Hygro-thermal coupling in earth building materials

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Abstract. Raw earth is emerging as a viable building material with lower carbon emissions than conventional concrete and fired bricks. Raw earth is as an excellent passive hygro-thermal regulator, which improves occupants' comfort while reducing the need for active heating/cooling installations. The coupled hygro-thermal response of earth materials is investigated by exploiting the principles of the thermodynamics of porous media and unsaturated soil mechanics. The degree of coupling between temperature and relative humidity (or water content) depends on the adopted simplifying assumptions. Some of these assumptions are valid for traditional building materials but may not be applicable to raw earth characterised by relatively high levels of liquid/gas permeability. The validity of current approaches is here assessed with reference to earth building via a simple one-dimensional transfer model, which simulates the behaviour of an unbounded earth wall subjected to time-dependent boundary conditions on the two faces. For typical values of water and vapour permeability, the complexity of the governing equations can be greatly reduced by neglecting variations of vapour mass and the dependency of suction on temperature without significantly reducing accuracy. Results are also strongly influenced by both initial state and water retention properties of the earth material.

1 Introduction

Earth is an ancient construction material that has recently been rediscovered as a low carbon alternative to conventional energy-intensive options such as concrete or fired bricks. Earthen materials may be manufactured directly on-site by compacting the soil at an optimum water to achieve high levels of dry density. After compaction, raw earth gains strength and stiffness over time as the manufacturing water content reduces due to evaporation. Earth materials are also excellent passive hygro-thermal regulators of indoor spaces, thus improving the comfort of occupants while reducing the need of economically and environmentally expensive air conditioning installations.

To date, however, the uptake of raw earth by mainstream construction has been hindered by the relatively poor durability of this material when exposed to atmospheric actions and by the absence of consolidated methods to estimate hygro-thermal performance [1]. To address this latter obstacle, the present work has developed a theoretical framework that combines the principles of unsaturated soil mechanics with the thermodynamics of porous media to analyse moisture and heat transfer in earth building materials.

A system of coupled partial differential equations has been derived based on the conservation of both energy and mass. These equations account for phase changes during evaporation and condensation of pore water while incorporating a retention curve, a moisture-dependent thermal capacity law and water/vapour permeability functions. The degree of hygro-thermal coupling, and thus the complexity of the governing equations, depends on the level of simplification introduced in the model. Among

simplifying assumptions, there is the disregard of both vapour mass changes and the dependency of suction on temperature, which are usually held valid for conventional building materials such as concrete or fired bricks. The validity of these two assumptions for earthen materials is however uncertain and is the object of the present work.

The resulting mathematical formulation has been implemented in a finite element code and then applied to the study of a one-dimensional transfer model, which simulates the behaviour of an infinite earth wall under ordinary service conditions, i.e. in the absence of capillary rise from the foundation ground and/or internal water sinks or sources.

2 Hygro-Thermal coupled model

Earth building materials often consist of highly compacted well-graded soils that remain in quasi-dry conditions during their entire service life (i.e. with a degree of saturation lower than 10%). Despite this rather stable hygroscopic state, earthen walls are very sensitive to environmental agents and the application of variable hygro-thermal boundary conditions at their boundaries may engender a flow of heat and water (in both liquid and gas form), which may in turn affect occupants' comfort.

This aspect is investigated in the present work by simulating the time-dependent action of temperature and relative humidity on the two faces of an unbounded earthen wall.

The earth is modelled as a three-phase material consisting of soil grains (solid phase), water (liquid and gas phase) and dry air (gas phase). It is assumed that the resistance to gas flow is negligible so that the air pressure

is always equal to the atmospheric value and the dry air mass balance is neglected. Also, the soil skeleton is incompressible and the dry density ρ_d remains constant.

The problem is therefore tackled by simultaneously solving the two balance equations of water mass and energy across the earth domain. These equations are strongly coupled as the pore water flow depends on both relative humidity and temperature while the heat flow depends on changes of water content.

The most general form of balance equation for any extensive quantity E is:

$$\frac{\partial e}{\partial t} = -\nabla(e\boldsymbol{V} + \boldsymbol{j}^E) + e\Gamma^E \tag{1}$$

where e is the intensive quantity of E, $\frac{\partial e}{\partial t}$ is the storage term, $e\boldsymbol{V}$ is the advective flux carried by the motion of E, $\boldsymbol{j}^E$ is the diffusive flux due to difference in concentration, and $e\Gamma^E$ is an external source term. In the following, eq. (1) will be specialised for both the water and thermal energy balance. The complete derivation of the coupling terms is outside the scope of this work and will be omitted here. Additional details can be found in Bear and Cheng [2].

2.1 Constitutive equations

The hydric state of a porous material may be alternatively defined in terms of liquid pressure p_l, vapor pressure p_v, water content w_l, degree of saturation S_l and relative humidity hr. These variables are linked to each other by constitutive and phase relations, so that any of them can describe the material state. Typically, earth materials work in the hygroscopic domain at very low water contents with moisture transport mainly driven by gradients of vapor pressure. Hence, relative humidity hr (-) is the preferred hydric variable as it is routinely measured in both laboratory and full-scale building applications. Conversely, the energetic state of the earth is typically defined in terms of the temperature T (K). At equilibrium, both relative humidity hr and temperature T are related to the total soil suction ψ by Kelvin's law:

$$\psi = s + \pi = -\rho_l \frac{R}{M_w} \cdot T \cdot \ln hr \tag{2}$$

where ρ_l is the density of liquid water (1000 kg/m³), R is perfect gas constant (8.314 J/mol K) and M_w is the molar mass of water (0.018 kg/mol).

The total suction ψ of eq. (2) is the sum of matric suction s and osmotic suction π. The matric suction s is the difference between pore air pressure p_a and pore water pressure p_l and, because the pore air pressure is often atmospheric (i.e. $p_a = 0$), it typically coincides with the pore water pressure p_l changed of sign:

$$s = p_a - p_l = -p_l \tag{3}$$

The osmotic suction π is instead a measure of dissolved salts in the pore water and is neglected in earth materials (i.e. $\pi = 0$), so that eq. (2) defines the dependency of

matric suction s (simply named suction hereafter) on temperature T and relative humidity hr (Fig. 1).
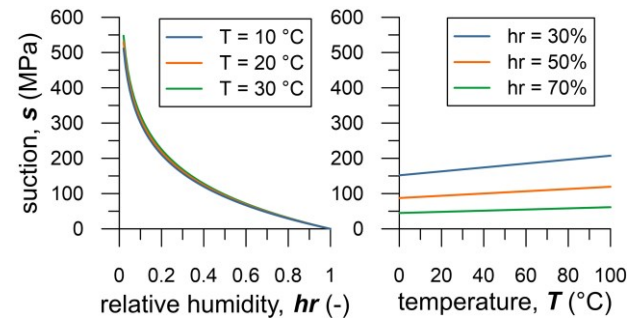

Fig. 1. Influence of relative humidity and temperature on suction.

The capacity of any porous material to store/release moisture upon changes of suction is described by a retention law. In this work, the well-known Van Genuchten retention law [3], linking water content w_l to suction s, is adopted:

$$w_l = w_{res} + (w_{sat} - w_{res})\left(1 + \left(\frac{s}{P}\right)^N\right)^{-M} \tag{4}$$

where w_{sat} and w_{res} are the saturated and residual water content respectively, whereas P (MPa), N (-) and M (-) are Van Genuchten fitting parameters. Water content and degree of saturation are linked as $S_l = w_l \rho_d / n \rho_l$ where n is the earth porosity. The relative humidity hr is instead:

$$hr = \frac{p_v}{p_{sat}(T)} \tag{5}$$

where p_v is the partial vapour pressure and $p_{sat}(T)$ is the saturated vapour pressure, which depends on temperature according to the following relationship:

$$p_{sat}(T) = 0.6108 \cdot 10^{\frac{7.5T}{T+273.15}} \quad (kPa) \tag{6}$$

The partial density and pressure of vapour are linked by the perfect gas relationship:

$$\rho_v = \frac{M_w}{RT} \cdot p_v \tag{7}$$

so that eq. (5) can be recast as:

$$hr = \frac{\rho_v}{\rho_{sat}(T)} \tag{8}$$

where ρ_v and $\rho_{sat}(T)$ are the vapour density at the current state and saturation, respectively.

2.2 Water mass balance

The water balance of both liquid and gaseous phases is:

$$
\frac{\partial}{\partial t}\left(n\rho_l S_l + n\rho_v(1-S_l)\right)
$$
$$
= -\nabla\left(\rho_l\left(-\frac{K_w(S_l)}{\rho_l g}\nabla(p_l + \rho_l gz)\right)\right) \quad (9)
$$
$$
- D_e\nabla\rho_v\right)
$$

On the right hand side of eq. (9), the advective flux of the liquid phase is described by Darcy's law where $K_w(S_l) = K_{sat} \cdot k_{rl}(S_l)$ is the hydraulic conductivity, which is the product of the constant saturated permeability, K_{sat} (m/s) and the relative permeability function, $k_{rl}(S_l)$. The latter depends on degree of saturation, S_l and can be estimated by Mualem [4] retention law as:

$$
k_{rl} = \sqrt{S_l} \cdot \left[1-\left(1-S_l^{M^{-1}}\right)^M\right]^2 \quad (10)
$$

where M is the Van Genuchten parameters of eq. (4).

The diffusive flux of the gas phase is instead described by Fick's law where $D_e = D_v \cdot n(1-S_l) \cdot \tau(S_l)$ is the effective diffusion coefficient, which is the product of the vapour diffusivity in still air D_v (m²/s), the gas porosity $n(1-S_l)$ and the voids tortuosity function $\tau(S_l)$. The latter depends on degree of saturation, S_l and can be estimated according to the following relationship:

$$
\tau(S_l) = n(1-S_l)^{2/3} \quad (11)
$$

The value of the effective diffusion coefficient D_e is often directly measured on building materials.

2.3 Thermal energy balance

Under the assumption of thermal equilibrium between different phases, the energy balance is imposed via a single differential equation as:

$$
\left(\rho C_p\right)_{eq}\frac{\partial T}{\partial t} = -\nabla(-\lambda\nabla T) - L_v m_{\to v} \quad (12)
$$

In eq. (12), λ is the thermal conductivity while $\left(\rho C_p\right)_{eq}$ is the equivalent heat capacity of the composite medium:

$$
\left(\rho C_p\right)_{eq} = (1-n)\rho_s C_{p,s} + nS_l\rho_l C_{p,l} \quad (13)
$$
$$
+ n(1-S_l)\rho_v C_{p,v}
$$

where ρ_s and $C_{p,s}$ are the density and specific heat of the earth grains, ρ_l and $C_{p,l}$ are the density and specific heat of liquid water and ρ_v and $C_{p,v}$ are the density and specific heat of water vapour. Reference values of $C_{p,l} =$4.183 kJ/kg·K and $C_{p,v} =$1.89 kJ/kg·K are assumed in the following.

In eq. (12), L_v is the latent heat of evaporation (here taken equal to 2.5·10⁶ J/kg) while $m_{\to v}$ is the evaporation or condensation mass, which is evaluated from the liquid phase balance as:

$$
-m_{\to v} = \frac{\partial}{\partial t}(n\rho_l S_l) + \nabla\rho_l\left(-\frac{K_w(S_l)}{\rho_l g}\nabla(p_l + \rho_l gz)\right) \quad (14)
$$

3 Numerical model

The water balance of eq. (9) and the energy balance of eq. (12) are recast into eqs. (15) and (16), respectively, in terms of temperature and relative humidity by using eqs. (2)-(8) and (14).

Both eqs. (15) and (16) have then been implemented in the Comsol Multiphysics finite element software, which has been used to reproduce the hygro-thermal response of an infinite earth wall subjected to identical cyclic variations of temperature and relative humidity on the opposite faces (Fig. 2a). Given the geometry and boundary conditions of the problem, only half wall thickness has been modelled because the middle depth constitutes a symmetry plane. The variations of temperature and relative humidity have therefore been applied only at one end of the one-dimensional finite element model while zero water and heat flow are imposed at the other end representing the symmetry plane.

The same boundary value problem has been simulated by Soudani *et al.* [5] via an alternative finite element formulation, whose results are compared with present study in the following part of the paper.

3.1 Comparison with Soudani *et al.* [5]

A daily sinusoidal variation of ambient relative humidity hr_e, between 0.5 and 0.7, was imposed on both sides of the wall while the ambient temperature T_e was maintained constant and equal to 30 °C. The initial conditions were uniform across the entire wall with a temperature $T_0 = 30$ °C and a relative humidity $hr_0 = 0.7$. This corresponds to an initial water content of 0.011 and degree of saturation of 0.056. The overall duration of the simulated experiment was set to 100 hours.

The parameter values used in the present model are listed in Table 1 and were determined based on the data in Soudani *et al.* [5].
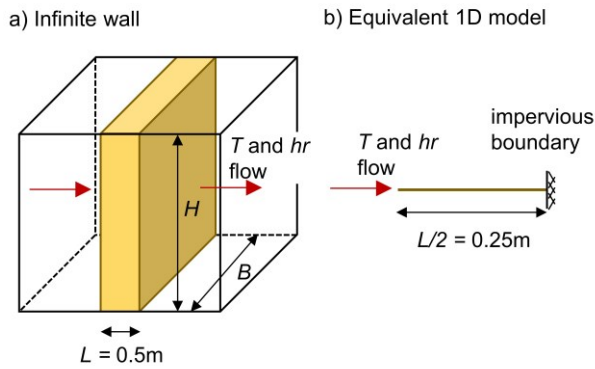
$$
\rho_d\frac{\partial w_l}{\partial hr}\frac{\partial hr}{\partial t} + n(1-S_l)\left(\frac{M_w}{RT}p_{sat}\frac{\partial hr}{\partial t} + hr\frac{M_w}{RT}\left(\frac{dp_{sat}}{dT} - \frac{p_{sat}}{T}\right)\frac{\partial T}{\partial t}\right) - \frac{\rho_d}{\rho_l}\frac{\partial w_l}{\partial hr}\frac{M_w}{RT}hr p_{sat}\frac{\partial hr}{\partial t}
$$
$$
= -\nabla(-\frac{K_w}{g}\rho_l\frac{R}{M_w}\left(\ln hr\,\nabla T + \frac{T}{hr}\nabla hr\right) + \left(-D_e\frac{M_w}{RT}\left(p_{sat}\nabla hr + hr\left(\frac{dp_{sat}}{dT} - \frac{p_{sat}}{T}\right)\nabla T\right)\right) \quad (15)
$$

$$
\left(\rho C_p\right)_{eq}\frac{\partial T}{\partial t} = -\nabla(-\lambda\nabla T) + L_v\left(\rho_d\frac{\partial w_l}{\partial hr}\frac{\partial hr}{\partial t} + \nabla(-\frac{K_w}{g}\rho_l\frac{R}{M_w}\left(\ln hr\,\nabla T + \frac{T}{hr}\nabla hr\right)\right) \quad (16)
$$

Fig. 2. Model of Soudani *et al.* [5]; b) present one-dimensional model.

Table 1. Parameters values of the present finite element model

Porosity (-)	n	0.35
Dry density (kg/m³)	ρ_d	1722
Grain density (kg/m³)	ρ_s	2650
Van Genuchten parameter (MPa)	P	0.55
Van Genuchten parameter (-)	N	1.64
Van Genuchten parameter (-)	$M = 1 - 1/N$	0.39
Saturated water content (-)	w_{sat}	0.20
Residual water content (-)	w_{res}	0
Saturated permeability (m/s)	K_{sat}	$1.3 \cdot 10^{-9}$
Effective diffusion coefficient (m²/s)	D_e	$2.7 \cdot 10^{-6}$
Heat capacity of the grains (kJ/(kg·K))	$C_{p,s}$	0.648
Thermal conductivity (W/(m·K))	λ	$0.6 + 9.22w_l$

The moisture and thermal flow across the two wall faces are written as:

$$
\begin{bmatrix}
(\boldsymbol{g}_l + \boldsymbol{g}_v) \cdot \boldsymbol{n} \\
(\boldsymbol{g}_T - L_v \boldsymbol{g}_v) \cdot \boldsymbol{n}
\end{bmatrix}
=
\begin{bmatrix}
\beta \left(p_{sat}(T_e)\, hr_e - p_{sat}(T_f)\, hr_f \right) \\
\alpha \left(T_e - T_f \right)
\end{bmatrix}
\tag{17}
$$

where $\boldsymbol{g}_T$, $\boldsymbol{g}_l$ and $\boldsymbol{g}_v$ are the heat, liquid and vapour flows, respectively, α and β are the heat and vapour mass transfer coefficients, which are equal to 8 W/(m²K) and $2.5 \cdot 10^{-8}$ kg/(m²sPa) as suggested by Künzel [6], while T_f and hr_f are the values of temperature and relative humidity in correspondence of the wall faces.

The above boundary conditions allow only for vapour flow, i.e. water can leave (or enter) the domain only in the gas phase, and the corresponding term $L_v \boldsymbol{g}_v$ accounts for the advective energy flow associated to phase change.

The comparison between the present model and that of Soudani *et al.* [5] is presented in Fig. 3 in terms of temporal variations of relative humidity and temperature on the symmetry plane (i.e. at a distance of 0.25 m from the external wall face). Qualitatively, the two models are in good agreement as they both predict that the centre of the wall starts to dry out after 24 hours and that the relative humidity reduces from an initial value of 0.7 to a final one of 0.68. This is consistent with the imposition of an average ambient relative humidity of 0.6, which is smaller than the initial relative humidity value inside the wall. In both models, the drying process induces heat and moisture flow from the inner wall towards the surrounding environment, thus causing a small temperature reduction on the symmetry plane. The maximum predicted temperature change is 1.5°C by Soudani *et al.* [5] and 1°C by the present model. The slight discrepancy between these two values may be due to differences in the finite element formulation and adopted retention law.
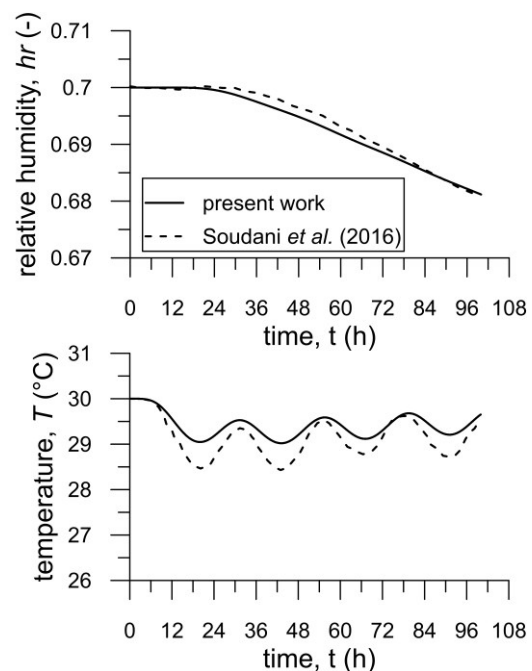

Fig. 3. Predicted hygro-thermal response at the wall centre.

3.2 Sensitivity study

A sensitivity analysis has been carried out to investigate the importance of i) the vapour mass change term and ii) the dependency of suction on temperature. The analysis has been performed for all combinations of the following initial values of relative humidity and temperature:

- hr_0 = 0.3, 0.5, 0.7, 0.9, 0.95
- T_0 = 5, 15, 30, 45 °C

with corresponding values of water contents from 0.5% (for hr_0 = 0.3 and T_0 = 45 °C) to 4% (for hr_0 = 0.95 and T_0 = 5 °C).

In the simulations, the imposed ambient temperature is set at a constant value $T_e = T_0$ while the ambient relative humidity is varied between $hr_e = hr_0$ and $hr_e - 0.2$ with a sinusoidal law, over a 24 h period, so that the wall experiences always a drying path.

3.2.1 Water vapour mass

The vapour mass change is given by eq. (15) as:

$$\frac{\partial}{\partial t}\left(n\rho_v(1 - S_l)\right)$$
$$= n(1 - S_l)\left(\frac{M_w}{RT}p_{sat}\frac{\partial hr}{\partial t}\right.$$
$$+ hr\frac{M_w}{RT}\left(\frac{dp_{sat}}{dT} - \frac{p_{sat}}{T}\right)\frac{\partial T}{\partial t}\right) \tag{18}$$
$$- \frac{\rho_d}{\rho_l}\frac{\partial w_l}{\partial hr}\frac{M_w}{RT}hr p_{sat}\frac{\partial hr}{\partial t}$$

which includes the time derivatives of both vapour density and degree of saturation. A simplified model, neglecting the variation of vapour mass (i.e. with eq. (18) set to zero), has here been compared with the complete formulations.

For the sake of brevity, Fig. 4 presents this comparison for different initial temperatures but only one initial relative humidity $hr_0 = 0.7$. The variation of both relative humidity and temperature at the wall centre are practically identical for both modelling approaches.

Similar results have been obtained for the other combinations of initial temperature and relative humidity, thus suggesting that vapour mass changes have a negligible effect on the corresponding balance equation. Fig. 4 also shows that hygro-thermal coupling is stronger for higher initial temperatures, thus suggesting a clear influence of the current thermal state on the material behaviour.
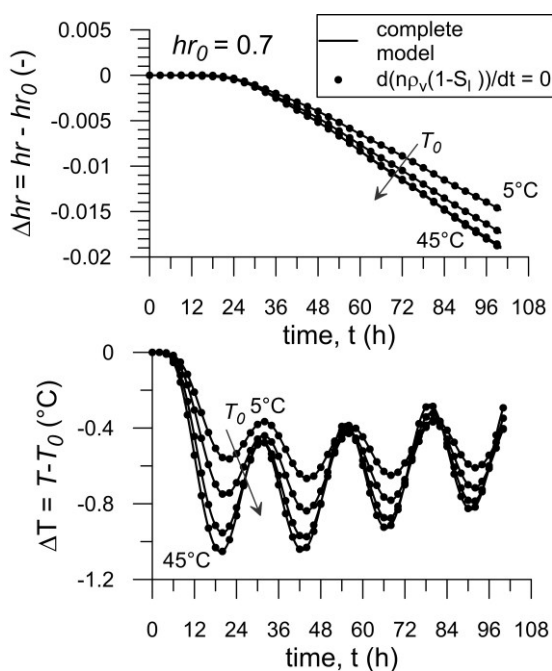

Fig. 4. Influence of term accounting for vapour mass changes at the centre of the wall.

3.2.2 Influence of temperature on suction

The influence of temperature on suction is instead examined by comparing the complete model with a simplified one where Darcy's liquid flow is driven exclusively by a relative humidity gradient while the effect of the thermal gradient is neglected, i.e. the term $ds/dT = K_w/K_w \rho_l R/M_w \ln hr \nabla T$ is neglected in both eqs. (15) and (16). This assumption is typically made in the capillary region, where changes of water content (or degree of saturation) influence the variation of suction much more than changes of temperature. The same assumption is, however, not well established in the hygroscopic domain where various approaches have been adopted by different researchers [5, 7 and 8].

In the sake of brevity, Fig. 5 presents the comparison for different initial values of relative humidity but only one initial temperature $T_0 = 30$ °C showing that, once again, the two modelling approaches yield to identical results. Interestingly, the centre of the wall dries to markedly different levels depending on the initial relative humidity. Relatively little drying is predicted for initial values of relative humidity smaller than 0.5 or greater than 0.9.
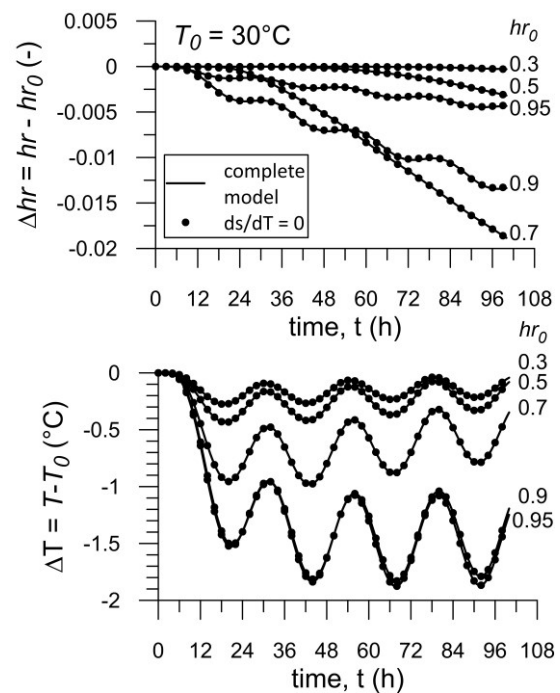

Fig. 5. Influence of the dependency of suction on temperature at the centre of the wall.

A marked cyclic variation of relative humidity, which reflects the periodicity of the boundary conditions, is observed at the wall centre only for an initial relative humidity larger than 0.7. This indicates that a wetter material accelerates the wall response to perturbations.

Moreover, as it may be expected, the level of hygro-thermal coupling increases with increasing material wetness. In a relatively dry material with an initial relative humidity of 0.3, the temperature at the centre of the wall reduces on average by only 0.1 °C. At the other extreme, in a relatively wet material with an initial relative humidity of 0.95, the temperature at the centre of the wall reduces on average by 1.5 °C approximately.

4 Discussion and conclusions

A coupled finite element model has been developed to simulate heat and mass transfer across earth building materials. The model accounts for the phase changes of pore water and the corresponding latent heat fluxes. Model results have been compared to the numerical simulations by Soudani *et al.* [5] obtaining a good agreement.

The proposed model can be used to investigate hygro-thermal coupling inside earthen building materials under exercise conditions, i.e. in absence of capillary rise from the ground and internal water sinks or sources.

Under these assumptions, the paper has investigated the influence of both vapour mass changes and thermal dependency of suction on the overall water balance for a range of initial temperature and relative humidity conditions. Results show that the influence is negligible in both cases, thus suggesting that the governing equations could be simplified as in eqs. (19) and (20).

These preliminary results are only valid for hygroscopic relative humidity values, i.e. values smaller than 0.95 and positive temperatures. Moreover, they are limited to the specific boundary conditions considered in the present work, which do not include cyclic variations of temperature. Further studies are needed to extend the proposed model to the capillary and frozen domain as well as different boundary conditions.

The influence of the hydric parameters, such as the saturated permeability and vapour diffusion coefficient, on the validity of the above simplifying assumptions has not been analysed in this study and constitutes matters for future research [9].

$$\rho_d \frac{\partial w_l}{\partial hr}\frac{\partial hr}{\partial t} = -\nabla\left(-\frac{K_w}{g}\rho_l\frac{R}{M_w}\frac{T}{hr}\nabla hr\right)$$
$$+ \left(-D_e\frac{M_w}{RT}\left(p_{sat}\nabla hr\right.\right. \tag{19}$$
$$\left.\left.+ hr\left(\frac{dp_{sat}}{dT}-\frac{p_{sat}}{T}\right)\nabla T\right)\right)$$

$$\left(\rho C_p\right)_{eq}\frac{\partial T}{\partial t} = -\nabla(-\lambda\nabla T)$$
$$+ L_v\left(\rho_d\frac{\partial w_l}{\partial hr}\frac{\partial hr}{\partial t}\right.$$
$$\left.+ \nabla\left(-\frac{K_w}{g}\rho_l\frac{R}{M_w}\frac{T}{hr}\nabla hr\right)\right) \tag{20}$$

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