords chosen/played by the user.

- the software provides the list of all the chords (and also scales) which are equal but have different names (inversion substitutions in case of chords and rotation of the main scale in case of scales).

- the software provides the graphic and time-dependent network constituted by the considered chords and scales which are melodically connected since similar in terms of common notes. This time-transient network can provide useful information for the real-time analysis of the played song or improvisation, according to the time interval adopted in the analysis. Then, deeper analyses by adopting different time intervals can be performed after the song or improvisation is performed.

The software represents, in a synthesized manner, a useful tool for harmonic and melodic analyses of songs. These analyses can be performed also through the pictorial consultation of the graphic network that changes with the evolution of time, contextually to the subjective choices made by the user in the setting of inputs to the program.

II. FUNDAMENTAL MUSIC SCALES AND CHORDS STRUCTURES

The scale types (and the related *modes*) considered in the present study are reported in the following list. The structure of these scales in terms of interval related to their tonic (namely the reference starting note of the scale and denoted by number 1) has been specified in Table 1:

Table 1: Structure of the scale types in terms of interval related to their tonic (1)

Scale type	Structure
<i>Major scale (ionian mode)</i>	1 2 3 4 5 6 7
<i>Melodic minor scale</i>	1 2 3b 4 5 6 7
<i>Harmonic minor scale</i>	1 2 3b 4 5 6b 7
<i>6 notes blues scale</i>	1 3b 4 4# 5 7b
<i>Whole-tone scale</i>	1 2 3 4# 5# 7b

<i>Half-step/whole step diminished scale</i>	<i>I 2b 3b 3 5b 5 6 7b</i>
<i>Whole step/half-step diminished scale</i>	<i>I 2 3b 4 5b 5# 6 7</i>
<i>Augmented half-step minor-third scale</i>	<i>I 2b 3 4 5# 6</i>
<i>Augmented minor-third half-step scale</i>	<i>I 3b 3 5 5# 7</i>
<i>Messiaen mode #3 (Rotation 1)</i>	<i>I 2b 2 3 4 4# 5# 6 7b</i>
<i>12 notes chromatic scale</i>	<i>I 2b 2 3b 3 4 5b 5 5# 6 7b 7</i>

It has to be noted that among the scales presented, each *major (ionian mode)*, *melodic minor*, *harmonic minor*, *6 notes blues* scale can be transposed within an octave by adopting twelve different tonics and thus obtaining twelve different scales for each scale type, each with a different tonic. The same does not happen for the *whole-tone*, the *half-step/whole step diminished*, the *whole step/half-step diminished*, the *augmented half-step minor-third*, the *augmented minor-third half-step*, the *Messiaen mode #3 (Rotation 1)*, the *12 notes chromatic scales*. In particular, the structure of these scales limits the transposition process (indeed these are also known as *Modes of limited transposition*, [7]). In particular, the *12 notes chromatic scale* does not allow any transposition since, if transposed, the same starting scale would be obtained. Therefore only one *12 notes chromatic scale* exists (whose tonic can be conventionally referred to *C*) that is the mother scale which includes and generates all the other scale and chord types. To cover an octave two *whole-tone* scales are needed and sufficient (whose tonic can be conventionally referred to *C* and *C#*). To the same aim, two *whole step/half-step diminished* scales are needed and sufficient (whose tonic can be referred to *C* and *D*), one *half-step/whole step diminished* scale is needed and sufficient (whose tonic can be referred to *C*), two *augmented half-step minor-third* scales are needed and sufficient (whose tonic can be referred to *C* and *D*), two *augmented minor-third half-step* scales are needed and sufficient (whose tonic can be referred to *C* and *D*), four *Messiaen mode #3 (Rotation 1)* scales are needed and sufficient (whose tonic can be conventionally referred to *C, C#, D, Eb*).

Many other typologies of scales exist, but for the sake of brevity they are not needed in order to introduce the present method. The scales incorporate, thus generate different types of chords. The types of chords considered in the present study are constituted by three and four notes.

They are here reported in terms of interval structures related to their fundamental (1) in Table 2:

Table 2: Structure of the chord types in terms of interval related to their fundamental (1)

Chord type	Structure
<i>dim</i>	<i>I 3b 5b</i>
<i>m</i>	<i>I 3b 5</i>
<i>5b</i>	<i>I 3 5b</i>
<i>maj</i>	<i>I 3 5</i>
<i>5#</i>	<i>I 3 5#</i>
<i>m/add9b</i>	<i>I 2b 3b 5</i>
<i>5b/add9b</i>	<i>I 2b 3 5b</i>
<i>add9b</i>	<i>I 2b 3 5</i>
<i>m/add9</i>	<i>I 2 3b 5</i>
<i>add9</i>	<i>I 2 3 5</i>
<i>m/add4b</i>	<i>I 3b 4b 5</i>
<i>dim/add4</i>	<i>I 3b 4 5b</i>
<i>m/add4</i>	<i>I 3b 4 5</i>
<i>m/add4#</i>	<i>I 3b 4# 5</i>
<i>dim7</i>	<i>I 3b 5b 7bb</i>
<i>m7/5b</i>	<i>I 3b 5b 7b</i>
<i>m/maj7/5b</i>	<i>I 3b 5b 7</i>
<i>m6</i>	<i>I 3b 5 6</i>
<i>m7</i>	<i>I 3b 5 7b</i>
<i>m/maj7</i>	<i>I 3b 5 7</i>
<i>m/maj7/5#</i>	<i>I 3b 5# 7</i>
<i>5b/add4</i>	<i>I 3 4 5b</i>
<i>add4</i>	<i>I 3 4 5</i>
<i>add4#</i>	<i>I 3 4# 5</i>
<i>7/5b</i>	<i>I 3 5b 7b</i>
<i>maj7/5b</i>	<i>I 3 5b 7</i>
<i>6</i>	<i>I 3 5 6</i>
<i>7</i>	<i>I 3 5 7b</i>
<i>maj7</i>	<i>I 3 5 7</i>
<i>7/5#</i>	<i>I 3 5# 7b</i>
<i>maj7/5#</i>	<i>I 3 5# 7</i>

The main chords constituted by three and four notes presented in the above list are needed and sufficient to completely denote the sound of each scale from which these chords are generated. Many other typologies of

chords exist (also constituted by five or more notes), but for the sake of brevity, they are not needed in order to introduce the present method (also because they can be thought as the superposition of two or more main chords constituted by three and four notes: for example, the chord *maj7/9* type has not needed to be presented in the above list since it can be easily obtained by mixing/superposing the *maj7* and *add9* types). Suspended chords (*sus* type) have not been considered, as well as incompletely chords without the third (*no3* type), without the fifth (*no5* type), some chord *inversions* (characterized by having a constituting chord note, which is different from the fundamental, placed as the bass note) and *slash chords* of the same chord type (characterized by having any different note from the fundamental which is placed as the bass note).

2.1 SCALES HARMONIC STRUCTURE

The harmonizations of the scales presented in the previous section can be obtained (not reported for the sake of brevity) by means of the main chords constituted by three and four notes introduced in the previous section. The tonic (the first degree of each scale) that can be chosen to present each scale is the *C* note (starting note and common reference of each scale in order to distinguish and highlight the differences among the scales in terms of structure).

By observing the scales' harmonic structure, it is possible to find out the property related to employing scales that do not generate/include the target chord. This property can be generalized as follows:

- scales (more in general generic groups of notes) that do not generate/include the target chord can, in general, be employed over the target chord together with some of the scales in which they are included into only if the containing (bigger) scales generate/include the target chord.

It has to be specified that the scales and chords considered in the present study are all included in the *12 notes chromatic scale*. Focusing on the scales, there are no further mutual inclusions among the other scales considered (*12 notes chromatic scale* excluded). More specifically, it has to be noted that the *12 notes chromatic scale* can generate any chord type. Thus, the *12 notes chromatic scale* can generate all the potential melody patterns that can be sustained by all the chord types which are all generated and included in the *12 notes chromatic scale*. Therefore the *12 notes chromatic scale* can be freely employed over all the chord types and, more in general, on all the potential chord progressions. The particular structure related to the *12 notes chromatic scale*, including

and generating all the harmonic and melodic material, would justify the employing of whatever scale on any chord type. This because all the scales and chords are generated and included in the *12 notes chromatic scale*. The musician's subjective taste will place a limit on the free-usage of the *12 notes chromatic scale* since very often it does not allow to confer a clear and organized sound to the harmonic and melodic processes. This is one of the reasons for which other scale types constituted by a lower number of notes exist.

It is important to notice that the *minor pentatonic* scale (not considered in the list reported in the present study for the sake of brevity) is included by the *6 notes blues* scale. For instance, the *C 6 notes blues* scale includes the *C minor pentatonic* scale that is constituted by the notes: *C Eb F G Bb*. The *C minor pentatonic* scale generates the *Eb6/9* chord on its third degree (its third degree is denoted by the name *Eb major pentatonic* scale that corresponds to the *Eb6/9* chord arpeggio).

Other two important functionalities related to the present method are reported hereafter:

- the method allows finding out all the potential scales that generate each chord, i.e. it is possible to detect all the different scales (among those considered by the system since chosen by the user-musician) that can be potentially employed over the same chord type and, more in general, on the same chord progression (see the examples reported in Section 6 of the present paper).
- Viceversa, it is also possible to detect all the scales (among those considered by the system and chosen by the user-musician) that include the chords that potentially can sustain the same melody pattern: the given melody pattern is in turn generated by the same different potential scales that generate its sustaining chords.

An investigation purposely limited to 4 chord types can be conducted by considering the *maj7*, *7*, *m7*, *m7/5b* chord types.

Considering each chord individually, according to the calculations based on the scale/chord relationship, all the available standard 12 notes could be used over such chord types even if from an aesthetic point of view some subgroups of notes could be better than others. This is because the *12 notes chromatic scale* generates these chords. Even if the *12 notes chromatic* would not be considered (bypassed/avoided) by the system, all the 12 notes can be covered in any case by considering as a whole the group of notes which constitute all the scales that generate/include each of these chord types. Once again it has to be noted how the *12 notes chromatic scale* can

generate all the potential melody patterns that can be sustained by all the chord types which are all generated and included in the *12 notes chromatic scale*.

The chord harmonization (starting from each "degree" of the chord) related to the main chords constituted by three and four notes presented so far can be obtained (not reported for the sake of brevity).

III. DESCRIPTION OF THE GENERAL ALGORITHM TO FIND OUT THE RELATIONSHIPS BETWEEN CHORDS/SCALES

The general algorithm adopted for finding the relationships between chords/scales/single or generic groups of notes is described hereafter:

- Starting from a certain note, whatever note can be chosen after any other one since two notes differ for 1 note.

Considering this statement it is possible to present the general algorithm based on the parsimonious voice-leading approach and building the related graph's network. More in general, the algorithm compares the single or generic groups of notes at the same k number of notes (if two compared groups of notes are originally characterized by a different k , the additional uncommon notes have not to be taken into consideration by the process). To be declared similar and thus connected, two generic notes or groups of k notes must have at least $(k-1)$ common notes. When two different single notes are compared $k=1$. Therefore whatever two different single notes result to be similar (thus connected) in any case since they obey the general criteria for which they have $(k-1)=0$ common notes.

- Considering different groups of notes of the same or a different number of notes, it is possible to present the general algorithm to derive some interesting relationships between different notes, scale types, and chord types.

Given two different sets A and B each having a certain number of elements (A can have the same or a different number of elements of B), the following relationships can be written by superposing or subtracting the two different sets:

$$A+B=A \text{ if } B \text{ already included in } A \quad (1)$$

$$A+B=B \text{ if } A \text{ is already included in } B \quad (2)$$

$$A+B=C \text{ if } A \text{ and } B \text{ are not included in each other even if they share some or no elements in common} \quad (3)$$

$$(A+B)-B=E \text{ if } A \text{ and } B \text{ share some or have all the elements in common} \quad (4)$$

$$(A+B)-A=F \text{ if } A \text{ and } B \text{ share some or have all the elements in common} \quad (5)$$

$$A-B=A \text{ if } B \text{ is not included in } A \text{ and do not share common elements} \quad (6)$$

$$B-A=B \text{ if } A \text{ is not included in } B \text{ and do not share common elements} \quad (7)$$

$$A-B=D \text{ if } A \text{ and } B \text{ share some or have all the elements in common} \quad (8)$$

$$(A-B)-B=D-B \text{ if } A \text{ and } B \text{ share some or have all the elements in common} \quad (9)$$

$$(A-B)-A=D-A \text{ if } A \text{ and } B \text{ share some or have all the elements in common} \quad (10)$$

$$(A-B)-B=A-B \text{ if } B \text{ already included in } A \quad (11)$$

$$(B-A)-A=B-A \text{ if } A \text{ already included in } B \quad (12)$$

From these general properties written among different sets A and B, it is possible to find out some interesting relationships between different notes, scale types, and chord types:

$$- A-B=D \text{ if } A \text{ and } B \text{ share some or have all the elements in common} \quad (13)$$

$$C_major\ scale - C_major\ pentatonic = \\ (F + B) \quad (14)$$

$$C_major\ scale - C_Melodic\ minor = E - Eb \quad (15)$$

$$C_major\ scale - D_Melodic\ minor = C - C\# \quad (16)$$

$$C_12\ notes\ chromatic\ scale - C\#_Whole-tone\ scale = C_Whole-tone\ scale \quad (17)$$

$$C_12\ notes\ chromatic\ scale - Eb\ minor\ pentatonic = C_major\ scale \quad (18)$$

In these examples the groups of notes A and B can be declared similar and thus connected since the two sets A and B have at least $(k-1)$ common notes when compared at the same k number of notes. The last examples concerning the *C_Whole-tone scale* and the *C_major scale* show how the resulting group of notes (D= *C_Whole-tone scale*; *C_major scale*) can be viewed as generated by the subtraction of two groups of notes (A= *C_12 notes chromatic scale* and B= *C#_Whole-tone scale*; *Eb minor pentatonic*) which are not necessarily connected to D (the *C#_Whole-tone scale* and the *Eb minor pentatonic* are not connected to *C_Whole-tone scale* and *C_major scale* respectively). In particular, it has to be noted how the *Eb minor pentatonic* is the farthest scale from the *C_major scale* since they have no common notes (when compared at the same k number of notes).

- A+B=C if A and B are not included in each other even if they share some or no elements in common (19)

$$C_Whole-tone\ scale + C_Augmented\ half-step\ minor-third\ scale = C_Messiaen\ mode\ #3\ (Rotation\ 1) \quad (20)$$

$$C_Whole-tone\ scale + D_Augmented\ minor-third\ half-step\ scale = C_Messiaen\ mode\ #3\ (Rotation\ 1) \quad (21)$$

These last examples, together with the one reported hereafter, show how the same resulting group of notes can be obtained by applying different operations between different sets which are all easily derived from the general relationships written so far.

$$Cmaj7 + D\ minor\ pentatonic = C_major\ scale \quad (22)$$

$$E\ minor\ pentatonic + D\ minor\ pentatonic = C_major\ scale \quad (23)$$

$$Cmaj + Cmaj7 + C\ major\ pentatonic + D\ minor\ pentatonic = C_major\ scale \quad (24)$$

$$Cmaj7 - Em = C \quad (25)$$

$$Cm7 - Ebmaj = C \quad (26)$$

$$Cmaj - (E + G) = C \quad (27)$$

$$Cm - (Eb + G) = C \quad (28)$$

$$(C+G) - G = C \quad (29)$$

$$C_12\ notes\ chromatic\ scale - D_major\ scale - A_major\ scale - F\#_major\ scale = C \quad (30)$$

$$C_12\ notes\ chromatic\ scale - D_Melodic\ minor - Ab_Melodic\ minor - F\#_Melodic\ minor = C \quad (31)$$

$$C_12\ notes\ chromatic\ scale - Ab_Harmonic\ minor - F\#_Harmonic\ minor = C \quad (32)$$

These examples demonstrate how the same note or group of notes emitted can be the result of different parallel processes. This also would highlight how what is not played secretly helps to define what is played.

IV. THE REAL HARMONIC CONNECTION DEGREE BETWEEN DIFFERENT SCALES

Even if the described general algorithm is adopted for finding the relationships between generic groups of notes, in the present section the generic groups of notes are constituted by scales.

- The present software allows finding the number of common chords (also known as pivot chords) between the scales. This number expresses the *harmonic connection degree number* between the different scales. The same procedure can be applied for finding the number of common chords between the chords. In this case, the number expresses the *harmonic*

connection degree number between the different chords.

A first example (not reported for the sake of brevity) can consider the above main chords constituted by three and four notes and the scale types presented so far. Only the *harmonic connection degree numbers* related to the *C major (ionian)* scale have been reported, even if the general algorithm implemented in the software tool can manage whatever type of scale having any tonic. Therefore the comparison process has been performed internally to the software among all the scales considered in the system and chosen by the user.

It has to be noted that the harmonic similarity level among scales (and in general among generic groups of notes) is expressed by the *harmonic connection degree number*. The compared scales are classified from the highest to the lowest *harmonic connection degree number*. The *C Ionian* scale (as well as any other considered scale, except for the *12 notes chromatic scale*) has a harmonic link with some of all the other scales when the above main chords of three and four notes are considered. It can be noticed that the *5b/add4* is the only chord type (among those chosen by the user) that does not allow any harmonic connection among the scales chosen by the user. The *5b/add4* chord type is not in common between scales since it is specific and proper of the *Messiaen mode #3 (Rotation 1)* scale besides of course the *12 notes chromatic scale*. Then the *real harmonic connection degree number* between the different scales presented so far can be introduced. Some general properties can be summarized in the following list:

- The *real harmonic connection degree number* is different from the *harmonic connection degree number*. The *real harmonic connection degree number* has been obtained by considering the *basic chords*.
- A *basic chord* is a particular chord type constituted at least by two and a maximum of three notes. The *basic chords* are conceptually different from the main chords constituted by three and four notes presented so far; the *basic chords* are not explicitly presented but only used and reported in the text as background information. By taking into account all the existent *basic chords* (whose names are not reported in a list for the sake of brevity) it is possible to derive the *real harmonic connection degree number* between the different scales since these chords constitute the base, the foundation for all the other chords (all the other chords constituted by four or more notes can be

considered as the superposition of two or more *basic chords*). Each *basic chord* contains the minimum group of notes needed to derive all the other chords and in general groups of notes by combining (superposing) different *basic chords*.

- The *real harmonic connection degree number* expresses how many *basic chords* are in common between two different scales, thus univocally represents the real harmonic connection's level between different scales.
- In general, the *real harmonic connection degree number* and the *harmonic connection degree number* do not coincide. These can coincide if the *basic chords* are exclusively taken into account in the harmonic comparison investigation. Only the *real harmonic connection degree number* is needed and significant to quantitatively represent the real harmonic connection's level between different scales.
- The classification related to the compared scales from the highest to the lowest *harmonic connection degree number* (not reported for the sake of brevity) would coincide with the one denoted by the *real harmonic connection degree number* (not reported for the sake of brevity) only if all the *basic chords* were taken into account and included in both the harmonic comparison investigation types. Only the *basic chords* are strictly needed to compute the *real harmonic connection degree number* because the *basic chords* constitute the base, the foundation, for all the other chord types.

In this case, since all the existent *basic chords* have been considered, the *C Ionian* scale presents a harmonic link with all the other scales considered in this study except with the *Eb 6 notes blues* scale. In this case, all the *basic chords* allow a harmonic connection among the scales chosen by the user since at least one *basic chord* is in common among all the scales. It has to be noted that, as clarified in the previous Section, it is no needed that the different scales (and in general the different generic groups of notes) have the same number of notes for the scale harmonic comparison process. The harmonic comparison process can be carried out also among scales constituted by a different number of notes. From a mathematical point of view, it is possible to highlight the following property:

- scales that originally have a lower number of notes also have a lower number of generated chords. Therefore the scales that originally

have a lower number of notes tend to have a lower *real harmonic connection degree number* by their nature, despite their high melodic connection level with the bigger scales to which they relate to.

In fact, the scales that have a lower number of notes are often included and similar to the bigger ones. This concept introduces the *melodic connection degree number*.

4.1 THE MELODIC CONNECTION DEGREE BETWEEN DIFFERENT SCALES

In the previous section, the scales have been harmonically compared detecting the *real harmonic connection degree number*.

- As seen, the scales generate, thus include, the chords. In turn, each scale, being the container of the chords that it generates, can be considered as a bigger chord constituted by the notes of the scale itself. The *12 notes chromatic scale* is the biggest container since it includes and generates all the other scale and chord types.
- In their turn chords can also be considered as groups of notes, thus as little scales contained (or generated) by bigger scales. Thus chords can be harmonically and melodically compared as well as the scales. Then chords (being little scales) can be directly compared also with scales. When the chords are harmonically compared, the *real harmonic connection degree number* and the *harmonic connection degree number* are detected (as well as for the scales).
- In general, the present method allows to melodically compare all the chords and scales, thus detecting the *melodic connection degree number*. From now on and within the present Section, the chords will be referred to the term scales indifferently. Groups of notes would represent a more general and precise term; the term "scales" has been chosen for the sake of brevity.
- The *melodic connection degree number* expresses the number of scales that are similar to each scale. The *melodic connection degree number* represents in an absolute manner, the melodic level of connection among scales. A scale characterized by a high *melodic connection degree number* has a high corresponding number of connected scales. Specifically, it has to be noted that the chords are

generated/included into the scales, thus unavoidably connected since similar to the scales from which are generated.

- It has to be specified that the scales and chords considered in the present study are all included (thus similar and connected) in the *12 notes chromatic scale*. Focusing on the scales, there are no further mutual inclusions among the other scales considered (*12 notes chromatic scale* excluded).

The melodic comparison process is carried out by the algorithm implemented in the software tool for all the scales considered in the system (these are chosen by the musician user while playing any instrument or typing on a pc-keyboard). The algorithm compares the scales at the same number of notes k (if two compared scales are originally characterized by a different k , the additional uncommon notes have not to be taken into consideration by the process). To be declared similar, two scales of k notes must have at least ($k-1$) common notes.

A first example (not reported for the sake of brevity) considers the melodic comparison among the scale types presented so far. For the sake of brevity, only the *melodic connection degree numbers* related to each scale type having the *C* note as the tonic have been reported. Anyhow, the general algorithm implemented in the software tool can manage whatever type of scale having any tonic. The comparison process involves all the scales taken into consideration by the system (these are chosen by the musician user while playing any instrument or typing on a pc-keyboard). The scale types are those presented so far.

- Remember that the scales that originally have a lower number of notes tend to have a lower *real harmonic connection degree number* by their nature, despite their high melodic connection level (*melodic connection degree number*) with the bigger scales to which they relate to.

The scales that have a lower number of notes are often included and similar to the bigger ones. For example, the *C major (ionian)* scale is melodically connected to the *D 6 notes blues*, *E 6 notes blues*, *A 6 notes blues*, *B 6 notes blues* scales, as well as the *F Ionian*, *G Ionian*, *C Melodic minor*, *D Melodic minor*, *A Harmonic minor*, *Eb Messiaen mode #3 (Rotation 1)* scales despite the *C major (ionian)* scale has a lower *real harmonic connection degree number* related to the *6 notes blues* scales (and a higher *real harmonic connection degree number* related to the *Ionian*, *Melodic minor*, *Harmonic minor*, *Messiaen mode #3 (Rotation 1)* scales).

- In most of the cases involving scales having the same number of notes, the scales melodically connected (for example those connected to the *C major (ionian)* scale) provide a corresponding high value of the *real harmonic connection degree number* (those related to the *C major (ionian)* scale). This because in general, scales melodically similar are also harmonically similar (compatibly with the number of notes and consequently of chords generated by each scale considered in the comparison process). Among the scales of 7 notes melodically connected to the *C major (ionian)* scale, the *F Ionian*, *G Ionian*, *C Melodic minor*, *D Melodic minor*, *A Harmonic minor* scales having the same number of notes (7), provide a correspondingly high value of the *real harmonic connection degree number*.

A second example (not reported for the sake of brevity) considers the melodic comparison process involving all the chords (that are treated as little scales) constituted by three notes presented so far. As for the previous example, for the sake of brevity, only the *melodic connection degree numbers* related to each chord having the *C* note as the fundamental have been reported, even if the general algorithm implemented in the software tool can manage whatever type of chord (scale/group of notes) having any fundamental. The comparison process involves only the chords of three notes presented so far and taken into consideration by the system (these are chosen by the user/the musician while playing any instrument or typing on a pc-keyboard).

- It has to be taken into account that the software allows detecting the chords (and also more in general the scales or any group of notes) that in some cases are characterized by the same notes despite their different names. These involved chords can be of the same family or not. In both cases, they constitute examples of the *inversion substitution* type. Among the chords of three notes presented, the *inversion substitutions* detected by the software and related to each chord having the *C* note as the fundamental have been reported hereafter:

$$C5\# = E5\# \quad (33)$$

$$C5\# = G\#5\# \quad (34)$$

It can be noted how the *melodic connection degree numbers* that denote the connection level among scales are preferably presented separately from those specific among chords (although at the beginning of the present Section it was stated that the chords would be referred to the term

“scale” indifferently). This because, as it will be presented in the next section, the graphs/diagrams that express the *melodic connection degree numbers* result to be conveniently represented distinctly in order to obtain an optimized graphical representation. Being the number of connections very high, their trajectories tend to excessively superpose each other.

- Therefore one or more graphs for the *melodic connection degree numbers* among chords and one or more graphs for the *melodic connection degree numbers* among scales have been traced separately. On the other hand, it has to be kept in mind that each chord has a melodic connection with each scale in which is included (hence similar) and more in general each chord has a melodic connection with each similar scale. These connections among chords and scales have not been explicitly presented in the graphs of the present study but they have been reported in the text as background information.

Remember that each chord takes part also in the *harmonic connection degree number* determination, as seen in the previous section.

- For the sake of completeness, among all the main chords constituted by three and four notes presented so far, the *inversion substitutions* related to each chord having the C note as the fundamental have been reported in the following list:

$$C5\# = E5\# \quad (35)$$

$$C5\# = G\#5\# \quad (36)$$

$$Cadd9b = Dbm/maj7/5b \quad (37)$$

$$Cm/add4b = Em/maj7/5\# \quad (38)$$

$$Cdim7 = Eb\dim7 \quad (39)$$

$$Cdim7 = Gb\dim7 \quad (40)$$

$$Cdim7 = Adim7 \quad (41)$$

$$Cm7/5b = Ebm6 \quad (42)$$

$$Cm7 = Eb6 \quad (43)$$

$$C7/5b = Gb7/5b \quad (44)$$

For the sake of completeness, among all the main chords constituted by three and four notes presented so far, the *Harmonic connection degree numbers* related to the *C major* chord can be obtained (not reported for the sake of brevity), even if the general algorithm implemented in the software tool can manage whatever type of chord having any fundamental. From the *Harmonic connection degree numbers* related to the *C major* chord inspection, it is straightforward to notice how the *Cmaj* chord has a

harmonic link with 12 of the 31 chord families presented at the beginning of the paper.

V. THE NETWORK OF MUSIC CONNECTIONS

In this section, the network of the music connections among chords and scales has been presented. As stated in the last section, these graphs and the related connections can be traced by taking into account the *melodic connection degree number*. These graphs/diagrams express the *melodic connection degree numbers*. The number of connections among scales/chords corresponds to the *melodic connection degree number* related to each scale/chord. The music universe is constituted by notes, scales, and chords. As stated in the previous Section, chords can also be considered as generic groups of notes, thus as little scales contained (or generated) by bigger scales. Thus chords can be harmonically and melodically compared as well as the scales. Then chords (being little scales) can be directly compared also with scales. The chords are generated/included into the scales, thus unavoidably connected since similar to the scales from which are generated. For the sake of representation clarity of the present study, it has been chosen to keep distinct the music universe as seen/focused on the scales from the music universe as seen/focused on the chords. Therefore two distinct graph types (one focused on the scales and one focused on chords) will be separately presented in order to have a good quality view of the whole music universe and its constituting mutual relationships/interconnections. The two graph types coexist and, even if distinctly presented as autonomous graphs, they are implicitly connected and constitute the network of the music universe. The connections graphs have been conveniently represented distinctly (i.e. one or more graphs for the melodic connections among chords and one or more graphs for the melodic connections among scales) in order to obtain an optimized graphical representation (being the number of connections very high, their trajectories tend to excessively superpose each other). It has to be stressed that the present approach has been followed exclusively for the sake of a better representation clarity related to the study reported in the present paper even if the software considers and treats the generic groups of notes in the same manner, regardless of whether the groups of notes are specifically chords or scales.

The existent connections among chords and scales have not been explicitly presented in the present paper but these can be detected as the result of employing the chords investigation function of the software (i.e. given the scale name this function gives the chords included in that scale while the trajectories between similar scales or similar

chords generated by that scale are highlighted). It has to be specified that the graphical feature related to the chords investigation function has been adopted ad-hoc exclusively for the sake of a better representation clarity related to the study reported in the present paper.

These graphs allow detecting all the potential melodic and harmonic choices, i.e. all the potential music paradigms and sonorities. These diagrams allow to map and analyze the human composition processes by observing their geometric superposed trajectories. By assuming a polar coordinate system (r, θ), scales/chords of the same family are arbitrarily placed at the same radial (r) coordinate (by adopting the same arbitrary symbol/marker type). The term family identifies the general name of the scale/chord type without specifying the tonic/fundamental of the scale/chord; for example the *Harmonic minor* scale family and the *maj* chord family. Scales/chords of the same family but characterized by a different tonic/fundamental assume a different arbitrary angular coordinate (θ). Therefore, scales/chords of different families have different radial coordinates (and different symbol/marker types). These scales/chords can be connected to scales/chords of other families according to the connections specified by the computed *melodic connection degree number*. Scales/chords of different families but having the same tonic/fundamental share the same color that is shown in the graphs (twelve different colors are used in general). The key concepts used to represent and decode the graphs presented in this section are summarized in the following list:

- Generic groups from 2 to 12 notes (chords and scales) are considered by the present method and can be represented as symbols and mutually interconnected according to the *melodic connection degree number* to generate the music network.
- Each group of notes (chords and scales) of the same family is represented in a condensed/synthetic manner by the same symbol/marker type.
- Generic groups of notes (chords and scales) of the same family and having a different tonic/fundamental are placed at the same radial coordinate (same symbol/marker type) but assume a different arbitrary angular coordinate (different color).
- Different families of chords and scales are arbitrarily placed at different radial coordinates (different symbols/markers).

- Chords and/or scales of whatever family having the same tonic/fundamental share the same color.
- Lines for tracing the connections between similar scales and similar chords are employed.
- Each connection line is constituted by the two colors of the corresponding connected symbols/markers.
- As the musician plays a scale (or digits the scale name), the software gives the chords included in that scale (chords investigation function). The played scale and the chords generated/included in that scale are highlighted in each graph together with the trajectories between similar scales or similar chords related to the scale played (or digitized). It has to be stressed that the graphical feature related to the chords investigation function has been adopted ad-hoc exclusively for the sake of a better representation clarity related to the study reported in the present paper.
- As the musician plays in real-time (or digits) a series of N notes or N chords (or a single $N=1$ note or chord), the software shows the scales (among those considered by the system and chosen by the user-musician) that include all the N notes or N chords of the series (or the single note or chord). This is the scales investigation function through which these scales are highlighted in each graph together with the trajectories between their similar scales or similar chords related to the played N notes or N chords.

The graph that expresses the *melodic connection degree numbers* among chords of the major (*maj*) and minor (*m*) families is presented in Fig. 1 as the first example:

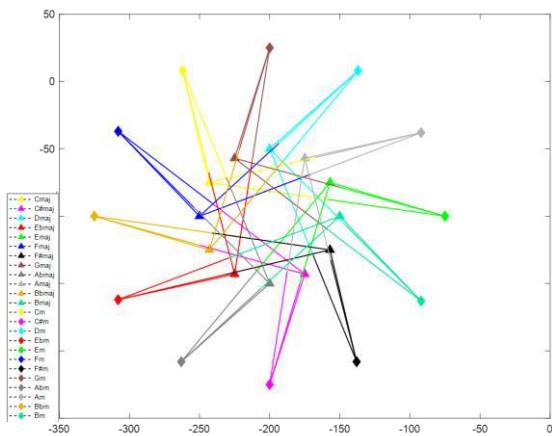


Fig. 1: Melodic connection degree numbers among chords of the major (maj) and minor (m) families.

The graph that expresses the *melodic connection degree numbers* among scales of the major (*ionian*) family is presented in Fig. 2.

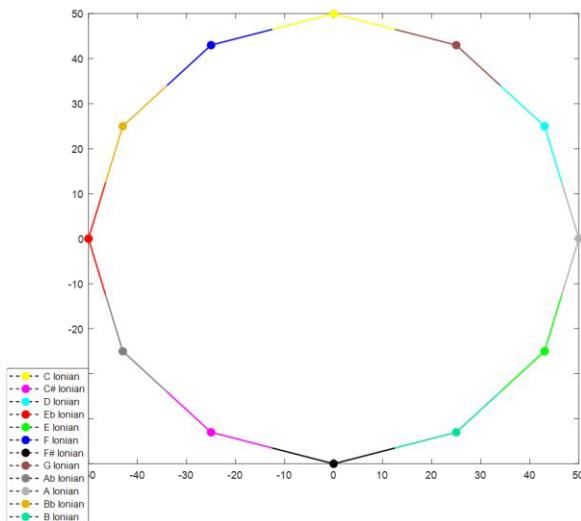


Fig. 2: Melodic connection degree numbers among scales of the major (*ionian*) family.

It can be noted that the graph in Fig. 2 represents the conventional circle of fifths (clockwise direction).

From these first two simple graphs, it is possible to detect and underline the existent background connections among chords and their similar scales, so far not explicitly presented. As the chords investigation function is employed, given the scale name (the software can also recognize the scale played by the musician in real-time) this function gives the chords included in that scale (scale harmonization starting from each degree of the scale). The played (or digitized) scale and the chords generated/included

in that scale are highlighted in each graph together with the trajectories between similar scales or similar chords related to the played scale. The graphical feature related to the chords investigation function has been adopted ad-hoc exclusively for the sake of a better representation clarity related to the study reported in the present paper. For instance, if the investigation is limited to the major (*ionian*) scales and the chords of the major (*maj*) and minor (*m*) families, when the *C major (ionian)* scale is played, it is possible to detect the chords included in the *C major (ionian)* scale together with all the highlighted trajectories between similar scales or similar chords related to the played scale. It has to be noted that the highlighted trajectories are exclusively between similar scales or similar chords and that only the played scale and the chords generated/included in that scale are highlighted (see Fig. 3). This is the representation of the *C major (ionian)* scale harmonization built with the chords of the major (*maj*) and minor (*m*) families (see Fig. 3). In addition, all the trajectories between similar major (*ionian*) scales or similar chords of the major (*maj*) and minor (*m*) families are highlighted when the *C major (ionian)* scale is played. This is a suitable way to explicitly represent the existent connections among chords and their similar scales avoiding the excessive superposition of connections trajectories, as shown by Fig. 3. It is interesting to notice that among the chords of the major (*maj*) and minor (*m*) families, all the highlighted trajectories between similar chords represent some examples of *diatonic substitution* type related to the played *C major (ionian)* scale (*Cmaj* is similar to *Em* and *Am*; *Fmaj* is similar to *Dm* and *Am*; *Gmaj* is similar to *Em*; *Dm* is similar to *Fmaj*; *Em* is similar to *Gmaj* and *Cmaj*; *Am* is similar to *Cmaj* and *Fmaj*).

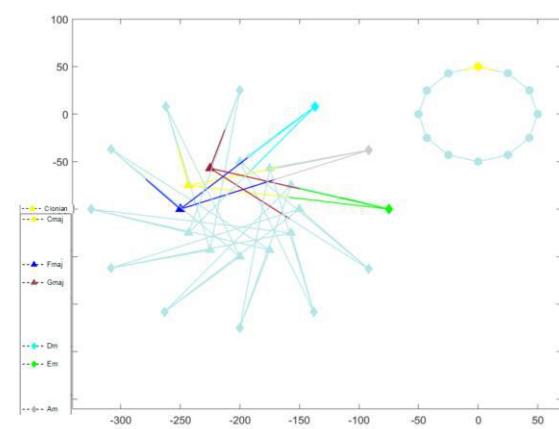


Fig. 3: Representation of the *C major (ionian)* scale harmonization built with the chords of the major (*maj*) and minor (*m*) families together with all the highlighted trajectories between similar scales or similar chords related to the *C major (ionian)* scale.

This kind of substitution involves similar chords included/generated by the same scale and takes place when the tonal scales of seven notes (*major/ionian, melodic minor, harmonic minor, harmonic major, double harmonic* and other scales) and all the existent bigger scales which contain them are considered. The graph that expresses the *melodic connection degree numbers* among the chords constituted by three notes of all the families considered in this paper is presented in Fig. 4 (the network of the music universe as seen/focused on the chords constituted by three notes of all the families considered in this paper).

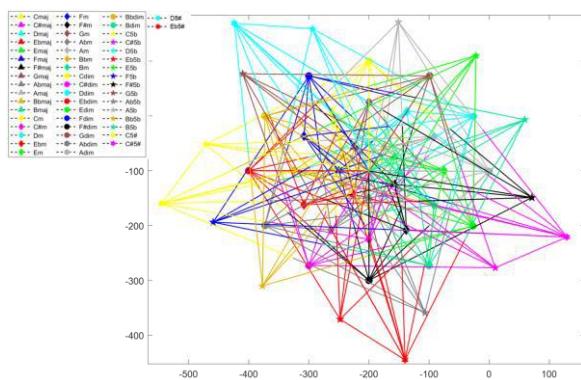


Fig. 4: The music universe as seen/focused on the chords constituted by three notes of all the families considered in this paper.

The graph that expresses the *melodic connection degree numbers* among scales of all the families considered in this paper is presented in Fig. 5a (the network of the music universe as seen/focused on the scales of all the families considered in this paper).

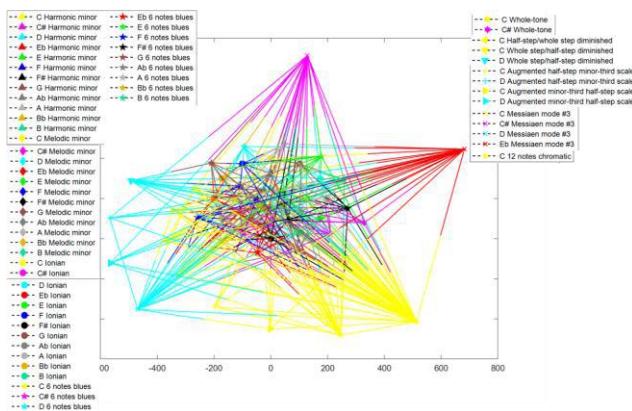


Fig. 5a: The music universe as seen/focused on the scales of all the families considered in this paper.

Remember that generic groups of notes (chords and scales) of the same family and having a different tonic/fundamental are placed at the same radial coordinate

(same symbol/marker) but assume a different arbitrary angular coordinate (different color). Different families of chords and scales are arbitrarily placed at different radial coordinates (different symbols). Therefore the shape assumed by each graph is arbitrary because it can be chosen by the musician-user subjectivity through the assignment of the polar coordinates (r, θ) for each scale/chord as previously explained. For this reason, the shape of each graph can vary but the connections among the nodes (the scales/chords represented by their marker and color) constituting the music network remain the same and consistent, according to the *melodic connection degree number* related to each scale/chord, regardless of the assignment of the polar coordinates (r, θ). For these reasons the same graph reported in Fig. 5a can be equivalently represented by the one shown in Fig. 5b since the symbols (and colors) related to the scales are arbitrarily chosen and placed.

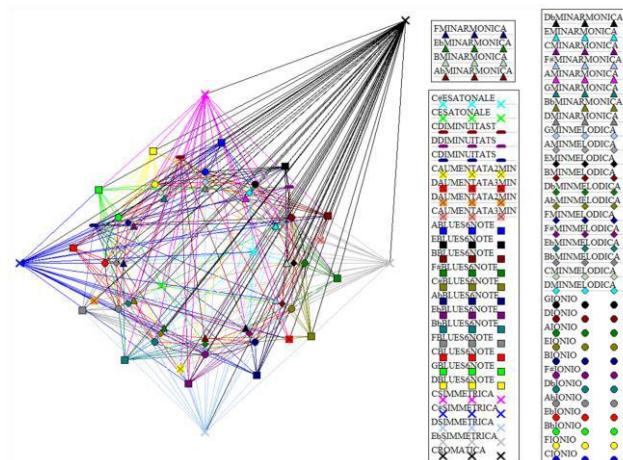


Fig. 5b: An alternative view of the music universe as seen/focused on the scales of all the families considered in this paper.

Fig. 5c shows the same graph shown by Fig. 5b without the 12 notes chromatic scale included.

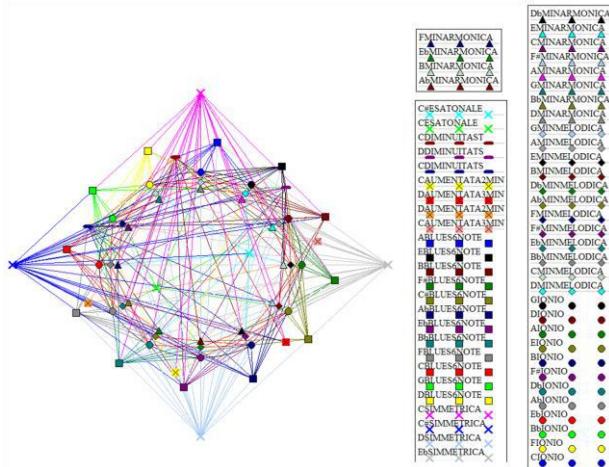


Fig. 5c: The music universe as seen/focused on the scales of all the families considered in this paper, without the 12 notes chromatic scale included.

It can be noticed how these graphs are not easy to visualize when all (or almost all) the scale/chord families are considered together in the same graph. In this case, the whole graph represents the result of the superposition of different graphs each having a lower number of scale/chord families. For this reason, each smaller graph expressing the *melodic connection degree numbers* among scales is analyzed and singularly presented hereafter as well as the one expressing the *melodic connection degree numbers* among chords. Through the series of the following smaller graphs, it is possible to demonstrate how the graph (Fig.5a) that expresses the *melodic connection degree numbers* among scales of all the families considered in this paper is the resultant of the sum (superposition) of different smaller graphs (Fig.2, Fig. 6 to 13) each containing a lower number of scale families. Fig. 14 summarizes this concept, where the superposition (+) of all the smaller graphs on the right-hand side of the equation confers the overall complete graph on the left-hand side of the equation (=), according to the relationships written in Section 3 about superposing or subtracting two different sets. Each smaller graph can be considered as a part and autonomously used in the composition process and music analysis.

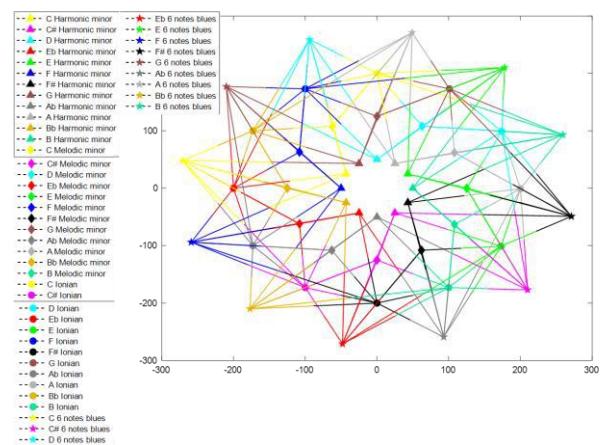


Fig. 6: Harmonic minor, Melodic minor, Ionian and 6 notes blues scales.

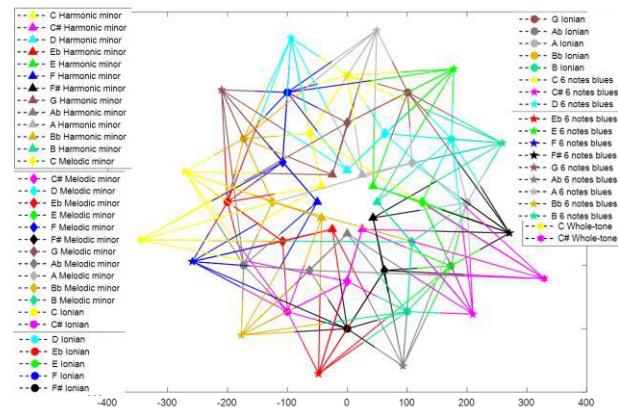


Fig. 7: Harmonic minor, Melodic minor, Ionian, 6 notes blues and Whole-tone scales.

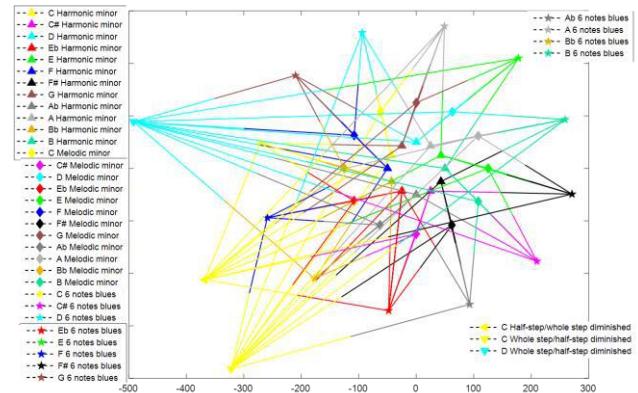


Fig. 8: Harmonic minor, Melodic minor, 6 notes blues, Half-step/whole step diminished and whole step/half-step diminished scales.

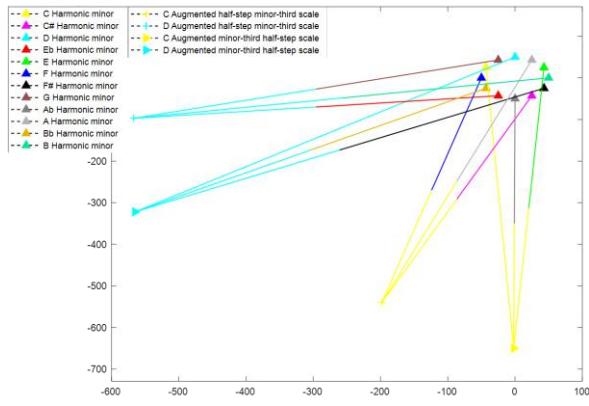


Fig. 9: Harmonic minor, Augmented half-step minor-third and Augmented minor-third half-step scales.

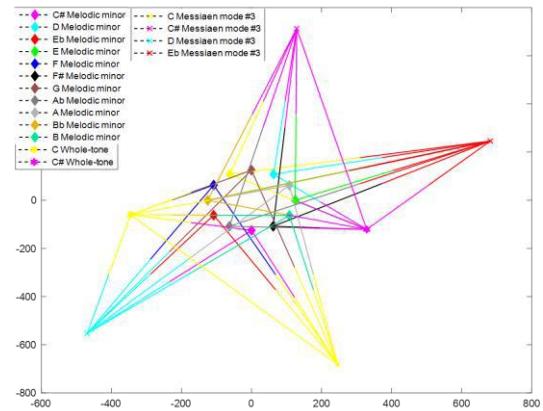


Fig. 12: Melodic minor, Whole-tone and Messiaen mode #3 (Rotation 1) scales.

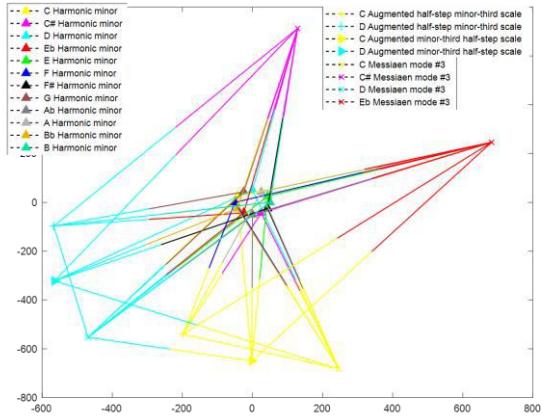


Fig. 10: Harmonic minor, Augmented half-step minor-third, Augmented minor-third half-step and Messiaen mode #3 (Rotation 1) scales.

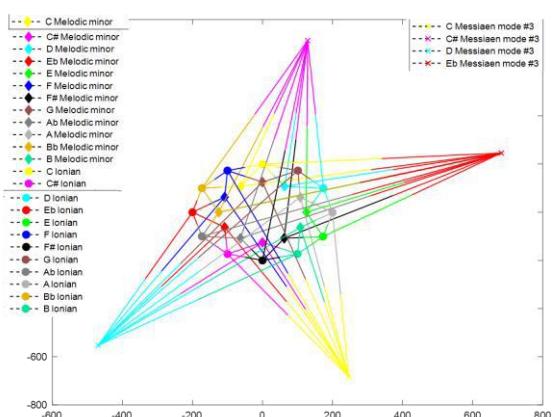


Fig. 13: Melodic minor, Ionian and Messiaen mode #3 (Rotation 1) scales.

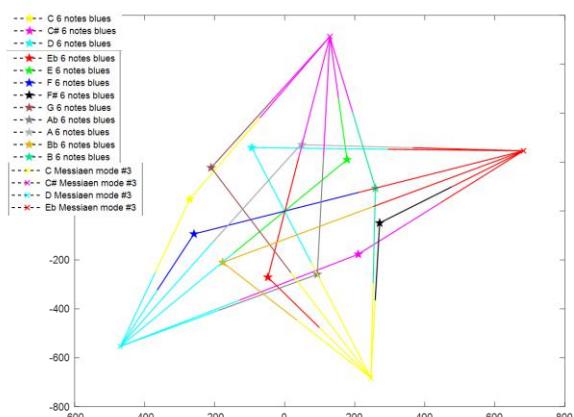


Fig. 11: 6 notes blues and Messiaen mode #3 (Rotation 1) scales.

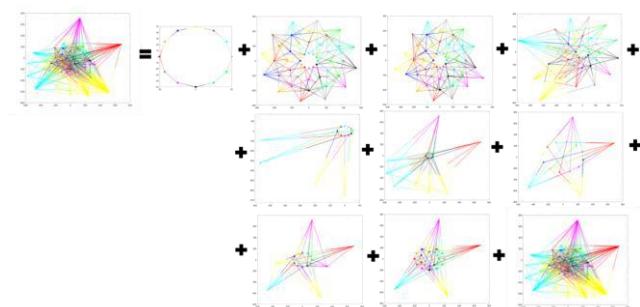


Fig. 14: The graph that expresses the melodic connection degree numbers among scales of all the families considered in this paper is the superposition (+) of different smaller graphs.

The same procedure is applied to demonstrate how the graph (Fig. 4) that expresses the *melodic connection degree numbers* among the chords constituted by three notes of all the families considered in this paper is the resultant of the sum (superposition) of different smaller graphs (Fig. 1, Fig.

15 to 20) each containing a lower number of chord families. Fig. 21 summarizes this concept, where the superposition (+) of all the smaller graphs on the right-hand side of the equation confers the overall complete graph on the left-hand side of the equation (=). Each smaller graph can be considered as a part and autonomously used in the composition process and music analysis.

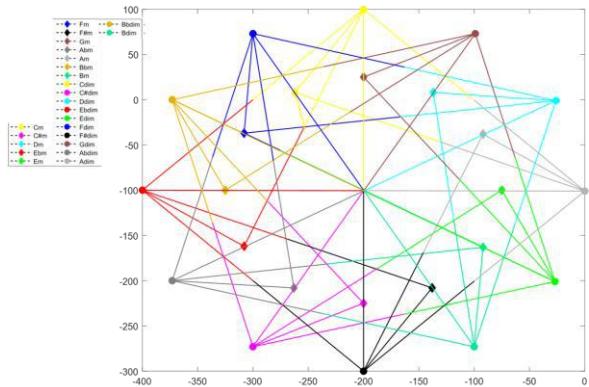


Fig. 15: Minor (m) and diminished (dim) chords.

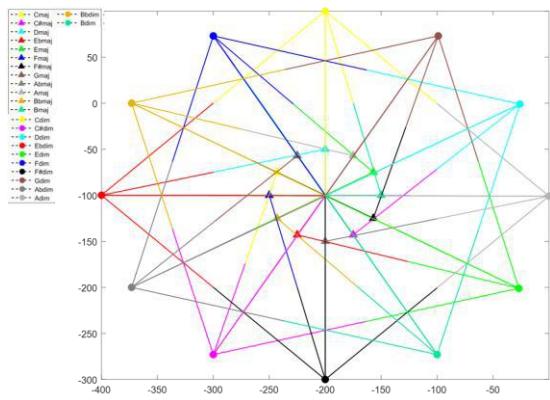


Fig. 16: Major (maj) and diminished (dim) chords.

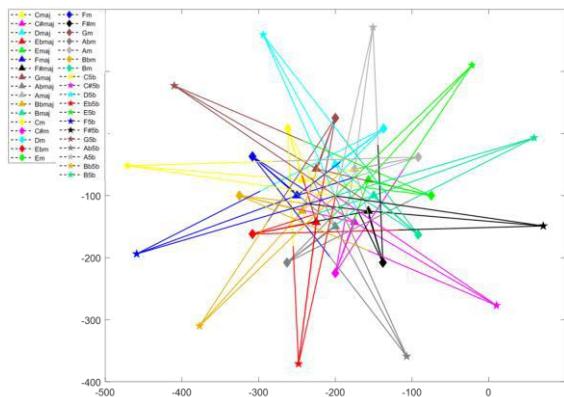


Fig. 17: Major (maj), minor (m) and diminished fifth (5b) chords.

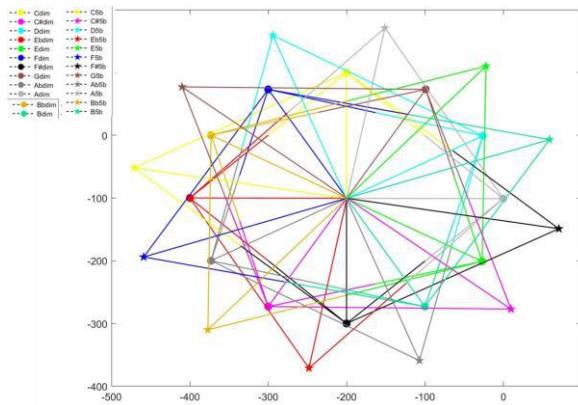


Fig. 18: Diminished (dim), and diminished fifth (5b) chords.

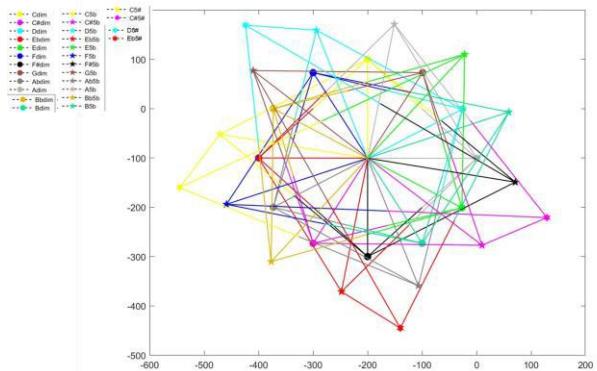


Fig. 19: Diminished (dim), diminished fifth (5b) and augmented fifth (5#) chords.

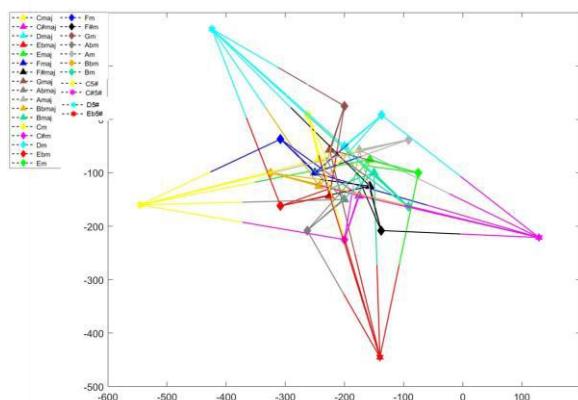


Fig. 20: Major (maj), minor (m) and augmented fifth (5#) chords.

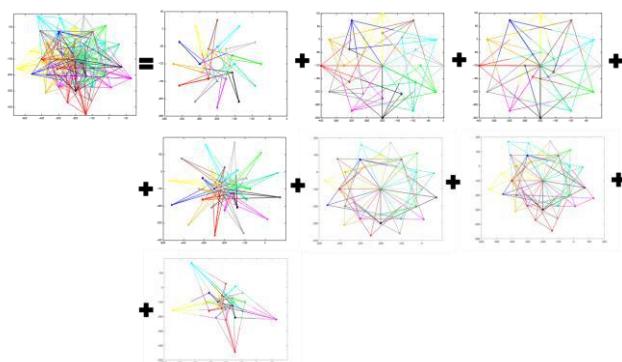


Fig. 21: The graph that expresses the melodic connection degree numbers among chords constituted by three notes of all the families considered in this paper is the superposition (+) of different smaller graphs.

A final consideration to the graph that expresses the *melodic connection degree numbers* among the chords constituted by two and three notes (*basic chords*). For the sake of brevity and simplicity, this graph has not been presented (the whole graph would be excessively full of symbols and superposed connections to be presented in a paper) but it is important to take into consideration the following concept: as stated, as the chords investigation function is employed, given (played or digitized) the scale name, this function gives and underlines the chords included in that scale while the trajectories between similar scales or similar chords related to the played (or digitized) scale are highlighted (exclusively for the sake of a better representation clarity related to the study reported in the present paper).

- It can be noted that each trajectory/pathway between the connected *basic chords* is part of one or more scales that include/generate those *basic chords*.
- Therefore to summarize, each symbol in each graph that expresses the *melodic connection degree numbers* among scales represents each scale in a condensed/synthetic form, as well as each linear feature related to the pathways/trajactories between connected *basic chords* is part of one or more scales (more than one scale can share the same pathway/trajactory between connected *basic chords*).

The same concept can be partially expressed by the graph that expresses the *melodic connection degree numbers* among the chords of the major (*maj*) and minor (*m*) families highlighting the *C major (ionian)* scale, as reported by the example in Fig. 3. Indeed, similar highlights to those shown in Fig. 3 are obtained if the major (*maj*) and minor (*m*) families highlighting the *G*

major (ionian) scale are considered in the graph, as shown by the following Fig. 22.

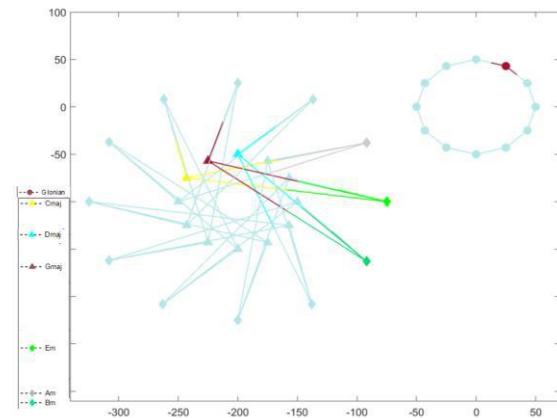


Fig. 22: Representation of the G major (ionian) scale harmonization built with the chords of the major (maj) and minor (m) families together with all the highlighted trajectories between similar scales or similar chords related to the G major (ionian) scale.

As it can be observed in Fig. 22 and Fig. 3, among the chords of the major (*maj*) and minor (*m*) families, the *C major (ionian)* and the *G major (ionian)* scales share the pathways/trajactories between the connected chords related to *Gmaj-Em*, *Cmaj-Am*, *Cmaj-Em*. Indeed these chords (and the related connections) can be generated/included by both the *C major (ionian)* and by the *G major (ionian)* scale.

The music universe is constituted by notes, scales, and chords. As stated before, for the sake of representation clarity, the network of the music universe as seen/focused on the scales is kept distinct from the network of the music universe as seen/focused on the chords. The present paragraph of the present Section is aimed to summarize the general concept related to the network of music connections focusing on the scales. Fig. 23 explicitly shows the main concept. For a better representation clarity, the arbitrary angular coordinates (θ) which distinguish the scales of the same family characterized by a different tonic adopted in Fig. 23 are different from those assumed and shown in the homologous Fig. 5a and Fig. 6 (which are similarly presented together in Fig. 23).

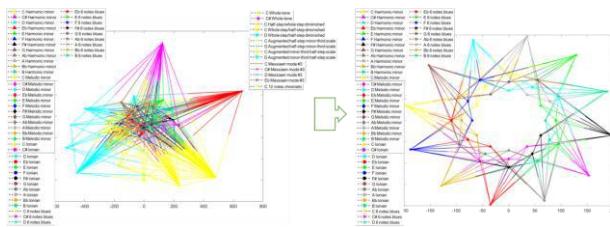


Fig. 23: The music universe as seen/focused on the scales of all the families considered in this paper (a). When appropriately filtered, the Harmonic minor, Melodic minor, Ionian and 6 notes blues scales turn out (b).

An example of the sound pathways constituting the music universe focused on the most useful scales adopted by a musician is reported hereafter. In particular, the sound pathways constituting the music universe focused on the most useful scales employed by Allan Holdsworth and reported in the “Just for the curious” book [8] are briefly presented and graphically reported. The names related to these scales have been assigned accordingly to those commonly employed (even if they are differently called in [8]). Some of these names have been already adopted in the previous sections of the present paper. The structures of these scales are summarized in Table 3:

Table.3: Structure of the scale types considered by [8] in terms of interval related to their tonic (1)

Scale type	Structure
<i>Major scale (ionian mode)</i>	<i>1 2 3 4 5 6 7</i>
<i>Melodic minor scale</i>	<i>1 2 3b 4 5 6 7</i>
<i>Harmonic minor scale</i>	<i>1 2 3b 4 5 6b 7</i>
<i>Harmonic major scale</i>	<i>1 2 3 4 5 6b 7</i>
<i>Whole-tone scale</i>	<i>1 2 3 4# 5# 7b</i>
<i>Half-step/whole step diminished scale</i>	<i>1 2b 3b 3 5b 5 6 7b</i>
<i>Whole step/half-step diminished scale</i>	<i>1 2 3b 4 5b 5# 6 7</i>
<i>Messiaen mode #3 (Rotation 1)</i>	<i>1 2b 2 3 4 4# 5# 6 7b</i>
<i>Bebop major scale</i>	<i>1 2 3 4 5 5# 6 7</i>
<i>Harmonic minor scale+6 notes blues scale</i>	<i>1 2 3b 4 4# 5 6b 7b 7</i>
<i>Bebop dominant scale</i>	<i>1 2 3 4 5 6 7b 7</i>
<i>Melodic minor scale+Dorian scale</i>	<i>1 2 3b 4 5 6 7b 7</i>
<i>Melodic minor scale+Harmonic minor scale</i>	<i>1 2 3b 4 5 6b 6 7</i>

<i>Melodic minor scale+4#</i>	<i>1 2 3b 4 4# 5 6 7</i>
<i>Melodic minor scale+Mixolydian scale</i>	<i>1 2 3b 3 4 5 6 7b 7</i>
<i>Minor pentatonic scale+3+6</i>	<i>1 3b 3 4 5 6 7b</i>

These scales have been suitably transposed and graphically presented in Fig. 24 that shows the network of music universe focused on the scales as seen by Allan Holdsworth. The subdivision of the entire graph shown in Fig. 24 into groups and subgroups of graphs (like the one presented in Fig. 14 of the present paper) will be presented hopefully in a subsequent study. Being the network of Fig. 24 constituted by a high number of symbols (markers) and lines of different colors which are difficult to distinguish, the legend has not been inserted in Fig. 24.

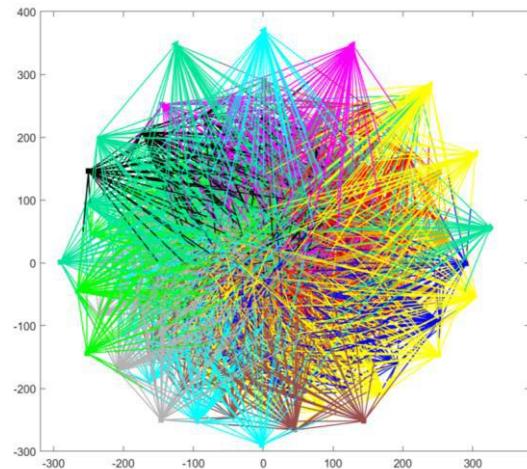


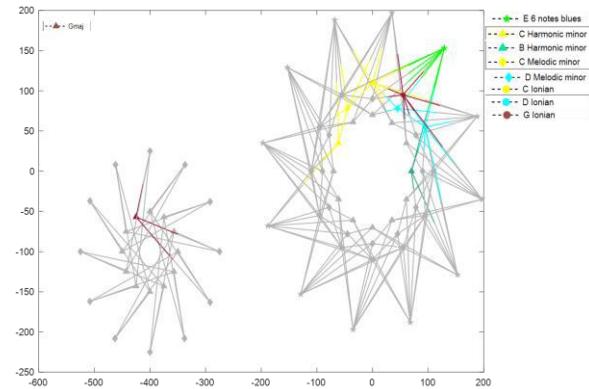
Fig. 24: The music universe as seen/focused on the scales of the families considered by Allan Holdsworth and reported in [8].

VI. THE FLOW OF SOUND PATHWAYS THROUGH THE MUSIC NETWORK

In this section, the transient process related to the flow of sound pathways through the music network is presented. As stated before, the connections graphs express the *melodic connection degree numbers* among scales/chords of all the families considered in this paper and include all the chromo-harmonic structures. These graphs allow representing all the potential melodic and harmonic choices, i.e. all the existent potential music paradigms and sonorities. The flow of sound pathways through the music network origins when a series of single notes or chords are played by the musician user. Under

these circumstances, some parts of the graph begin to be intercepted by the sound emitted by the instrument played by the musician. Therefore some parts of the graph begin to be highlighted like a target as the sound is emitted in real-time by the instrument. This because the present software application tool is connected to the instrument played by the musician via a hardware interface. As the musician plays (or types on a pc-keyboard) a series of N notes or N chords or a single note or chord (in this case $N=1$), the software receives and recognizes the signal constituted by the series of N notes or N chords from the instrument via the hardware interface (unless typed from the pc-keyboard). The signal receiving and recognizing process takes place for every finite number N of notes or chords (N can be varied during time). Among the functions implemented in the software, this is aimed to show the scales (more in general generic groups of notes that can be scales and chords) which include the series of N notes or N chords (or a single note or chord) played in real-time by the musician (or typed on a pc-keyboard). As the scales investigation function is employed, it gives and underlines the scales, among those considered by the system since chosen by the user-musician for the analysis, that include all the N notes or N chords of each series. These scales are highlighted in each graph together with the trajectories between their similar scales or similar chords (related to the played series of N notes or N chords).

For instance, when the *G major* chord ($N=1$) or arpeggio (that is the series of $N=3$ notes related to the *G major* chord) is played within each finite time interval Δt , it is possible to detect the scales among the families considered in this paper that include/generate the *G major* chord or arpeggio within each finite time interval Δt , together with the trajectories between similar scales (related to those which include/generate the played *G major* chord or arpeggio) or similar chords (related to those which include/generate the played *G major* chord or arpeggio and constituted by three notes of the major (*maj*) and minor (*m*) families; the graphical feature related to the highlighted chords constituted by three notes of the major (*maj*) and minor (*m*) families which are exclusively considered and distinctly represented in the scale investigation function has been adopted ad-hoc for the sake of a better representation clarity related to the present study in order to obtain an optimized graphical representation).



*Fig. 25: Representation of the *G major* chord or arpeggio through the pathways/trajectories between scales. The trajectories between similar scales (related to those which include/generate the played *G major* chord or arpeggio) or similar chords (related to the played *G major* chord or arpeggio and constituted by three notes of the major (*maj*) and minor (*m*) families) are highlighted.*

For the sake of brevity and a better representation clarity, the scales reported in Fig. 23 (b) are exclusively considered in this section for the present investigation. Fig. 25 shows the representation of the *G major* chord or arpeggio (played within each finite time interval Δt) through the pathways/trajectories among the nodes of the network of scales and chords (generic groups of notes). In addition, the trajectories between similar scales (related to those which include/generate the played *G major* chord or arpeggio) or similar chords (related to the played *G major* chord or arpeggio and constituted by three notes of the major (*maj*) and minor (*m*) families, for the sake of brevity and a better representation clarity) are highlighted when the *G major* chord or arpeggio (more in general when one or more notes/chords) is played within each finite time interval Δt .

The finite interval Δt is the time in which each series of N notes or N chords has to be analyzed. The musician user can choose and set the finite number N of notes or chords in the software tool instead of Δt . This last procedure corresponds (it is equivalent) to set the finite time interval Δt . The lower is the finite number N of notes or chords (the time interval Δt is too short) compared to the time signature of the executed tune (if any), the higher is the detail level provided by the scales investigation function which detects the sound pathways among the nodes of the network of scales and chords. If $N=1$ (the Δt is so short that it corresponds to $N=1$ executed note/chord for each Δt), the completed and detailed list containing all the scales that include each played (or typed) note/chord is provided and figured in the graphs. This high level of

detail could be excessively punctual and unuseful for the music analyses. On the other hand, if the finite number N of notes or chords is too high (the time interval Δt is too long) compared to the time signature of the executed tune (if any), the provided list containing all the scales and chords that include the series of notes/chords could be too poor and again unuseful for music analyses. The finite number N of notes or chords (or equivalently the time interval Δt) has to be sufficiently great and suitably matchable to the time signature of the tune (more in general, of any group of notes played) to allow the related music melodic and harmonic deep analyses. In this last case, a suitable average of the sound pathways is equivalently applied, since only the scales (and chords) that include each series of N notes/chords (within each suitable Δt) are provided and usefully figured in the graphs.

Four test cases aimed at showing the scales investigation function are reported here below. For better representation clarity, the graph that expresses the *melodic connection degree numbers* among scales reported in Fig. 23 (b) has been chosen to represent the music-transient process related to the chord progression of each Case. This graph does not include the scales related to the *Modes of limited transposition* considered in the present paper but exclusively involves the *Harmonic minor*, *Melodic minor*, *Ionian* and 6 notes blues scales. Additional analyses that include the *Modes of limited transposition* considered in the present paper will be addressed hopefully in a subsequent paper.

Case#1 is characterized by the following quite ordinary chord progression:

Table.4: Chord progression related to Case#1

Am	Cmaj	Gmaj	Em
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It is possible to graphically represent the chord progression related to Case#1 for three different N values ($N=1$, $N=2$, $N=4$). Since Case#1 is characterized by the chords of the major (*maj*) and minor (*m*) families, similarly to what presented in Fig. 25, the graph that expresses the *melodic connection degree numbers* among chords of the major (*maj*) and minor (*m*) families and the graph that expresses the *melodic connection degree numbers* among scales reported in Fig. 23 (b) are presented together in this investigation. These graphs are aimed at representing the music-transient process related to the chord progression of Case#1 for the three different N values. Case#1 related to $N=1$ is shown in Figs. 26 to 29. It has to be specified that in this Section, some colors used for highlighting markers and the related connection lines are different from those previously reported in Fig. 1 and Fig. 23 because the grey

color has already been employed as the transparency neutral color for all the no highlighted markers and lines. It has to be noted that the graphs which exclusively show the no highlighted markers conceptually represent the mutual interconnections and logical patterns that have already been traced and that had been waiting to be intercepted/tracked like a target as the sound is emitted in real-time by the instrument played by the musician.

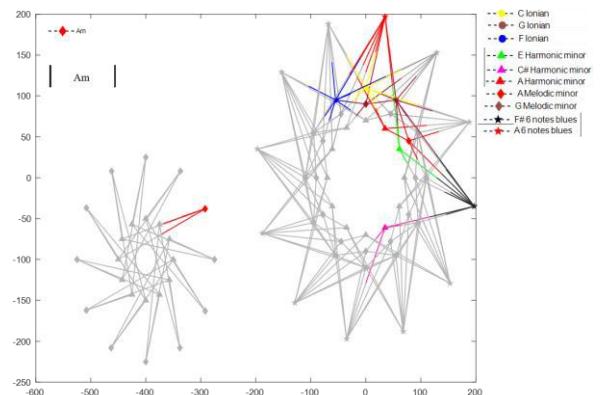


Fig. 26: Case#1 related to $N=1$, 1st bar.

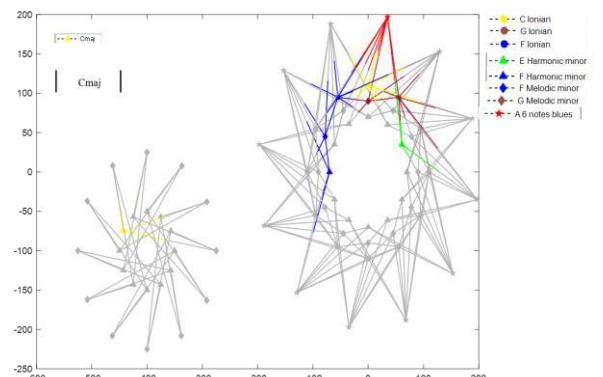


Fig. 27: Case#1 related to $N=1$, 2nd bar.

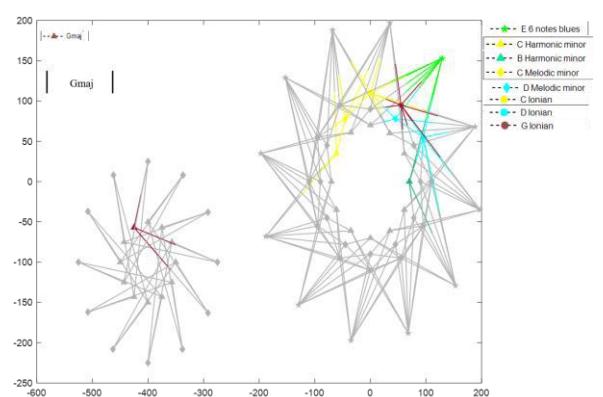


Fig. 28: Case#1 related to $N=1$, 3rd bar.

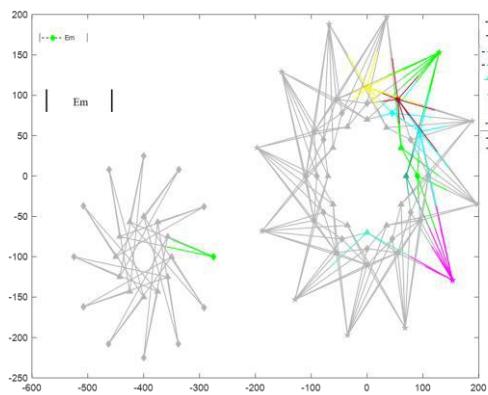
Fig. 29: Case#1 related to N=1, 4th bar.

Fig. 30 summarizes the whole music-transient process related to the chord progression denoted by Case#1 for N=1.

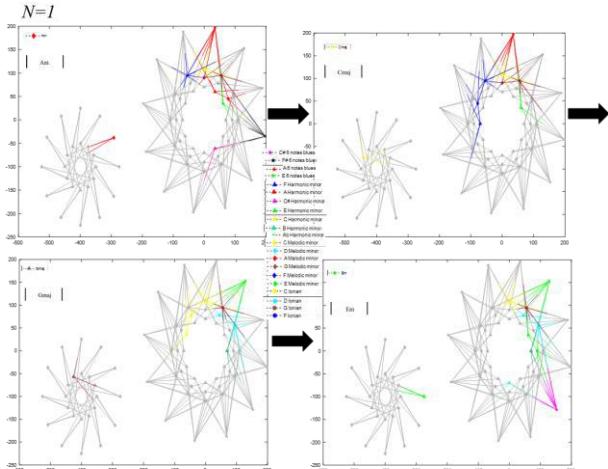


Fig. 30: The flow of sound pathways through the music network related to the chord progression denoted by Case#1 for N=1.

Case#1 related to N=2 is shown in Figs. 31 and 32.

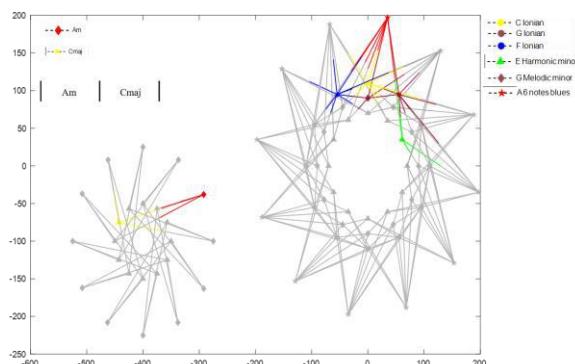


Fig. 31: Case#1 related to N=2, first two bars.

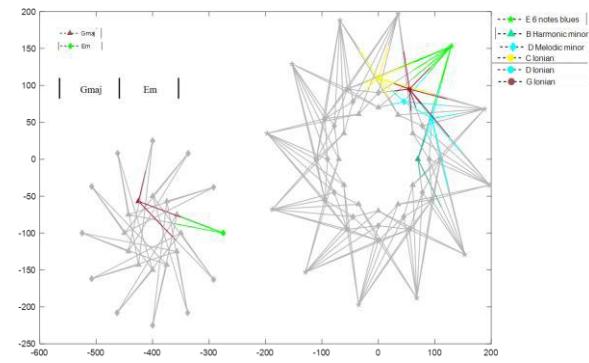


Fig. 32: Case#1 related to N=2, last two bars.

The whole music-transient process related to the chord progression denoted by Case#1 for N=2 is summarized by Fig. 33.

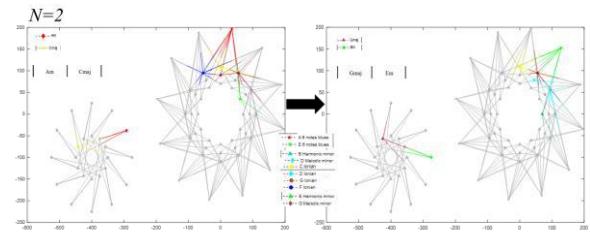


Fig. 33: The flow of sound pathways through the music network related to the chord progression denoted by Case#1 for N=2.

Case#1 related to N=4 is shown in Fig. 34.

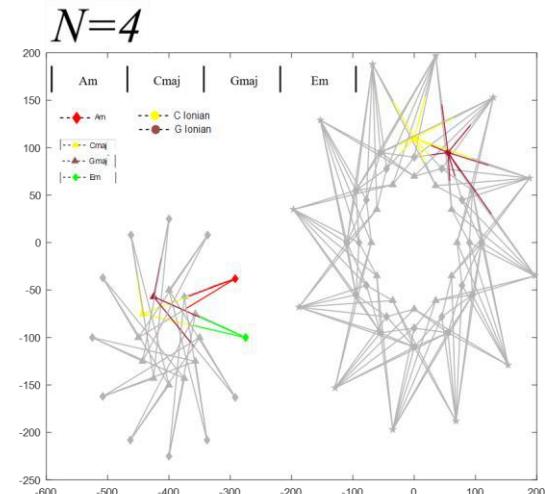


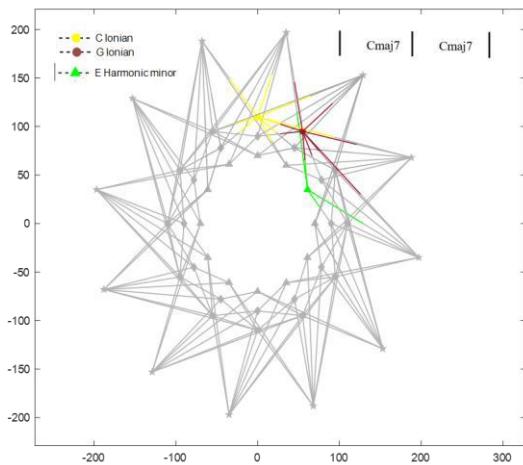
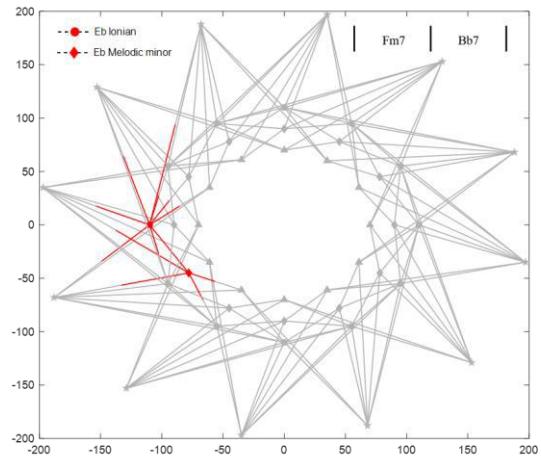
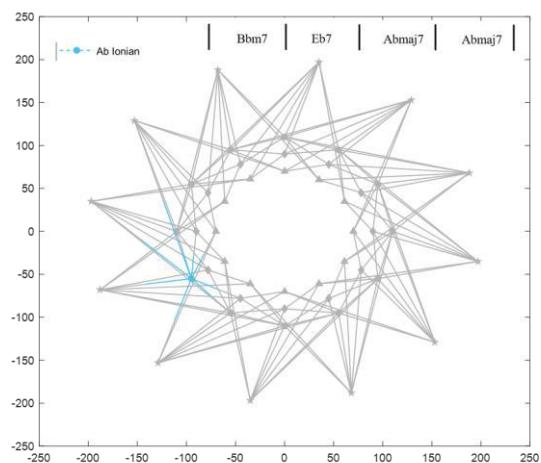
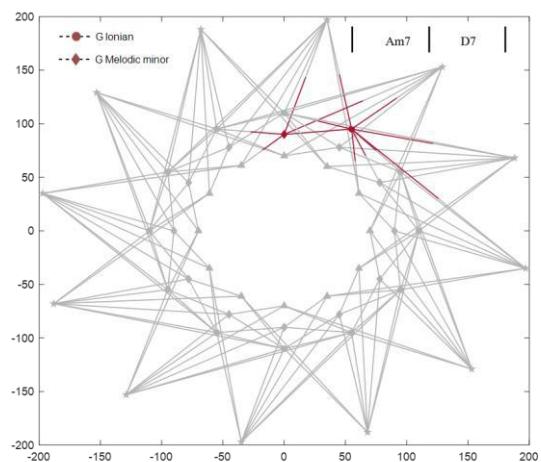
Fig. 34: Case#1 related to N=4.

Case#2 is characterized by the following chord progression:

Table.5: Chord progression related to Case#2

<i>Cmaj7</i>	<i>Cmaj7</i>	<i>Fm7</i>	<i>Bb7</i>
<i>Cmaj7</i>	<i>Cmaj7</i>	<i>Bbm7</i>	<i>Eb7</i>
<i>Abmaj7</i>	<i>Abmaj7</i>	<i>Am7</i>	<i>D7</i>
<i>Dm7</i>	<i>G7</i>	<i>Cmaj7 Eb7</i>	<i>Abmaj7G7/5#</i>

It is possible to graphically represent the chord progression related to Case#2 by adopting different N values. In particular, the value related to $N=2$ has been employed for the first six bars (Figs. 35, 36), the value related to $N=4$ has been employed from the 7th to 10th bar (Fig. 37), the value related to $N=2$ has been employed from the 11th to 12th bar (Fig. 38), the value related to $N=3$ has been employed from the 13th to 15th bar (Fig. 39), the value related to $N=2$ has been employed from the 15th to 16th bar (Figs. 40 and 41). The whole music-transient process related to the chord progression denoted by Case#2 is summarized by Fig. 42 for the different N values.

Fig. 35: Case#2 related to $N=2$: 1st, 2nd, 5th and 6th bars.Fig. 36: Case#2 related to $N=2$: 3rd and 4th bars.Fig. 37: Case#2 related to $N=4$: 7th, 8th, 9th and 10th bars.Fig. 38: Case#2 related to $N=2$: 11th and 12th bars.

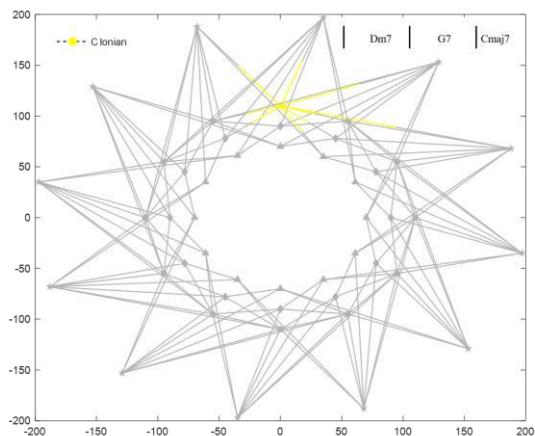
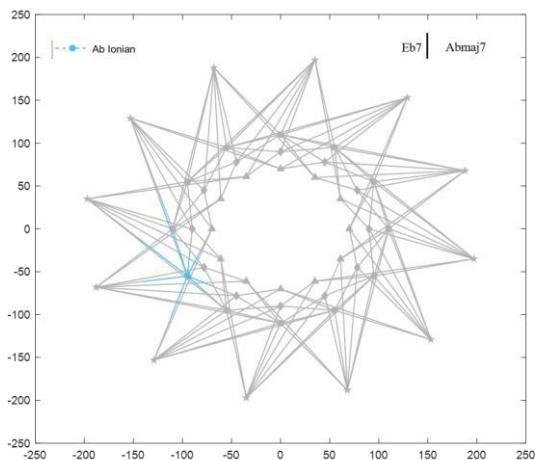
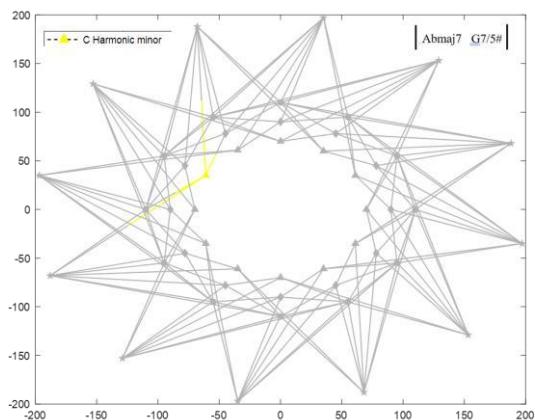
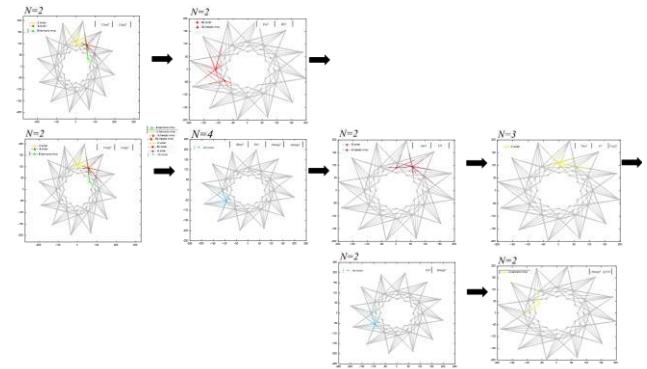
Fig. 39: Case#2 related to N=3: 13th, 14th and 15th bars.Fig. 40: Case#2 related to N=2: 15th and 16th bars.Fig. 41: Case#2 related to N=2: 16th bar.

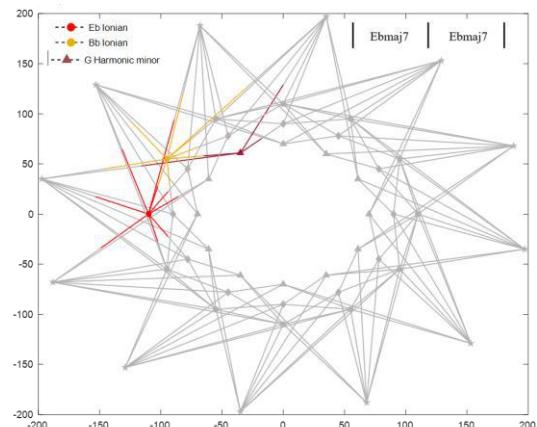
Fig. 42: The flow of sound pathways through the music network related to the chord progression denoted by Case#2 for different N values.

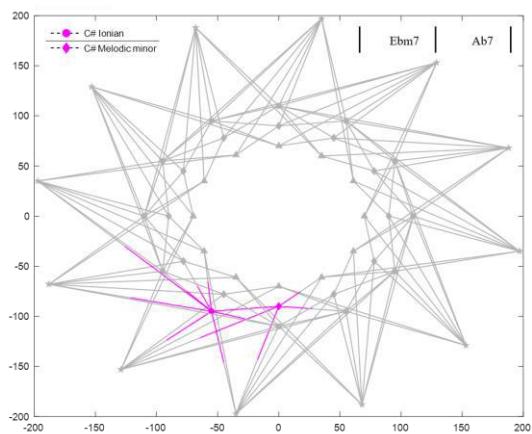
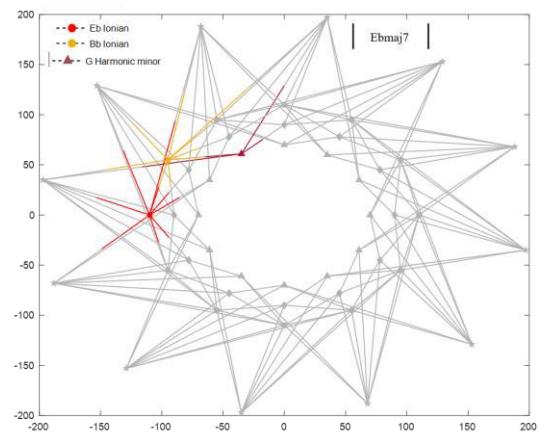
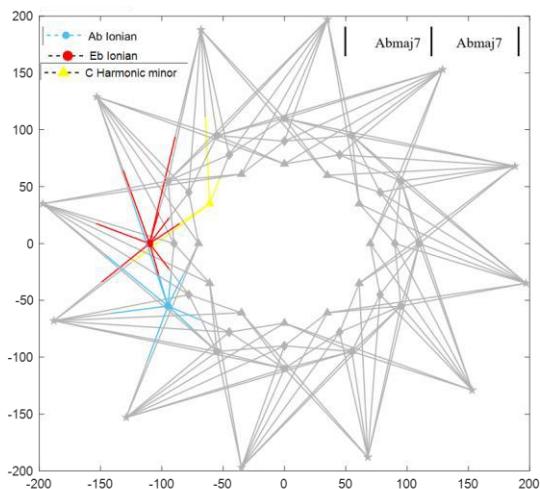
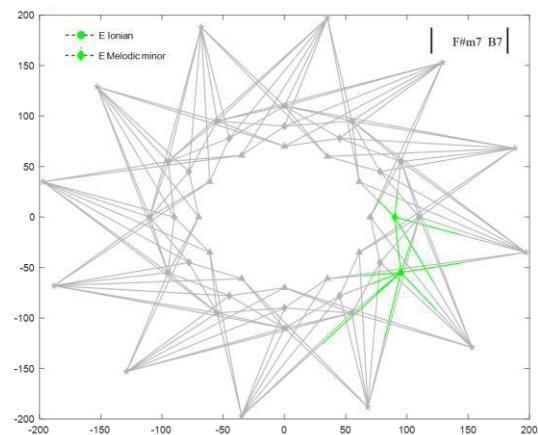
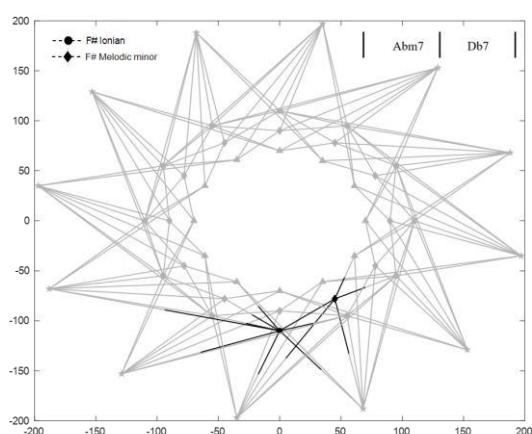
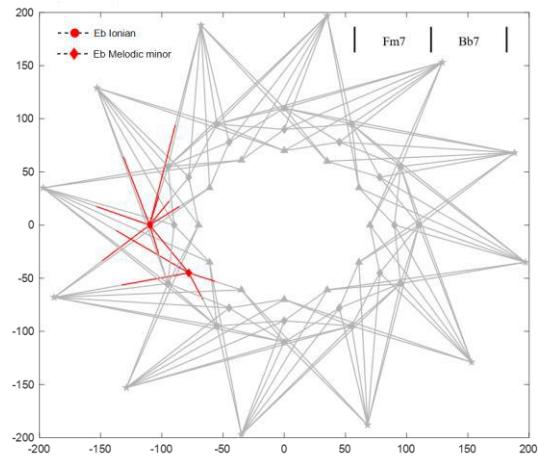
Case#3 is characterized by the following chord progression:

Table.6: Chord progression related to Case#3

Ebmaj7	Ebmaj7	Ebm7	Ab7
Abmaj7	Abmaj7	Abm7	Db7
Ebmaj7	F#m7 B7	Fm7	Bb7
Ebmaj7	F#m7 B7	Fm7	Bb7

It is possible to graphically represent the chord progression related to Case#3 by adopting different N values. In particular, the value related to $N=2$ has been employed for the first eight bars (Figs. 43 to 46), the value related to $N=1$ has been employed for the 9th and 13th bar (Fig. 47), the value related to $N=2$ from the 10th to 12th bar and from the 14th to 16th bar (Figs. 48 and 49). The whole music-transient process related to the chord progression denoted by Case#3 is summarized by Fig. 50 for the different N values.

Fig. 43: Case#3 related to N=2: 1st and 2nd bars.

Fig. 44: Case#3 related to N=2: 3rd and 4th bars.Fig. 47: Case#3 related to N=1: 9th and 13th bars.Fig. 45: Case#3 related to N=2: 5th and 6th bars.Fig. 48: Case#3 related to N=2: 10th and 14th bars.Fig. 46: Case#3 related to N=2: 7th and 8th bars.Fig. 49: Case#3 related to N=2: 11th, 12th, 15th and 16th bars.

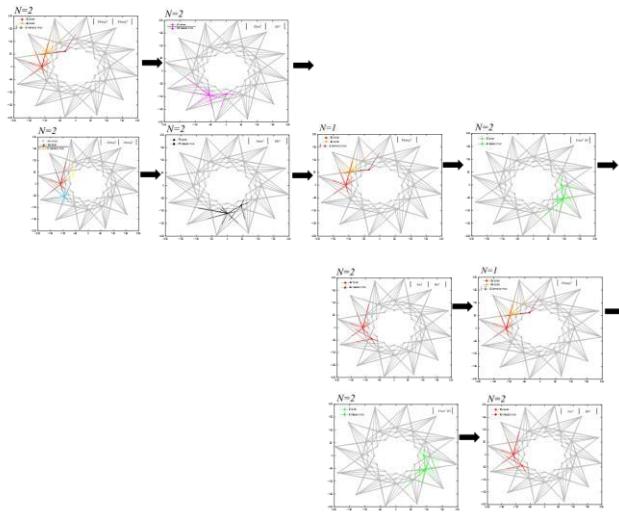


Fig. 50: The flow of sound pathways through the music network related to the chord progression denoted by Case#3 for different N values.

Case#4 is characterized by the following quite ordinary chord progression:

Table.7: Chord progression related to Case#4

<i>Em</i>	<i>Cmaj</i>	<i>Gmaj</i>	<i>Dmaj</i>
-----------	-------------	-------------	-------------

It is possible to graphically represent the chord progression related to Case#4 for three different N values ($N=1$, $N=2$, $N=4$). Case#4 related to $N=1$ is shown in Figs. 51 to 54. It has to be specified that the colors adopted for markers and connection lines could be slightly different than those of other cases due to graphic issues.

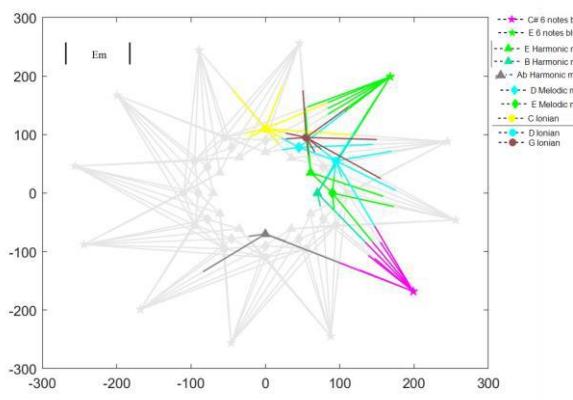


Fig. 51: Case#4 related to $N=1$, 1st bar.

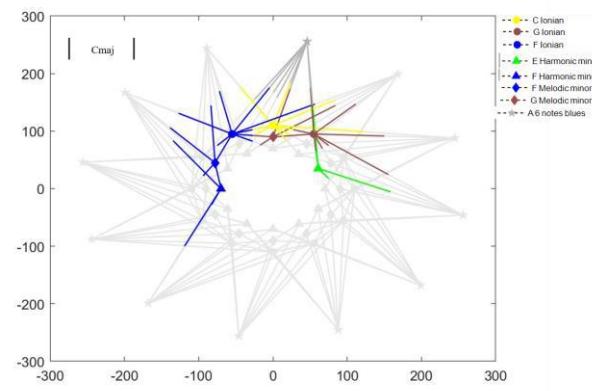


Fig. 52: Case#4 related to $N=1$, 2nd bar.

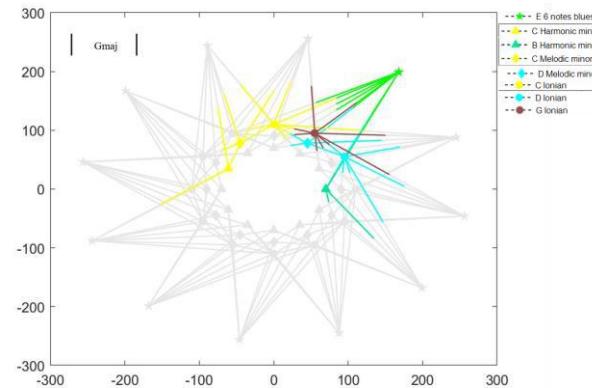


Fig. 53: Case#4 related to $N=1$, 3rd bar.

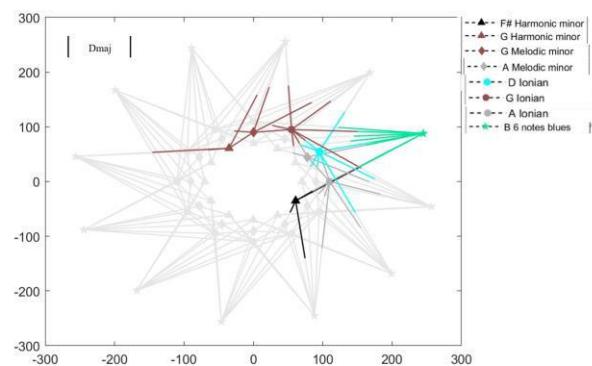


Fig. 54: Case#4 related to $N=1$, 4th bar.

Fig. 55 summarizes the whole music-transient process related to the chord progression denoted by Case#4 for $N=1$.

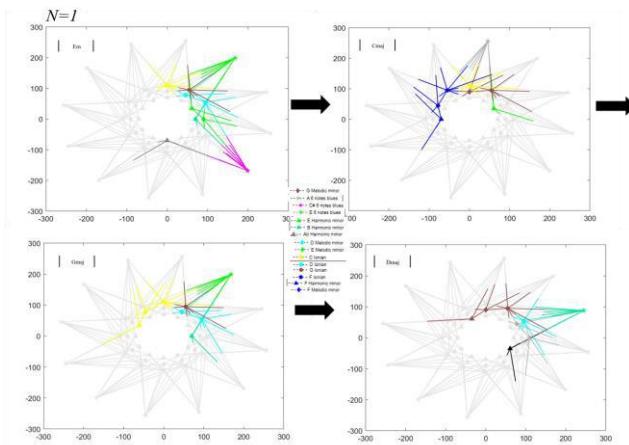


Fig. 55: The flow of sound pathways through the music network related to the chord progression denoted by Case#4 for $N=1$.

Case#4 related to $N=2$ is shown in Figs. 56 and 57.

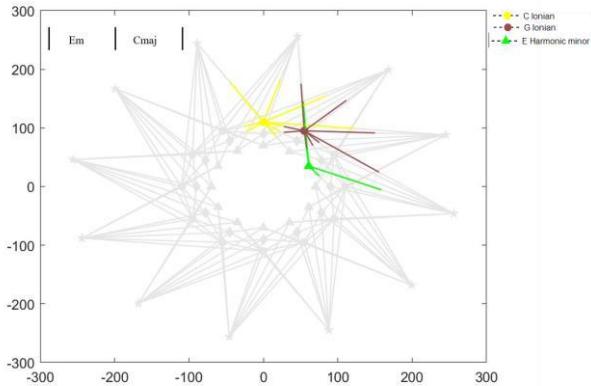


Fig. 56: Case#4 related to $N=2$, first two bars.

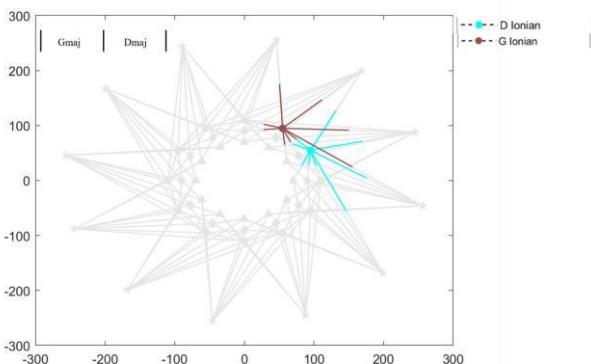


Fig. 57: Case#4 related to $N=2$, last two bars.

The whole music-transient process related to the chord progression denoted by Case#4 for $N=2$ is summarized by Fig. 58.

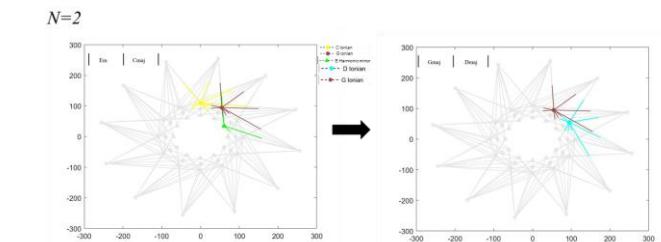


Fig. 58: The flow of sound pathways through the music network related to the chord progression denoted by Case#4 for $N=2$.

Case#4 related to $N=4$ is shown in Fig. 59.

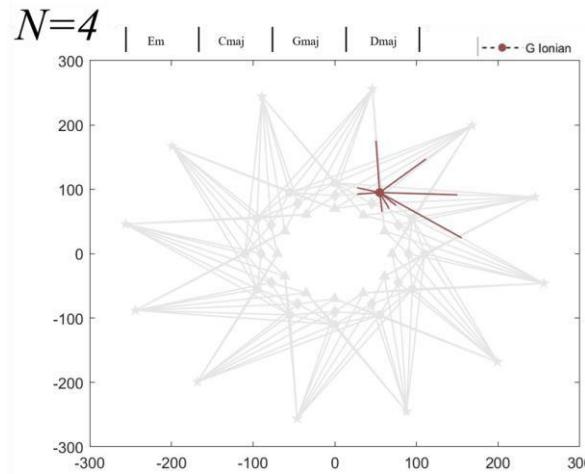


Fig. 59: Case#4 related to $N=4$.

It is interesting to notice that if the last bar had featured the Dm chord instead of $Dmaj$, the analysis related to $N=4$ would have yielded a different response despite the first three bars remaining the same, as shown in Fig. 60.

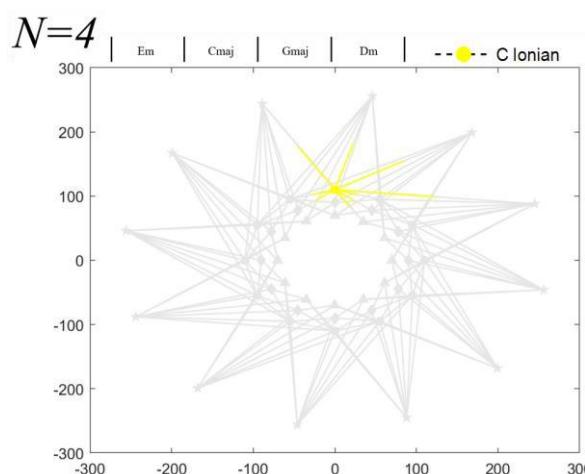


Fig. 60: Case#4 related to $N=4$ when the Dm chord replaces the $Dmaj$ in the last bar.

In general, the scales related to the *Modes of limited transposition* should be included in the analysis (*12 notes chromatic scale* included), in order to better describe and justify some of the harmonic and melodic passages related to the chord progressions reported in the present paper. In this case, deeper harmonic analyses will be presented hopefully in a subsequent study. This paper has to be intended as a general presentation, an introduction of the main concepts related to the utilization of the music graphs (on how these graphs and the related approach work).

VII. CONCLUSIONS

A specific algorithm has been designed to detect the harmonic and melodic transient passages involving any generic group of notes arbitrarily played in real-time (or digitized on a pc-keyboard). An interactive software application tool (via hardware connected to the instrument of each musician) has been realized. The geometrical music graphs are automatically traced by the process as any generic group of notes is played. These graphs constitute a network whose nodes are reciprocally interconnected. Each node symbolically represents a specific music scale or a specific chord (more in general any generic group of notes). The music network is constituted by a set of maps in which the melodic passages and harmonic progressions can be traced and analyzed on the graphs instead of on a pentagram. The shape of these graphs is determined by the interconnections between the nodes which are arbitrarily placed by the user. These graphs have not to be intended for replacing the pentagram but they have to be intended as an auxiliary device to see and analyze the music transient passages through the music geometrical network. In particular, these graphs can be employed for performing the melodic and harmonic analysis (any sort of music investigation) of any kind of tune, to recognize recurring patterns in music, and also to represent and better explain the reasons behind specific melodic and harmonic choices. These graphs can be also intended as an auxiliary device for the visual detection of innovative unexplored harmonic and melodic solutions in the human composition process. More in general, the present system is not related to a specific instrument, representing a synthetic geometrical view of all the potential connections between different groups of frequencies. The number and typology of groups and the shape of the related network depend on the musician's subjective knowledge and taste. The more the musician learns and increases his music-theoretical and practical knowledge, the more the size and shape of the related music graph network can change and grow.

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NOMENCLATURE

Notes names:

$C = (B\#)$

$C\# = Db$

D

$D\# = Eb$

$E = (Fb)$

$F = (E\#)$

$F\# = Gb$

G

$G\# = Ab$

A

$A\# = Bb$

$B = (Cb)$

Interval names:

$1/8 = \text{perfect unison}/\text{perfect octave}$

$(1\#) = (\text{augmented unison})$

$2b/9b = \text{minor second}/\text{minor ninth}$

$2/9 = \text{major second}/\text{major ninth}$

$2\#/9\# = \text{augmented second}/\text{augmented ninth}$

$3b = \text{minor third}$

$3 = \text{major third}$

$(3\#) = (\text{augmented third})$

$(4b/11b) = (\text{diminished fourth}/\text{diminished eleventh})$

$4/11 = \text{perfect fourth}/\text{perfect eleventh}$

$4\#/11\# = \text{augmented fourth}/\text{augmented eleventh}$

$5b = \text{diminished fifth}$

$5 = \text{perfect fifth}$

$5\# = \text{augmented fifth}$

$6b/13b = \text{minor sixth}/\text{minor thirteenth}$

$6/13 = \text{major sixth}/\text{major thirteenth}$

(6#/13#)=(augmented sixth/augmented thirteenth)

(7bb)=(diminished seventh)

7b= minor seventh

7 = major seventh

(7#)= (augmented seventh)

(8b) =(diminished octave)

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