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The Determination of the Price of Capital Goods: A Differential Game Approach

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Abstract

In this paper, building upon a q-model of investment with adjustment costs, we address the strategic determinants of the price of newly installed productive capacity. Specifically, we develop a differential game in which a competitive producer of consumption goods deals with a seller of capital goods endowed with market power. From a theoretical perspective, we show that an open-loop Stackelberg equilibrium with non-cooperative features requires the producer of consumption goods to be more impatient than the seller of capital goods. Thereafter, relying on some numerical simulations, we show that our theoretical setting is able to replicate the countercyclical pattern of the relative price of capital goods as well as its negative relationship with the investment-output ratio.

JEL Classification: C72; D25; E22; G31.

Keywords: Price of capital goods; Tobin's q; Internal and external adjustment costs; Differential games.

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1 Introduction

In the market for machine tools and industrial equipment are traded all the items that allow active firms to increase their productive capacity. As it always happens for actually traded goods, such a market can hardly be thought as a competitive one for a number of sound reasons. First, the machine tool industry is one of the smallest sectors of the manufacturing industry in most industrial countries both in terms of employment and added value, so it is unlikely to be populated by a large number of firms on its supply side (cf. Carlsson, 1983; Kim and Lee, 2008). Second, in order to meet the specific needs of the involved buyers, the capital goods traded in that market are usually characterized by a strong degree of differentiation and customization (cf. Chen, 2018). Third, no matter the political orientation of the country in which such a market may be actually functioning, public authorities are often active in regulating some of its terms of trade by fixing tariffs, technological trajectories and/or import-export shares (cf. Fransman, 1986; Dinopoulos and Kreinin, 1991, Eaton and Kortum, 2001; Estevadeordal and Taylor, 2013).[1] Moreover, given the technological complexity of the involved items, even the market for capital goods is often affected by informational asymmetries that bear both on buyers and sellers (cf. Hünemberg and Hüttman, 2003; Stanula et al. 2020).

A tiny market in which are traded differentiated goods under imperfect information, and where government interventions are not unusual, is unlikely to see the presence of price-taking sellers as implicitly assumed within the intertemporal q-model of corporate productive investment with adjustment costs (cf. Yoshikawa, 1980; Summers, 1981; Hayashi, 1982; Cooper and Ejarque, 2003). Consequently, drawing on such a dynamic framework, it may be interesting to show how the price of tangible capital goods is determined in a proper non-competitive environment in which the sellers of these items are in the position to set an optimal price strategy by observing the demand of the buyers. To the best of our knowledge, with the sole exception of a contribution sketched by Mino (1987), this paper is the first to attempt such an extension which appears promising from different perspectives.

On the macroeconomic ground, the determination of the price of the newly installed productive capacity is important because the price of capital goods is often considered as an essential driver of capital accumulation and economic growth (cf. Greenwood et al., 1997; Mutreja et al. 2018). In addition, from a microeconomic perspective, the determinants of the price of capital goods are equally relevant for the distinction and the relationship between internal and external adjustment costs. As it is well known, the former type of costs arises when firms bear some contingent expenditures to adjust their capital stocks such as the costs of installing the new capital, reorganizing the production lines and/or training employees to operate the new machines (cf. Eisner and Strotz, 1963; Lucas, 1967). On the other side, the latter arises when the firms' decision to invest or disinvest lead to an adjustment in the price set by the sellers of

[1] Again on the public side, the recognition and the protection of patents – crucial for the incentives to innovation and growth – is certainly another element that distances the market of capital goods from perfect competition by creating the conditions for setting predatory prices and implementing other non-competitive practices (cf. Kim and Lee, 2008).

new capital goods (cf. Keynes, 1936, Chapter 11; Foley and Sidrauski, 1970). Consequently, the joint determination of the amount of investment and its relative price allows us to assess how internal and external adjustment costs are related each other (cf. Christiano and Fisher, 2003).

In this paper, we address the strategic determinants of the price of newly installed productive capacity by developing a non-cooperative differential game in which a competitive producer of consumption goods continuously interacts over an infinite time-horizon with a seller of capital goods endowed with market power that behaves as a Stackelberg leader (cf. van der Ploeg, 1987). Specifically, we develop an analytically tractable q-model of investment with internal and external adjustment costs in which the producer of consumption goods is assumed to take its decisions on additional productive capacity with the aim of maximizing its discounted stream of profits by taking as given the dynamic trajectory of the price of capital goods set by the seller of these items and considering the relevant law of capital accumulation. At the same time, the seller of capital goods is assumed to set the price sequence of its competence by maximizing the present value of its stream of profits and taking into account the optimal investment decisions of the producer of consumption goods, the law of capital accumulation and the way in which its customer values its capital stock over time.

Within this theoretical setting, we demonstrate that an economically meaningful open-loop Stackelberg long-run equilibrium with non-cooperative features requires the producer of consumption goods to be more impatient than the seller of capital goods (cf. Rubinstein, 1982; Guerrazzi, 2020; Bressan and Jiang, 2020). Thereafter, relying on the outcome of some numerical simulations calibrated on the US economy, we evaluate the empirical performance of our non-competitive q-model of investment by showing that it is fairly able to replicate the countercyclical pattern displayed by actual data on the relative price of tradable capital goods as well as its negative relationship with the investment-output ratio (cf. Restuccia and Urrutia, 2001; Fisher, 2006; Justiniano et al. 2011; Lian et al. 2020). Obviously, at the micro level, this pattern implies that internal and external adjustment costs would tend – at least partially – to compensate each other in a short-run perspective. In addition, by making a comparison with a companion setting in which no player has market power, we show that a non-competitive market for capital goods bends but does not undermine the positive linear relationship between the marginal q and the investment-capital ratio and it is also responsible for a sluggish adjustment of the capital stock (cf. Andrei et al. 2019; Fuchs et al. 2016).

The paper is arranged as follows. Section 2 provides the theoretical framework. Section 3 explores its numerical properties. Finally, Section 4 concludes.

2 The theoretical framework

We consider a deterministic model economy in which time (t) is continuous – so that $t \in \Re^{+}$ – and where a competitive producer of consumption goods interacts over an infinite horizon with a seller of capital goods endowed with full market power just like a monopolist. On the one side, the revenues of the producer of consumption goods logarithmically depends on the level achieved by its capital stock (K) whose accumulation, in turn, is boosted by purchasing new capital goods (I) in order to counteract depreciation and increase output (cf. Mino, 1987; Cooper and Ejarque, 2003). Such a producer of consumption goods takes the trajectory of the price of investment goods (p) charged by their seller as given and it also bears a convex internal adjustment cost arising from the installation of the newly purchased productive capacity (cf. Lucas, 1967; Gould, 1968; Hayashi, 1982).[2] The price-taking behaviour of the producer of consumption goods on the market for machine tools and equipment can be thought as the upshot of a binding provision contract countersigned by the seller of capital goods that the two parties cannot renege. In other words, on the account of the stark degree of customization that usually characterizes this kind of industrial provisions, we assume that the seller of capital goods is not allowed to restart the game after the initial stage by setting a price strategy different from the one selected at the beginning (cf. Guerrazzi, 2020).

On the other side, the seller of investment goods is assumed to face the whole demand and to bear a constant cost ($c > 0$) for each unit of the capital goods sold to the producer of consumption goods (cf. Kultti, 2021). No matter the actual traded quantities, in a competitive market for machine tools and equipment such a constant value of the marginal selling cost would be identically and constantly equal to the price of capital goods.

Working on backward induction, we will solve first the problem of the producer of consumption goods and then the one the seller of capital goods. Moreover, all the times in which it does not detract from the clarity of the exposition, in the remainder of this section we will omit the functional dependence of variables on time.

2.1 The problem of the producer of consumption goods

Recalling that it takes as given the trajectory of the price of investment goods set by their seller as well as all the other hypotheses put forward above, the intertemporal problem of the producer of consumption goods can be written as

$$\max_{I} \int_{t=0}^{\infty} \exp\left(-r_I t\right) \left(\alpha \ln K - pI - \frac{h}{2} I^2\right) dt \tag{1}$$

s.to

$$\dot{K} = I - \delta K \qquad K(0) = K_0 > 0 \tag{2}$$

where $r_I > 0$ is its discount rate, $\alpha > 0$ is the price of consumption goods or a measure of capital productivity, $h > 0$ is the slope of marginal adjustment costs, $\delta > 0$ is the in-use depreciation

[2]Since fixed and non-convex adjustment costs tend to smooth each other at the aggregate level, a model with only convex costs appears as the natural candidate to explore the behaviour of investment for the whole economy (cf. Cooper and Haltiwanger, 2006).

rate of capital whereas K_0 is the initial capital stock.[3]

Given the time-path of p, the first-order conditions (FOCs) for the problem in (1) and (2) are given by

$$-p - hI + q = 0 \qquad \text{for all } t \tag{3}$$

$$\dot{q} = (r_I + \delta)\, q - \frac{\alpha}{K} \tag{4}$$

$$\lim_{t \to \infty} \exp(-r_I t)\, q(t)\, K(t) = 0 \tag{5}$$

where q is the costate variable associated to the capital accumulation constraint in eq. (2) that in the present setting mirrors the shadow value of capital whereas the ratio q/p is the marginal Tobin's q, i.e., the ratio of the market value of an additional unit of capital for the producer of consumption goods to its replacement cost (cf. Hayashi, 1982; Andrei et al., 2019).

The expression in eq. (3) is the FOC with respect to I and it provides – after a trivial manipulation – the instantaneous demand for capital goods undertaken by the producer of consumption goods that represents its optimal open-loop strategy. As illustrated by the downward linear schedule in Figure 1, its Marshallian representation reveals that the actual value taken by q fixes the boundary between investment and disinvestment. Specifically, whenever p is lower (higher) than the prevailing value of q, the producer of consumption goods decides to invest (disinvest) by purchasing a positive (negative) amount of investment goods in order to counteract (accelerate) capital depreciation and increase (decrease) its productive capacity.[4]

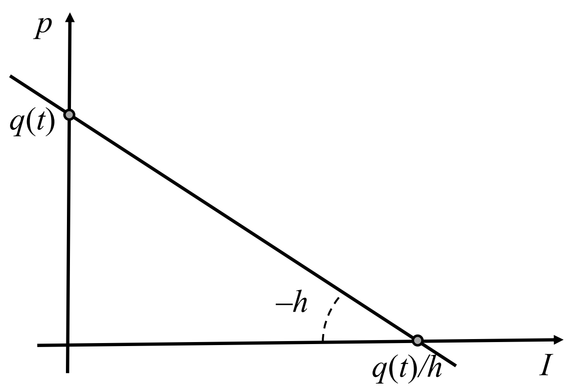

Figure 1: The instantaneous demand for capital goods

[3]Without loss of generality, we abstract from describing the decision about the employment of labour (cf. Andrei et al. 2019).

[4]In the context of the present model, negative values of I mean that the producer of consumption goods is allowed to return the capital goods to the seller by receiving their current value. In other words, we assume that there is no difference between the purchasing price of capital goods and their retirement price.

The remaining FOCs provide, respectively, the optimal evolution of q over time and an endpoint limit on the value of the state variable. The former, i.e., eq. (4) implies that in each instant the marginal productivity of capital (α/K) for the producer of capital goods must equal to the user cost of capital ($\left(r_I + \delta - \dot{q}/q\right) q$), whereas the latter, i.e., eq. (5) is the required transversality condition (cf. van der Ploeg, 1987).

For given values of p, the problem in (1) and (2) has two asymmetric stationary solutions, namely, one characterized by a positive value of the capital stock in the long run and another with a negative value. Obviously, the economically meaningful one is only the former in which the producer of consumption goods purchases a positive amount of capital goods just to provide for their physical depreciation. Therefore, in the long run, we can rule out disinvestment paths so that the steady-state equilibrium value of q will be strictly higher than the price of the capital goods set by their seller augmented by the marginal internal adjustment costs.[5]

2.2 The problem of the seller of capital goods

As recalled above, the seller of capital goods sets p in each instant by taking into account the manner in which the producer of consumption goods takes its investment or disinvestment decisions, the law of capital accumulation and the way in which it values capital over time. Therefore, recalling the hypotheses put forward above and considering the expressions in eq.s (3) and (4), its dynamic problem can be written as

$$\max_p \int_{t=0}^{\infty} \exp\left(-r_p t\right) \left((p - c) \frac{q-p}{h} \right) dt \tag{6}$$

s.to

$$\dot{K} = \frac{q-p}{h} - \delta K \qquad K(0) = K_0 > 0 \tag{7}$$

$$\dot{q} = \left(r_I + \delta\right) q - \frac{\alpha}{K} \tag{8}$$

where $r_p > 0$ is the discount rate of the seller of capital goods that does not necessarily coincide with the one of the producer of consumption goods.

The FOCs for the problem in (6) – (8) are given by

$$q - 2p + c - \mu = 0 \qquad \text{for all } t \tag{9}$$

$$\dot{\mu} = \left(r_p + \delta\right) \mu - \eta \frac{\alpha}{K^2} \tag{10}$$

[5] In addition, it would be possible to show that the stationary solution with positive investment and positive capital stock is characterized by a saddle-path dynamics. Formal details are given in Appendix.

$$\lim_{t\to\infty} \exp(-r_p t)\,\mu(t)\,K(t) = 0 \tag{11}$$

$$\dot{\eta} = (r_p - r_I - \delta)\,\eta - \frac{p+\mu-c}{h} \qquad \eta(0) = 0 \tag{12}$$

where μ and η are the costate variables associated, respectively, to the capital accumulation constraint in eq. (7) and to the optimal evolution of the shadow price of capital for the producer of consumption goods in eq. (8).

The expression in eq. (9) is the FOC with respect to p and it allows us to derive the instantaneous supply of capital goods; indeed, solving eq. (3) with respect to q and plugging the result into eq. (9) leads to the following upward-sloped linear expression:

$$p = c - \mu + hI \qquad \text{for all } t \tag{13}$$

Eq. (13) reveals that the selling reservation price for capital goods is given by $c - \mu$ whereas the slope of the corresponding supply-price schedule has the same value of the marginal adjustment cost borne by the producer of consumption goods but with the opposite sign. Furthermore, as illustrated in Figure 2, combining eq. (13) with eq. (3) allows us to retrieve the instantaneous value of the investment undertaken by the producer of consumption goods as well the instantaneous value of the price for charged these items.

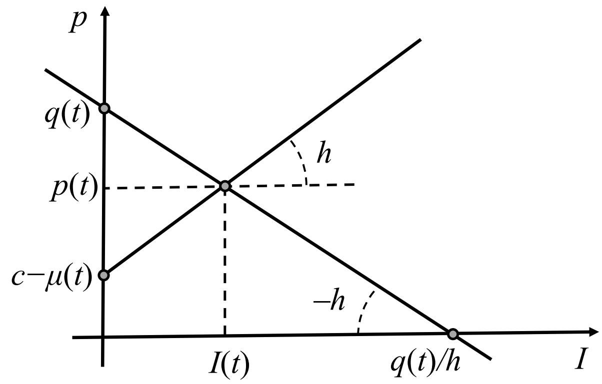

Figure 2: The instantaneous equilibrium of the market for capital goods

The diagram in Figure 2 straightforwardly reveals that the producer of consumption goods will invest (disinvest) whenever the prevailing value of q is higher (lower) than the corresponding value of $c-\mu$. Obviously, whenever the prevailing value of q is exactly equal to $c-\mu$ the producer of consumption goods nor invest nor disinvest by allowing the reduction of its capital stock implied by the depreciation rate.[6] According to eq.s (3) and (13), the analytical expressions for the instantaneous values of I and p are, respectively, given by

[6] Once again, in the economically meaningful steady-state equilibrium with a positive capital stock, the long-run value of q will be strictly higher than the long-run value of $c - \mu$.

$$I = \frac{q - (c - \mu)}{2h} \qquad \text{and} \qquad p = \frac{c - \mu + q}{2} \tag{14}$$

Interestingly, the two expressions in (14) reveal that the investment in new productive capacity carried out by the producer of consumption goods and its unit price set by the seller of capital goods are both positively indexed to the current value of q. As it will become apparent in the numerical analysis, the link between p and the shadow value of capital is a distinguishing feature of the model developed in this paper and it will shape the cyclical behaviour of the price of capital goods.

The remaining FOCs in eq.s (10) – (12) complete the description of the solution of the problem faced by the seller of capital goods. First, eq. (10) pins down the optimal dynamics of the costate variable associated to the capital accumulation constraint and it implies that the marginal productivity of capital for the producer of consumption goods (α/K) must be equal – in each instant – to the corresponding marginal revenue for the seller of capital goods ($-(K/\eta)\left(\dot{\mu}/\mu - r_p - \delta\right)\mu$). As argued by van der Ploeg (1987), η – and probably μ – will be typically negative, since the producer of consumption goods and the seller of capital goods have conflicting objectives in a proper non-cooperative setting where the respective payoffs move in opposite directions.[7] Second, eq. (11) is the required transversality condition. Moreover, eq. (12) describes the optimal dynamics of the costate variable associated to the likewise optimal evolution of the shadow price of capital for the producer of consumption goods. As far as such a FOC is concerned, it is also worth noticing that the negative marginal profit for the seller of capital goods triggered by variations of the shadow price of capital for the producer of consumption goods has to be equal to zero at the beginning of the optimization horizon (cf. Başar and Olsder, 1999, Chapter 7; Dockner et al. 2000, Chapter 5). Such an additional initial condition for the model economy is required because the shadow value of capital – and therefore the investment expenditure carried out by the producer of consumption goods – is free to jump in the initial stage of the game. Therefore, setting $\eta(0) = 0$ allows the price strategy of the seller of capital goods to be optimal independently of the choice of $q(0)$ (cf. Calvo, 1978; Lipton et al. 1982; van der Ploeg, 1987).

Summing up, eq.s (7) – (14) implies that the solution of the intertemporal problems of the two player described above returns the following 4×4 non-linear system of differential equations:

$$\begin{pmatrix} \dot{q} \\ \dot{\mu} \\ \dot{K} \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\frac{c}{2h} \\ \frac{c}{2h} \end{pmatrix} + \begin{bmatrix} r_I + \delta & 0 & \frac{\alpha}{K^2} & 0 \\ 0 & r_p + \delta & 0 & -\frac{\alpha}{K^2} \\ \frac{1}{2h} & \frac{1}{2h} & -\delta & 0 \\ -\frac{1}{2h} & -\frac{1}{2h} & 0 & r_p - r_I - \delta \end{bmatrix} \begin{pmatrix} q \\ \mu \\ K \\ \eta \end{pmatrix} \tag{15}$$

It is worthwhile to notice that the non-linearity of the system in (15) involves only the common state variable for the two players which is obviously given by the stock of capital owned by the producers of consumption goods.

[7]A visual appraisal of the non-cooperative character of the game under scrutiny is given in Appendix.

2.3 Steady state

In the model economy described above, steady-state allocations are defined as the set of quadruplets $\overline{\mathcal{S}} := \{\overline{q}, \overline{\mu}, \overline{K}, \overline{\eta}\} \in \mathbb{R}^4_+$ such that $\dot{q}(\overline{q}, \overline{\mu}, \overline{K}, \overline{\eta}) = \dot{\mu}(\overline{q}, \overline{\mu}, \overline{K}, \overline{\eta}) = \dot{K}(\overline{q}, \overline{\mu}, \overline{K}, \overline{\eta}) = \dot{\eta}(\overline{q}, \overline{\mu}, \overline{K}, \overline{\eta}) = 0$. Obviously, in order to have economically meaningful stationary allocations, we require that the eligible values of $\overline{K}$ to be strictly positive. Moreover, in case of asymptotic stability, some elements of that set will be also characterized by the fact that $\lim_{t\to\infty} q(t) = \overline{q} \wedge \lim_{t\to\infty} \mu(t) = \overline{\mu} \wedge \lim_{t\to\infty} K(t) = \overline{K} \wedge \lim_{t\to\infty} \eta(t) = \overline{\eta}$ by leading also to the convergence of the flow of investment carried out by the producer of consumption goods and the price charged by the seller of capital goods.

The elements of the set $\overline{\mathcal{S}}$ can be derived as follows. First, setting $\dot{q} = \dot{\mu} = 0$, the first and the second row of the dynamic system in (15) respectively imply that

$$\overline{q} = \frac{\alpha}{(r_I + \delta)\overline{K}} \tag{16}$$

$$\overline{\eta} = \frac{(r_p + \delta)\overline{K}^2 \overline{\mu}}{\alpha} \tag{17}$$

Second, setting $\dot{K} = 0$, the third row of (15) and the expression in eq. (16) imply that

$$\overline{\mu} = 2h\delta\overline{K} + c - \frac{\alpha}{(r_I + \delta)\overline{K}} \tag{18}$$

Moreover, setting $\dot{\eta} = 0$, the fourth row of (15) and the in eq.s (17) and (18) leads to the following expression:

$$2h\delta\overline{K}^2 + c\overline{K} - \frac{\alpha}{(r_I + \delta)} \frac{r_p (r_p - r_I)}{(r_p - r_I - \delta)(r_p + \delta)} = 0 \tag{19}$$

Eq. (19) is simply a quadratic function of the long-run equilibrium stock of capital owned by the producer of consumption goods which implies that – in general – the quadruplets of the set $\overline{\mathcal{S}}$ are two only. Descartes' rule, however, straightforwardly implies that the existence of an economically meaningful positive long-run solution requires the constant term on the LHS of eq. (19) to be negative and this can happen in two completely distinct cases which are characterized by the actual degree of impatience of the two involved players. Specifically, the constant term of eq. (19) results in being negative (i) when the seller of capital goods is less impatient of the producer of consumption goods, i.e., whenever r_p is lower than r_I, and (ii) when the degree of impatience of the seller of capital goods exceeds the one the producer of consumption goods by more than the value of the depreciation rate of physical capital, i.e., whenever r_p is higher than $r_I + \delta$.

A selection between the two cases mentioned above can be done by focusing on the existence of a non-cooperative stationary solution characterized by a positive stock of capital; indeed, the expressions in eq.s (2), (12) and (14) imply that

$$(r_p - r_I - \delta)\,\overline{\eta} = \delta\overline{K} \tag{20}$$

Eq. (20) reveals that a positive value of $\overline{K}$ is consistent with a negative value of $\overline{\eta}$ only when r_p is lower than r_I by ruling out the case in which r_p is higher than $r_I + \delta$. Consequently, the existence of an economically meaningful stationary solution with non-cooperative features requires the producer of consumption goods to be more impatient than the seller of capital goods. In addition, eq. (17) suggests that when $\overline{\eta}$ is negative, $\overline{\mu}$ is also negative and this implies – through eq. (14) – that the equilibrium price of capital goods ($\overline{p}$) is always higher than the corresponding marginal selling cost (c) by allowing the seller these items to realize long-run positive profits, a typical pattern of a non-competitive market (cf. Guerrazzi, 2023).

The result on the different magnitude of the discount rates of the two players is important for the definition of the leadership of the differential game under scrutiny and is deeply rooted in the existing literature on non-cooperative dynamic games. For instance, in the sequential bargaining model with alternate offers developed by Rubinstein (1982), the more patient agent is the one with the dominant position because in the subgame perfect equilibrium she/he obtains the largest piece of the pie to share. Similarly, developing a differential game in which a union of insiders that sets the wage of its members by acting as the leader and a firm that sets employment by acting as the follower, Guerrazzi (2020) shows that in an open-loop Stackelberg equilibrium increasing values of the discount rate on the side of the firm tend to weaken its labour market power to the benefit of the union. Moreover, Bressan and Jiang (2020) show that the existence of self-consistent Stackelberg equilibria for stochastic games in infinite-time horizon requires the discount rate of the leader (follower) to be sufficiently low (high) because this weakens the incentives to renege. Consequently, having regard to these arguments, it should not be surprising that the role of leader in our differential game is played by the less impatient agent.[8]

Before addressing the local dynamics of the model economy, it might be interesting to compare the stationary solutions delivered by eq. (19) with the corresponding steady-state values of the stock of capital – say $\overline{\overline{K}}$ – achieved in a competitive situation in which capital goods are always sold and purchased at their constant marginal selling cost c. As shown in Appendix, in such a competitive setting the two stationary solutions for the stock of capital are given by the roots of the following quadratic expression:

$$h\delta\overline{\overline{K}}^2 + c\overline{\overline{K}} - \frac{\alpha}{r_I + \delta} = 0 \tag{21}$$

The quadratic term as well as the constant term of eq. (21) are lower with respect to the corresponding coefficients of eq. (19); indeed, the second factor of the constant term of eq. (19) – namely, $r_p\,(r_p - r_I) \,/\, (r_p - r_I - \delta)\,(r_p + \delta)$ – is strictly higher than 1. Obviously, as illustrated in Figure 3, this means that in non-competitive setting the economically meaningful long-run

[8]According to the standard theory, lower values of the discount rate are usually associated with a better access to the capital market (cf. Huber and Gormsen, 2023).

equilibrium level of capital accumulation will be strictly lower with respect to the one prevailing in a competitive situation.

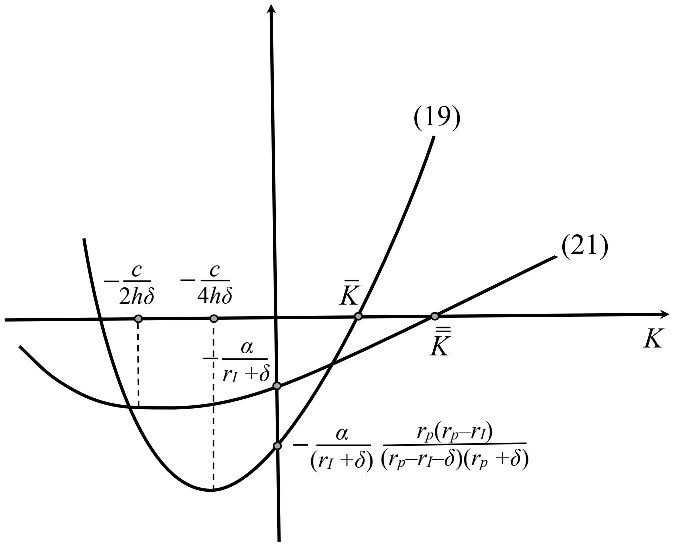

Figure 3: Stationary solutions in the competitive and the non-competitive setting

The horizontal distance between $\overline{K}$ and $\overline{\overline{K}}$ plotted in Figure 3 represents a measure of the market power owned by the seller of capital goods and its length depends on the parameters entering the second factor of the constant term of eq. (19). Specifically, whenever r_I is higher than r_p, such a distance is unambiguously an increasing function of the discount rate of the producer of consumption goods and a decreasing function of the depreciation rate of capital. The former pattern corroborates the idea that – ceteris paribus – the impatience of the producer of consumption goods strengthens the market power of the seller of capital goods. By contrast, the effect of the discount rate of the seller of capital goods on the extent of its market power is not unambiguous. In detail, whenever r_I is higher (lower) than $2r_p$, the distance between $\overline{K}$ and $\overline{\overline{K}}$ is an increasing (decreasing) function of the discount rate of the seller of capital goods.[9]

2.4 Local dynamics

The system in (15) and the expressions in eq.s (16) – (20) imply that the local dynamics of the model economy around the economically meaningful element of $\overline{\mathcal{S}}$ is described by the following 4×4 linear system:

$$\begin{pmatrix} \dot{q} \\ \dot{\mu} \\ \dot{K} \\ \dot{\eta} \end{pmatrix} = \begin{bmatrix} r_I+\delta & 0 & \frac{\alpha}{\overline{K}^2} & 0 \\ 0 & r_p+\delta & \frac{2\alpha\delta}{(r_p-r_I-\delta)\overline{K}^2} & -\frac{\alpha}{\overline{K}^2} \\ \frac{1}{2h} & \frac{1}{2h} & -\delta & 0 \\ -\frac{1}{2h} & -\frac{1}{2h} & 0 & r_p-r_I-\delta \end{bmatrix} \begin{pmatrix} q-\overline{q} \\ \mu-\overline{\mu} \\ K-\overline{K} \\ \eta-\overline{\eta} \end{pmatrix} \tag{22}$$

[9]By continuity, such a distance is not influenced by r_p whenever r_I is exactly equal to $2r_p$.

Let $\Lambda := \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ be the set of the four eigenvalues of the Jacobian matrix – say J – in (22). Thereafter, according to the decomposition suggested by Bosi and Desmarchelier (2019), the corresponding characteristic equation can be written as

$$\lambda^4 - 2r_p\lambda^3 + \mathcal{P}_2\lambda^2 - \mathcal{P}_3\lambda + \frac{2h\delta\left(r_p+\delta\right)\left(r_I+\delta\right)\left(r_I-r_p+\delta\right)\overline{K}^2 + \alpha r_p\left(r_I-r_p\right)}{2h\overline{K}^2} = 0 \tag{23}$$

where $\mathcal{P}_2 \equiv \lambda_1\left(\lambda_2+\lambda_3+\lambda_4\right)+\lambda_2\left(\lambda_3+\lambda_4\right)+\lambda_3\lambda_4$, $\mathcal{P}_3 \equiv \lambda_1\lambda_2\left(\lambda_3+\lambda_4\right)+\lambda_3\lambda_4\left(\lambda_1+\lambda_2\right)$ whereas the constant term is given by the positive value of the determinant of J.

Whenever $\mathcal{P}_2$ and $\mathcal{P}_3$ are negative, Descartes' rule implies that the characteristic equation has two positive real roots and two negative real roots so that the unique economically meaningful element of $\overline{\mathcal{S}}$ is represented by a convergent saddle point. Such a characterization of the steady-state solution is corroborated by the turnpike property of optimal growth models; indeed, given the initial conditions for K and η, the problem of seller of capital goods in eq.s (6) – (8) consists in discounting at a positive rate a concave instantaneous profit function over an infinite horizon by complying with two convex dynamic constraints. Consequently, such a problem must have a unique meaningful saddle-path stationary solution towards which all the endogenous variables asymptotically tend to converge in order to verify the transversality conditions in (5) and (11) (cf. Cass, 1966).

From a dynamic perspective, a saddle-path implies there is only one trajectory that satisfies the dynamic system in (22) that converges to the steady-state allocation, whereas all the others tend to diverge by vanishing the commitment of the price-strategy set by the seller of capital goods. Strictly speaking, in the proposed dynamic model in which the two players continuous interacts, the equilibrium path is locally determinate, i.e., taking a given initial value of the stock of capital (K_0) and the initial condition for η (0), there is a unique vector $\begin{bmatrix} q(0) & \mu(0) \end{bmatrix} \in \Re^2_+$ in the neighbourhood of the relevant element of $\overline{\mathcal{S}}$ that generates a converging trajectory. Specifically, the values of $q(0)$ and $\mu(0)$ should be selected to satisfy the transversality conditions in (5) and (11) by placing the system in (22) on the stable branch of the saddle point. In the remainder of the paper, the stable saddle path followed by q, μ , η, K – and, indirectly, by I and p – will be taken as the perfect-foresight (or the self-consistent) path of the model economy and its shape will be explored numerically.

3 Numerical properties

In this section we explore the numerical properties of the theoretical framework developed above. For this purpose, the model is calibrated on an annual basis by taking as reference the economy of the US which is one of the countries with the highest ratio between equipment production and GDP (cf. Eaton and Kurtun, 2001). Specifically, the productivity parameter (α) is set at the value suggested by Kydland and Prescott (1982) for the elasticity of output with respect to capital.[10] The depreciation rate (δ) is fixed by averaging the values of physical obsolescence attached to different capital items retrieved by Gomme and Lkhagvasuren (2013). The value of the discount rate of the seller of capital goods (r_p) is set by taking the lower figure of the degrees of impatience suggested by Itskhoki and Moll (2019) whereas the discount rate of producer of consumption goods (r_I) is set by respecting the condition for an economically meaningful stationary solution that neutralizes the effect of r_p on market frictions. Moreover, aiming at keeping the profits of the producer of consumption goods at non-negative values, the slope of the internal marginal adjustments costs (h) born by the producer of consumption goods is set at the lower bound of the figures for the convex component of these expenditures retrieved by Cooper and Haltiwanger (2006). Furthermore, consistently with the hypothesis of perfect competition in the market for consumption goods, the marginal selling cost (c) born by the seller of capital goods is set at a value that in the long run drives to zero the profits of the buyer of these items. All the parameter values together with their respective description are collected in Table 1.

Parameter	Description	Value
α	Productivity of capital	0.3600
h	Slope of marginal adjustment costs	0.0490
r_I	Discount rate of the producer of consumption goods	0.0600
δ	Depreciation rate of capital	0.0884
r_p	Discount rate of the seller of capital goods	0.0300
c	Marginal selling cost of capital goods	0.0705

Table 1: Calibration

Exploiting the baseline calibration in Table 1 and setting the initial capital stock 1% below its long-run equilibrium value, a suitable discretization allows us to simulate our model by generating artificial series for all the involved variables.[11] In order to test the empirical performance of our theoretical framework, we begin our numerical exploration by tracking the implied trajectories followed by the variables for which there exists an observable counterpart. Specifically, the saddle path-dynamics of I, p, K and q are illustrated in the two panels of Figure 4.

[10]The implicit hypothesis is that the production function of the producer of consumption goods in a log-linearization of a standard Cobb-Douglas function.

[11]Recall that the initial condition for η is given by the FOCs in (12). Moreover, the baseline calibration is integrated by setting to 1 the time-step of simulation and simulating the model for 180 periods in order to allow the full convergence of all the variables. MAT LAB codes are available from the authors upon reasonable request.

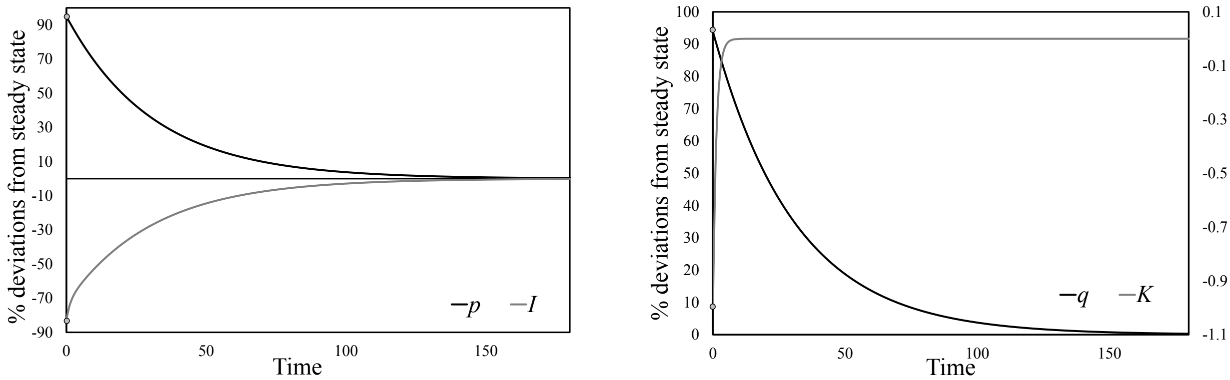

Figure 4: Saddle-path dynamics

The plot on the left-hand-side of Figure 4 shows that when the stock of capital undershoots its equilibrium value by 1%, the price of capital goods (level of investment) overshoots (undershoots) its steady-state value by 95% (83%). Obviously, this means that p and I move in opposite directions during their respective adjustment process. Consequently, our non-competitive model implies a procyclical behaviour for productive investment and a countercyclical behaviour for the price of capital goods.[12] These two dynamic patterns represent distinguishing attributes of the model developed in this paper. On the one hand, the procyclical behaviour displayed by I is at odds with the implications of the textbook q-model of investment with adjustment costs and it reproduces a well-known regularity of business cycles (cf. Alogoskoufis, 2019, Chapter 11; Kydland and Prescott, 1982). On the other hand, the countercyclical pattern of p mirrors in a clear manner the path followed by actual available data on the relative price of capital goods. For instance, surveying US data over the period 1964-2009, Justiniano et al. (2011) document a persistent fall of the relative price of investment which is robust to the adoption of alternative measure of the deflator for consumption goods. Moreover, the plot on the right-hand-side of Figure 4 reproduces the standard dynamics of the textbook q-model of investment by showing that when K undershoots its equilibrium value by 1% (left scale), the shadow value of capital overshoots its steady-state value by 94% (right scale).

In the existing macroeconomic literature, the progressive decline of the relative price of capital goods has been often rationalized with the advancement of endogenous or exogenous technical progress and the ongoing integration of the markets in which these items are traded (cf. Eaton and Kortum, 2001; Estevadeordal and Taylor, 2013; Lian et al. 2020). Our model offers a fully-endogenous complementary explanation grounded on the strategic interplay between buyers and sellers of capital goods that can be summarized as follows. In the early stages of the accumulation process, the productivity of employed capital – because of its scarcity – is relatively high by leading to high values of its shadow value for the buyers of capital goods who clearly will find profitable to increase their productive capacity. Aware of this pattern,

[12] In terms of the diagram in Figure 2, this means that in each instant the demand and the price schedules for capital goods tend to shift downwards. The drop of the latter, however, is always higher than the drop of the former.

the sellers of these items will then tend to charge high prices when they are endowed with market power. Thereafter, as the accumulation process proceeds, the productivity of capital and its shadow value tend to reduce so that the sellers of capital goods will have incentive to cut charged prices by reducing their markup over the marginal cost. In this way, even without technical progress or any trade liberalization, the price of capital goods will tend to display a countercyclical pattern like the one shown in Figure 4 which is confirmed by the available empirical evidence (cf. Fisher, 2006).[13]

The declining behaviour of the price of capital goods triggers some consequences even at the micro level; indeed, the progressive reduction of p seems to prefigure that internal and external adjustment costs should – at least in part – counterbalance each other during the adjustment process to the stationary solution. This pattern is at odds with the findings derived within a q-model of investment with only external adjustment costs developed by Mino (1987) in which increasing (decreasing) investment always puts upward (downward) pressure on the price of capital goods. By contrast, the dynamics implied by our model appears to be consistent with the theoretical framework developed by Christiano and Fisher (2023) in which a fall (rise) in the price of capital goods leads to a simultaneous increase (decrease) of investment and installation costs. As it will be apparent in a moment, an evaluation of the dominating effect on the actual path followed by new productive capacity can be assessed from the implied relation between the marginal q and the investment-capital ratio.

As we noticed in the introduction, the price of capital goods is an important driver of growth because its reductions (increases) tend to boost (decrease) investment in new productive capacity (cf. Greenwood et al., 1997; Mutreja et al. 2018). The theoretical relationship tracked over the adjustment path between the price of capital goods and investment-output ratio – that in our model is given by $I/\left(\alpha \ln K + I\right)$ – is illustrated on a logarithmic scale in Figure 5.

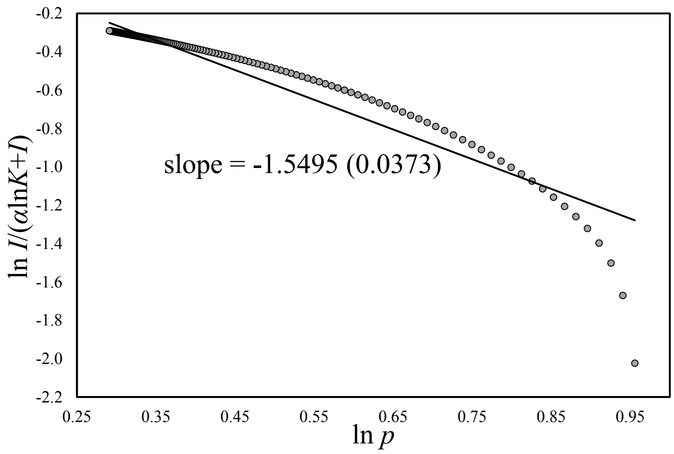

Figure 5: Price of capital goods and investment-output ratio (logarithmic scale)

[13]A more agnostic view on the cyclical behaviour of the price of capital goods is given by Beaudry et al. (2015).

The plot of Figure 5 shows a plain negative relationship between the price of capital goods and the investment-output ratio which is effectively summarized by the negative sign of the slope of the OLS coefficient reported together with its standard error (in parenthesis) on the diagram. Interestingly, estimating the same relationship on a sample of 125 countries over the period 1960-1985, Restuccia and Urritia (2001) find coefficients on the range $[-0.98, -1.36]$, an interval which is not too distant from the point estimation retrieved from our artificial observations plotted in Figure 5.[14]

We conclude our numerical exploration by comparing some critical magnitudes generated by our non-competitive framework with the matching values retrieved from a companion competitive setting as the one described in Appendix. In this direction, ignoring the value of r_p, we calibrate the 2×2 competitive model by exploiting the parameter values in Table 1 with one only exception. Specifically, exactly as we assumed to happen when the seller of capital goods has market power, the value of the marginal selling cost of capital goods – that in a competitive environment should coincide with their constant unit price – is set to a point value that drives to zero the long-run level of the profit achieved by the producer of consumption goods. In this way, all the rents are removed from the model and the value of the marginal q converges to the same level in both scenarios. Straightforward algebra revels that such a level of c is higher than the one collected in Table 1 but lower than $\overline{p} = 1.3342$. More precisely, such a value of c amounts to 1.3311, so that $\overline{q}/\overline{p} = \overline{\overline{q}}/1.3311$.[15] Under the suggested calibration, the equilibrium output gap between an economy with a non-competitive market for capital goods and an economy in which no player has market power, namely the ratio $\left(\alpha \ln \overline{K} + \overline{I}\right) / \left(\alpha \ln \overline{\overline{K}} + \overline{\overline{I}}\right)$, is equal to 45.90%, a figure close to the 40% estimated by Mutreja et al. (2018) through a Ricardian model of international trade as the average value of the output distance between a market for capital goods with frictions and a corresponding frictionless setting.

The q-theory of investment has often been used to assess the link between the marginal q, i.e., the ratio between q and p, and the investment-capital ratio (cf. Hayashi, 1982). In order to assess how the presence of market power on the side of the seller of capital goods may affect the shape of such a critical relationship, the panel of the left-hand-side of Figure 6 plots the theoretical link between q/p and I/K in the competitive setting whereas the panel on the right-hand-side plots the corresponding relation in the non-competitive one.

[14]Similar findings are also found by Lian et al. (2020) on a sample of developed and developing countries over the period 1995-2011.

[15]In the 2×2 competitive model, setting $p = c = 0.0705$ as we did in the non-competitive setting would simply shifts the rent from the seller of capital goods to the producer of consumption goods by creating an unrealistic gap between $\overline{K}$ and $\overline{\overline{K}}$. Further details are available from the authors upon request.

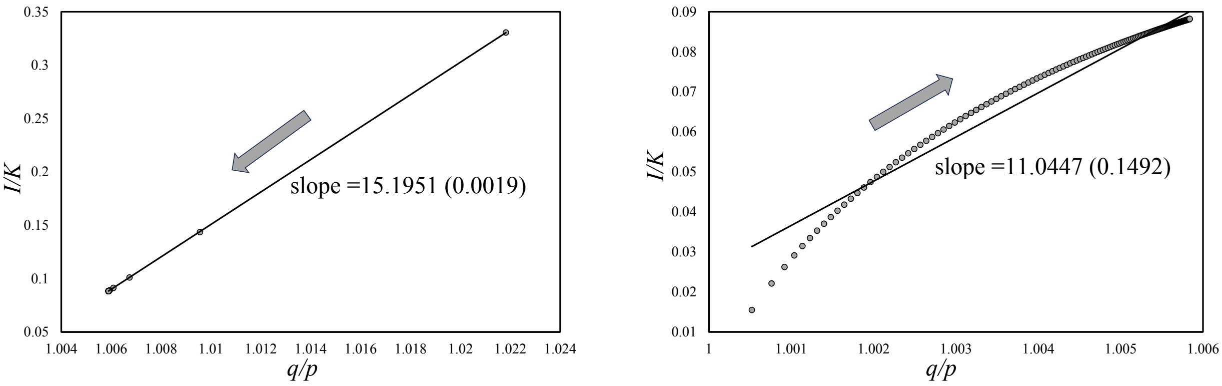

Figure 6: Marginal q and the investment-capital ratio in the competitive and the non-competitive setting

The two diagrams in Figure 6 show that in both scenarios there is a significant positive link between q/p and I/K which is mirrored in the coefficients of the OLS regressions reported together with their respective standard errors on each chart (cf. Andrei et al. 2019).[16] The presence of market power on the side of the seller of capital goods, however, alters the shape of the relationship between the marginal q and investment-capital ratio. In the competitive setting such a relationship is pretty linear – as it happens in the textbook q-model of investment with internal adjustment costs – whereas in the non-competitive case it becomes strictly concave.

The pattern described above can be explained as follows. In the competitive setting, as indicated by the arrow, the marginal q and the investment-capital ratio are both countercyclical and their growth rate are directly proportional the one to the other over their respective path of adjustment as confirmed by the linear relationship in the left-hand side panel of Figure 6. By contrast, as implied by the diagrams of Figure 4, in the non-competitive setting q/p and I/K are both procyclical. In other words, when the seller of capital goods is endowed with market power, all-over the adjustment path the increase (decrease) of investment is always above the increase (decrease) of the capital stock and, at the same time, the reduction (increase) of the price of capital goods always exceeds the reduction (increase) of the shadow value of capital. Because of the operation of the internal adjustment costs and a persistent markup over the marginal selling cost, however, the increase (reduction) of investment is lower than the corresponding reduction (increase) of the price of capital goods and this sluggish response of I is responsible for the concave shape of the relation plotted in the right-hand-side panel of Figure 6. As a consequence, according to our non-competitive model the reduction (increase) of the external adjustment costs does not completely offset the increase (reduction) of installation costs associated with increasing (declining) investment rates.

Another intriguing issue that can be addressed within our theoretical framework is how the degree of competition in the market for capital goods affect the speed of adjustment to the long-

[16] The point estimates of the OLS coefficients reported on the two panels of Figure 6 are quite above the estimates usually retrieved by regressing the average q over the investment-capital ratio. This result is due to the low value of h used to calibrate both versions of the model to ensure non-negative profits for the producer of consumption goods (cf. Cummis et al. 1994; Cooper and Ejarque, 2003).

run equilibrium. For that purpose, in Figure 7 we plot the adjustment of the stock of capital owned by the producer of consumption goods in the competitive and the non-competitive setting under the assumption that they start with the same deviation from their respective steady-state allocation.

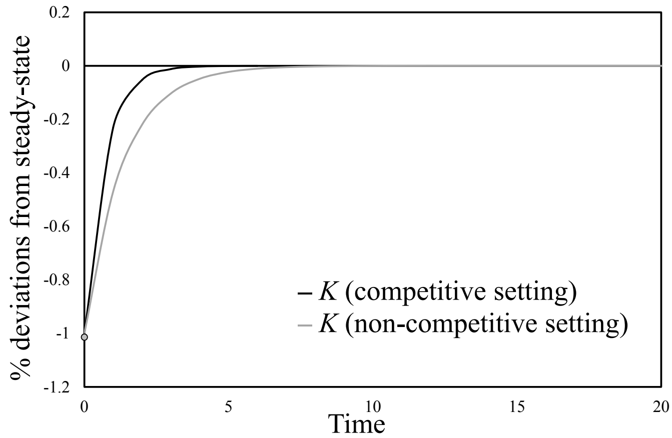

Figure 7: Capital adjustment in the competitive and the non-competitive setting

Unsurprisingly, the plot of Figure 7 reveals that the adjustment of capital is definitely faster in the competitive setting. As one may have expected from the arguments above, the adjustment of relative prices that takes place in the non-competitive framework interacts with the internal adjustment costs and it tends to delay the achievement of the stationary solution via a lower investment rate.[17] As shown by Fuchs et al. (2016), this feature of the adjustment process is typical of markets for capital goods with frictions and it may also represent a source of shock amplification (cf. Cui, 2022).

4 Concluding remarks

In this paper, we developed a differential game in which a competitive producer of consumption goods continuously interacts with a seller of capital goods endowed with full market power. Under the assumption that the latter acts as the leader and the former as the follower, we shown that an economically meaningful binding commitment between the two players – in the form of an open-loop Stackelberg long-run equilibrium with a positive capital stock – requires the seller of capital goods to be endowed with a lower discount rate with respect to the one of

[17]This finding would be completely overturned by shifting the economic rent from the seller of capital goods to the producer of consumption goods. In this case, the non-competitive setting would still imply a lower capital accumulation but a faster adjustment to the steady-state solution. The latter pattern is due to the fact that the profit function of the seller is concave in its control by mirroring its risk-aversion. Consequently, when endowed with market power, the seller of capital goods adjusts the price of these items in order to have a sooner convergence to the steady-state allocation (cf. Guerrazzi, 2011).

the producer of consumption goods (cf. Rubinstein, 1982; Guerrazzi, 2020; Bressan and Jiang, 2020). Moreover, on a numerical perspective, our model reconciled some relevant empirical regularities such as the countercyclical (procyclical) pattern of the relative price of capital goods (investment) that the textbook q-model of investment with only internal adjustment costs is unable to replicate (cf. Fisher, 2006; Justiniano et al. 2011; Alogoskoufis, 2019, Chapter 11). In addition, comparing artificial series with actual date, our theoretical framework easily reproduced the negative relationship observed between the relative price of capital goods and the investment-output ratio as well as the positive relation between the marginal q and the investment-capital ratio (cf. Restuccia and Urritia, 2001; Andrei et al. 2019; Lian et al. 2020).

As it usually happens in controllable Stackelberg games, a limitation of the model developed in Section 2 is the fact that the implied trajectory tracked by the price of capital goods is dynamically inconsistent. From an analytical perspective, such a feature of the dynamic path followed by p is mirrored in the negative long-run value of η (cf. Dockner et. al. 2000, Chapter 5). Therefore, from an empirical point of view, we should not expect to observe a falling trajectory of the relative price of capital goods for too long because their sellers have clear incentives to deviate (cf. Lambertini, Chapter 9). As a matter of fact, in our setting, the seller of capital goods moves along a linear demand schedule so that a path of increasing investment will lead to allocations in which the elasticity of a such a schedule is progressively lower. Aware of this, at a certain instant of time, the seller of capital goods will find profitable to renege on its former commitment by increasing the price charged to the producer of consumption goods for its purchases of new productive capacity.[18]

The arguments on dynamic inconsistency developed above are indubitably sound. In the real world, however, there may be forces that work against the deviation from the declining price-strategy set at the beginning of the game. As we mentioned in Section 3, technical progress and the integration of the markets for capital goods – that in our model can be mirrored by a reduction of c – have both a downward pressure on the price of the traded items that certainly tends to offset the gains from reneging (cf. Eaton and Kortum, 2001; Estevadeordal and Taylor, 2013; Lian et al. 2020). In addition, whenever the producer of consumption good is in the position to punish a deviating seller of capital goods by stopping, reducing or delaying its purchases, the incentives to renege on its starting commitment may be very feeble, especially when the latter is not too impatient as actually required by our model to have a meaningful stationary solution (cf. van der Ploeg, 1987). As a consequence, despite of its dynamic inconsistency, a continually falling trajectory of the price of capital goods like the one implied by our theoretical model and tracked on the left-hand-side panel of Figure 4 is likely to be easily observed over a long period of time as the available data seem to confirm.

The analysis summarized above could be extended in different directions. For instance, it could be interesting to consider a situation in which there is an explicit technology to produce capital goods. In this case, capital accumulation in the sector of consumption goods will be

[18]Formally speaking, when the leader reneges, it will re-set the value of η to zero so that the new price strategy will be different from the one announced at the outset of the game.

distinct from the one that occurs in the sector of capital goods and there will be room for a sectoral evaluation of the relationship between the marginal q and the investment-capital ratio (cf. Christiano and Fisher, 2003). Furthermore, it may be more realistic to explore the case in which the purchase of new productive capacity is constrained by the existence financial frictions (cf. Cooper and Ejarque, 2003). The inability to borrow is indeed important also for the design of monetary and fiscal policies. These will be the promising avenues for further developments.

Appendix A: Stationary solution and local dynamics in the competitive setting

Setting $p = c > 0$ for all t, the FOCs for the problem of the producer of consumption goods imply that the dynamics of q and K is given by the following non-linear differential system:

$$\begin{pmatrix} \dot{q} \\ \dot{K} \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{c}{h} \end{pmatrix} + \begin{bmatrix} r_I + \delta & -\frac{\alpha}{K^2} \\ \frac{1}{h} & -\delta \end{bmatrix} \begin{pmatrix} q \\ K \end{pmatrix} \tag{A1}$$

In the present setting, steady-state allocations are defined as the set of pairs $\overline{\overline{\mathcal{S}}} := \left\{ \overline{\overline{q}}, \overline{\overline{K}}, \right\} \in \mathbb{R}^2_{++}$ such that $\dot{q}\left(\overline{\overline{q}}, , \overline{\overline{K}}, \right) = \dot{K}\left(\overline{\overline{q}}, \overline{\overline{K}}\right) = 0$. The elements of the set $\overline{\overline{\mathcal{S}}}$ can be derived as follows. First, setting $\dot{q} = 0$, the first row of the dynamic system in (A1) implies that

$$\overline{\overline{q}} = \frac{\alpha}{(r_I + \delta)\, \overline{\overline{K}}} \tag{A2}$$

Thereafter, setting $\dot{K} = 0$, the second row of (A1) leads to the quadratic expression in the main text which is given by

$$h\delta \overline{\overline{K}}^2 + c\overline{\overline{K}} - \frac{\alpha}{r_I + \delta} = 0 \tag{A3}$$

Finally, the system in (A1) and the expressions in eq.s (A2) and (A3) imply that the local dynamics of the competitive model economy around the economically meaningful element of $\overline{\overline{\mathcal{S}}}$ is described by the following 2×2 linear system:

$$\begin{pmatrix} \dot{q} \\ \dot{K} \end{pmatrix} = \begin{bmatrix} r_I + \delta & -\frac{\alpha}{\overline{\overline{K}}^2} \\ \frac{1}{h} & -\delta \end{bmatrix} \begin{pmatrix} q - \overline{\overline{q}} \\ K - \overline{\overline{K}} \end{pmatrix} \tag{A4}$$

The trace of the Jacobian matrix in (A4) is equal to $r_I > 0$ whereas its determinant is given by $-\left(\delta\left(r_I + \delta\right) + \alpha / h\overline{\overline{K}}^2\right) < 0$. Consequently, the system in (A4) has only one negative converging root so that the unique economically meaningful element of $\overline{\overline{\mathcal{S}}}$ is characterized by a saddle-path dynamics.

Appendix B: The payoffs of the two players

Exploiting the baseline calibration in Table 1, in Figure B1 we provide some evidence of the non-cooperative features of the differential game developed in Section 2 – which are driven by the persistent negative sign of η and μ – by plotting the path of the point value of the profits realized by the two players in the non-competitive setting.

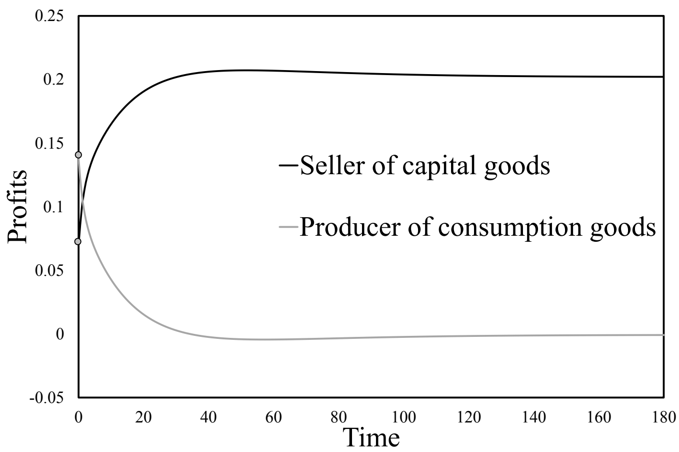

Figure B1: The profits of the two players in the non-competitive setting

The plots of Figure B1 corroborates the fact that the producer of consumption goods and the seller of capital goods actually have conflicting objectives, indeed, before achieving the stationary solution, the dynamic behaviour of the profit of the former is exactly the mirror-imagine of the dynamic behaviour of the profit of the latter. Interestingly, reasoning in terms of representative firm, the actual path of the profit of the producer of consumption goods retraces an entry/exit process of identical firms which is typical of a competitive market as we actually assumed. Specifically, at the beginning of the game, the producer of consumption goods enters the relative market and purchases capital goods to boost its output attracted by relatively high profits. Thereafter, the perspective of positive profits enhances the entrance of other investing firms that progressively increases the profits of the producer of capital goods at the expenses of the incumbent firms operating in the market for consumption goods. In the steady-state equilibrium, the representative producer of consumption goods realizes no profit whereas the producer of capital goods – because of its market power – realizes persistently positive profits (cf. Guerrazzi, 2023).

References

[1] Alogoskoufis, G. (2019), *Dynamic Macroeconomics*, MIT Press: Cambridge, MA.

[2] Andrei, D., Mann, W., Moyen, N. (2019), Why Did the q Theory of Investment Start Working?, *Journal of Financial Economics*, Vol. 133, No. 2, pp. 251-272.

[3] Basar, T., Olsder, G.J. (1999), *Dynamic Noncooperative Game Theory*, Second Edition, SIAM Classic in Applied Mathematics, No. 23, New York.

[4] Beaudry, P., Moura, A., Portier, F. (2015), Reexamining the Cyclical Behavior of the Relative Price of Investment, *Economics Letters*, Vol. 135, No. C, pp. 108-111.

[5] Bosi, S., Desmarchelier, D. (2019), Local Bifurcations of Three and Four-Dimensional Systems: A Tractable Characterization with Economic Applications, *Mathematical Social Science*s, Vol. 97, No. C, pp. 38-50.

[6] Bressan, A., Jiang, Y. (2020), Self-Consistent Feedback Stackelberg Equilibria for Infinite Horizon Stochastic Games, *Dynamic Games and Applications*, Vol. 10, No. 2, pp. 328-360.

[7] Carlsson, B. (1983), The Machine Tool Industry: Problems and Prospects in an International Perspective, *Working Paper Series of the Research Institute of Industrial Economics*, No. 96.

[8] Calvo, G.A., (1978), On the Consistency of Optimal Policy in a Monetary Economy, *Econometrica*, Vol. 46, No., pp. 1411-1428.

[9] Cass, D. (1966), Optimum Growth in an Aggregative Model of Capital Accumulation: A Turnpike Theorem, *Econometrica*, Vol. 34, No. 4, pp. 833-850.

[10] Chen, J-L. (2018), Status Quo and Development of Cutting Tool Industry in Taiwan and China, *International Business Research*, Vol. 11, No. 8, pp. 90-96.

[11] Christiano, L.J., Fisher, J.D.M. (2003), Stock Market and Investment Goods Prices: Implications for Macroeconomics, *NBER Working Paper Series*, No. 10031.

[12] Cooper, R.W., Haltiwanger, J.C. (2006), On the Nature of Capital Adjustment Costs, *Review of Economic Studies*, Vol. 73, No. 3, pp. 611-633.

[13] Cooper, R.W., Ejarque, J.M. (2003), Financial Frictions and Investment: Requiem in *Q*, *Review of Economic Dynamics*, Vol. 6, No. 4, pp. 710-728.

[14] Cui, W. (2022), Macroeconomic Effects of Delayed Capital Liquidation, *Journal of the European Economic Association*, Vol. 20, No. 4, pp. 1683-1742.

[15] Cummis, J.G., Hassett, K.A., Hubbard, R.G. (1994), A Reconsideration of Investment Behavior Using Tax Reforms as Natural Experiments, *Brookings Papers on Economic Activity*, Vol. 1994, No. 2, pp. 1-74.

[16] Dinopoulos, E., Kreinin, M.E. (1991), The US VER on Machine Tools: Causes and Effects, in Baldwin, R.E. (ed.), *Empirical Studies of Commercial Policy*, University of Chicago Press: Chicago, pp. 113 -134.

[17] Dockner, E., Jorgensen, S., Van Long, N., Sorger, G. (2000), *Differential Games in Economics and Management Science*, Cambridge University Press: New York.

[18] Eaton, J., Kortum, S. (2001), Trade in Capital Goods, *European Economic Review*, Vol. 45, No. 7, pp. 1195-1235.

[19] Eisner, R., Strotz, R.H. (1963), Determinants of Business Fixed Investment, in *Commission on Money and Credit, Impacts of Monetary Policy*, Englewood Cliffs, NJ: Prentice Hall, pp. 59-337.

[20] Estevadeordal, A., Taylor, A.M. (2013), Is the Washington Consensus Dead? Growth, Openness, and the Great Liberalization, 1970s-2000s, *Review of Economics and Statistics*, Vol. 95, No. 5, pp. 1669-1690.

[21] Fisher, J.D.M. (2006), The Dynamic Effects of Neutral and Investment-Specific Technology Shocks, *Journal of Political Economy*, Vol. 114, No. 3, pp. 413-451.

[22] Fransman, M. (1986), International Competitiveness, Technical Change and the State: The Machine Tool Industry in Taiwan and Japan, *World Development*, Vol. 14, No. 12, pp. 1375-1396.

[23] Foley, D.K., Sidrauski, M. (1970), Portfolio Choice, Investment and Growth, *American Economic Review*, Vol. 60, No. 1, pp. 44-63.

[24] Fuchs, W., Green, B., Papanikolaou, D. (2016), Adverse Selection, Slow-Moving Capital, and Misallocation, *Journal of Financial Economics*, Vol. 120, No. 2, pp. 286-308.

[25] Gomme, P., Lkhagvasuren, D. (2013), Calibration and Simulation of DSGE Models, in Hashimzade, N., Thornton, M.A. (ed.s), *Handbook of Research Methods and Applications in Empirical Macroeconomics*, Cheltenham (UK): Edward Elgar Publishing, pp. 575-592.

[26] Gould, J.P. (1968), Adjustment Costs in the Theory of Investment of the Firm, *Review of Economic Studies*, Vol. 35, No. 1, pp. 47-55.

[27] Greenwood, J., Hercowitz, Z., Krusell, P. (1997), Long-Run Implications of Investment-Specific Technological Change, *American Economic Review*, Vol. 87, No. 3, pp. 342-362.

[28] Guerrazzi, M. (2023), Optimal Growth with Labor Market Frictions, *International Journal of Economic Theory*, Vol. 19, No. 4, pp. 961-987.

[29] Guerrazzi, M. (2020), Wage and Employment Determination in a Dynamic Insider-Outsider Model, *Evolutionary and Institutional Economics Review*, Vol. 17, No. 1, pp. 1-23.

[30] GUERRAZZI, M. (2011), Wage Bargaining as an Optimal Control Problem: A Dynamic Version of the Right-to-Manage Model, *Optimal Control Applications and Methods*, Vol. 32, No. 5, pp. 505-526.

[31] HAYASHI, F. (1982), Tobin's Marginal q and Average q: A Neoclassical Interpretation, *Econometrica*, Vol. 50, No. 1, pp. 213-224.

[32] HUBER, K., GORMSEN, N.J. (2023), Corporate Discount Rates, *CEPR Discussion Papers*, No. 18221.

[33] HUNEMBERG, R., HUTTMAN, A. (2003), Performance as a Basis for Price-setting in the Capital Goods Industry: Concepts and Empirical Evidence, *European Management Journal*, Vol. 21, No. 6, pp. 717-730.

[34] ITSKHOKI, O., MOLL, B. (2019), Optimal Development Policies with Financial Frictions, *Econometrica*, Vol. 87, No. 1, pp.139-173.

[35] JUSTINIANO, A., PRIMICERI, G.E., TAMBALOTTI, A. (2011), Investment Shocks and the Relative Price of Investment, *Review of Economic Dynamics*, Vol. 14, No. 1, pp. 102-121.

[36] KEYNES, J.M. (1936), *The General Theory of Employment, Interest and Money*, London: MacMillan and Company.

[37] KIM, Y-Z., LEE, K. (2008) Sectoral Innovation System and a Technological Catch-up: The Case of the Capital Goods Industry in Korea, *Global Economic Review*, Vol. 37, No. 2, pp. 135-155.

[38] KULTTI, K. (2021), Monopoly and Demand Uncertainty, *Research in Economics*, Vol. 75, No. 4, pp. 354-364.

[39] KYDLAND, F.E., PRESCOTT, E.C. (1982), Time to Build and Aggregate Fluctuations, *Econometrica*, Vol. 50, No. 6, pp. 1345-1370.

[40] LAMBERTINI, L. (2018), *Differential Games in Industrial Economics*, Cambridge University Press: New York.

[41] LIAN, W.,NOVTA, N., PUGACHEVA, E., TIMMER, Y., TOPALOVA, P. (2020), The Price of Capital Goods: A Driver of Investment Under Threat, *IMF Economic Review*, Vol. 68, No. 3, pp. 509-549.

[42] LIPTON, D., POTERBA, J., SACHS, J., SUMMERS, L. (1982), Multiple Shooting in Rational Expectations Models, *Econometrica*, Vol. 50, No. 5, pp. 1329-1333.

[43] LUCAS, R.E., JR. (1967), Adjustment Costs and the Theory of Supply, *Journal of Political Economy*, Vol. 75, No. 4, pp. 321 334.

[44] Mino, K. (1987), A Model of Investment with External Adjustment Costs, *Economic Studies Quarterly*, Vol. 38, No. 1, pp. 76-85.

[45] Mutreja, P., Ravikumar, B., Sposi, M. (2018), Capital Goods Trade, Relative Prices, and Economic Development, *Review of Economic Dynamics*, Vol. 27, pp. 101-122.

[46] Restuccia, D., Urrutia, C. (2001), Relative Prices and Investment Rates, *Journal of Monetary Economics*, Vol. 47, No. 1, pp. 93-121.

[47] Rubinstein, A. (1982), Perfect Equilibrium in a Bargaining Model, *Econometrica*, Vol. 50, No. 1, pp. 97-109.

[48] Summers, L.H. (1981), Taxation and Corporate Investment: A q-Theory Approach, *Brookings Papers on Economic Activity*, No. 1, pp. 67-127.

[49] Stanula, P., Praetzas, C., Kohn, O., Metternich, J., Weigold, M., Buchwald, A. (2020), Stress-Oriented, Data-Based Payment Model for Machine Tools, *Procedia CIRP*, Vol. 93, pp. 1526-1531.

[50] van der Ploeg, F. (1987), Trade Unions, Investment and Employment. A Non-Cooperative Approach, *European Economic Review*, Vol. 31, No. 7, pp. 1465-1492.

[51] Yoshikawa, H. (1980), On the "q" Theory of Investment, *American Economic Review*, Vol. 70, No. 4, pp. 739-743.