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On the Expected Present Value of the Dividend Payments under a Dependence Structure Assumption

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Abstract

We consider the classical risk theory model modified by a dependence structure between the amount of payments related to each claim and the time elapsed between two subsequent claims. In the classical Poisson model, the surplus definition is based on a Poisson compound process, which assumes independence between the arrival times of two subsequent claims and the amount of the related claims. In this paper, the classical model is modified to achieve more flexibility and adaptability to reality through a dependence structure. This dependence is defined by a generalized Farlie-Gumbel-Morgenstern copula (FGM). The classical risk theory model is further modified by the presence of a linear upper reflecting barrier; under different assumptions, we obtain the equations for the discounted dividend payments.

Keywords

Risk Theory Models, Dependence Structure, Farlie-Gumbel-Morgenstern Copula, Dividends, Integral-Differential Equations

1. Introduction

More than a century ago, thanks to the contributions of Lundberg (1909) and Cramer (1930), the classic model of risk theory was introduced. Over the years, the model has been modified in several ways. De Finetti (1957) proposed an alternative approach to the theory of collective risk, focusing on maximizing the expected value of dividends paid until the time of ruin. Since the second half of the last century, this problem has been addressed as part of an optimal dividend strategy for an insurance company (see Gerber (1979, 1981); Gerber & Shiu (2006); Lin & Pavlova (2006)) through the introduction of various types of upper barriers within the model.

Dickson and Gray (1984), Gerber (1974, 1981), and Vittal and Vasudevan (1987) consider a model modified by the presence of an upper reflecting barrier. The introduction of an upper barrier ensures that as soon as the surplus reaches the barrier, the difference between the surplus and the barrier is paid by the insurance company as a dividend, so the surplus remains at the barrier until a new claim occurs.

In the classical Poisson model, the surplus is based on a Poisson compound process, which assumes independence between the arrival times of two subsequent claims and the amounts of the related claims (Grandell, 1991; Rolski et al., 1999).

However, we observe that in real contexts, this assumption is too restrictive. Therefore, the classical model can be modified to achieve more flexibility and better adapt to reality. In this context, Cossette et al. (2008, 2011, 2018) studied the expected discounted penalty function, assuming a dependence structure and the presence of a constant dividend strategy.

In this paper, we study the expected present value of dividends in a model with a dependence structure, assuming a barrier strategy. We consider a dependence structure between the arrival times of two subsequent claims and the amounts of the related claims, defined by an FGM generalized copula. Additionally, we consider the model modified by the presence of a linear superior barrier.

The paper is organized as follows: Section 2 presents the classic risk theory model. Section 3 introduces the dependence structure between the amount of payments related to each claim and the time elapsed between two subsequent claims, defined by a generalized Farlie-Gumbel-Morgenstern copula. In Section 4, the barrier strategy is introduced, and integral and integral-differential equations of the expected present value of dividends with a constant dividend barrier are studied. Finally, Section 5 concludes the findings of the paper.

2. The Classic Model

Referring to an insurance portfolio, we made the following assumptions. We consider the classical model of collective risk theory, where $\underline{N} = \{N(t), t \geq 0\}$ the process "number of claims up to time t", is a Poisson process with parameter λ (expected number of claims per unit time).

We denote by $\{W_j, j = 1, 2, \cdots\}$ the random variables representing the "elapsed time between two successive claims", with W_1 being the arrival time of the first claim. These random variables are independent and identically distributed as the canonical random variable W. We denote by f_w the probability density function of W, F_W as the respective distribution function and f_w^* as its Laplace transformation. If $f_w(t) = \lambda e^{-\lambda t}$, then $F_W(t) = 1 - e^{-\lambda t}$ and $f_w^*(s) = E\left[e^{-sW}\right] = \frac{\lambda}{\lambda + s}$.

We denote by $\{X_j, j = 1, 2, \cdots\}$ the random variables representing "individual claim size", which we assume to be positive, independent, and identically distributed as the canonical random variable X, with X_1 being the first claim amount. We denote by f_X the probability density function of X, F_X as the respective distribution function and f_X^* as its Laplace transformation. Let us assume that the amount concerning each claim has a finite expectation μ, $\mu = \int_0^{+\infty} x\mathrm{d}F(x)$.

According to Cossette (2008), we assume that $\{(X_j, W_j), j = 1, 2, \cdots\}$ form a sequence of random vectors that are independent and identically distributed as the canonical vector (X, W), whose component can be dependent. We denote $f_{X,w}$ as the joint probability density function, $F_{X,W}$ as the respective joint distribution function, and $f_{X,w}^*$ as its bivariate Laplace transformation.

Let the process $\underline{S} = \{S(t), t \ge 0\}$, where

$$S(t) = \sum_{j=1}^{N(t)} X_j$$

is the stochastic process "aggregate claims amount up to time *t*" (with $S(t) = 0$ if $N(t) = 0$.

We denote by $\underline{U} = \{U(t), t \ge 0\}$ the stochastic process representing the "surplus of the Insurance Company at time *t*". According to the classical model, it results in:

$$U(t) = u + pt - S(t),$$

where $U(0) = u, u \ge 0$ is the initial surplus, known and nonnegative, and p, $p > 0$, is the premium flow received continuously per unit time.

We consider the time of ruin of the insurance company, which occurs whenever the surplus becomes negative, defined as follows:

$$T = \inf_{t \ge 0} \{t, U(t) < 0\},$$

with $T = +\infty$ if $U(t) \ge 0, \forall t \ge 0$, where no ruin occurs.

3. The Dependence Structure

The classical model of collective risk theory assumes an independence structure between the inter-claim times and the amount of claims.

In this paper, we introduce a dependence structure between inter-claim times and claim amounts using a copula approach. For this, we assume that the joint distribution of (X, W) is based on a generalization of the FGM copula, given by:

$$\overline{C}(u,v) = uv + \theta uv(1-u)(1-v), \quad -1 \le \theta \le 1, (u,v) \in [0,1]^2,$$

according to Cossette et al., this copula is defined by

$$C(u,v) = uv + \theta u^a (1-u)^b v^c (1-v)^d, \quad (u,v) \in [0,1]^2. \tag{1}$$

The Authors Rodríguez-Lallena and Ubeda-Flores (2004) mentioned that the range for θ contains the interval $[-1,1]$. They proved that the range for θ depends on the parameters a,b,c,d, $a,b,c,d \ge 1$; for example, if $a,b,c,d = 2$, they show that $-27 \le \theta \le 27$.

We consider the case $a,b \ge 1$, $c = 2,3,\cdots$ and $d > 1$.

If we consider in Equation (1) the functions

$$h(u) = u^a (1-u)^b, \quad g(v) = v^c (1-v)^d,$$

as considered in Cossette et al., we have

$$C(u,v) = uv + \theta h(u) g(v). \tag{2}$$

And the probability density function associated to Equation (2) is

$$c(u,v) = 1 + \theta h'(u) g'(v).$$

Therefore, the joint distribution function of X and W yields

$$\begin{aligned} F_{X,W}(x,t) &= C(F_X(x), F_W(t)) \\ &= F_X(x) F_W(t) + \theta (F_X(x))^a (1 - F_X(x))^b (F_W(t))^c (1 - F_W(t))^d \end{aligned} \tag{3}$$

with probability function f

$$\begin{aligned} f_{X,W}(x,t) &= c(F_X(x), F_W(t)) f_X(x) f_W(t) \\ &= f_X(x) f_W(t) + \theta h'(F_X(x)) g'(F_W(t)) f_X(x) f_W(t). \end{aligned}$$

Let

$$K_X(x) = f_X(x) h'(F_X(x)) \quad \text{and} \quad K_W(t) = f_W(t) g'(F_W(t)), \tag{4}$$

therefore, it yields:

$$f_{X,W}(x,t) = f_X(x) f_W(t) + \theta K_X(x) K_W(t). \tag{5}$$

4. The Dividend's Integral and Integral-Differential Equations in the Presence of a Constant Dividend Barrier

As stated previously, we assume a barrier strategy introducing a constant superior reflecting barrier, obtaining integral and integral-differential equations satisfied by the expected present value of the dividend payments resulting from the barrier presence.

According to several authors (see, for example, Dickson & Gray, 1984; Gosio & Lari, 2001; Gosio, Lari, Ravera, & Torrente, 2018), let us assume that when the surplus reaches the barrier, the difference between the surplus and the barrier is paid out, generating the so-called "dividend payment" to subscribers. Therefore, the surplus remains over the barrier until the next claim occurs.

We consider the classical Lundberg Risk Theory model, modified by the presence of a constant barrier $D(t)$, such that $D(t) = b$, with $u \le b < +\infty$.

Even with the barrier, dividend payments stop immediately if ruin occurs, which happens when the surplus becomes negative (see Gerber (1981)).

We denote $V(u,b)$ the expected present value, at time 0, of the dividend payments, evaluated using a constant force of interest δ, $\delta > 0$.

We recall that, as stated in Section 2, W_1 is the arrival time of the first claim, and X_1 is the first claim amount. Let $W_1 = t$ be the length of the time interval before the first claim occurs, and $X_1 = x$ be the first claim amount. Depending on when the first claim occurs and its amount, the following scenarios may occur: if the first claim occurs before the surplus reaches the barrier, no dividends are paid; whereas if the first claim occurs after the surplus reaches the barrier, dividends are paid. In both scenarios, the amount of the first claim can be so high as to cause the ruin of the company, meaning it is higher than $u+pt$ or b, respectively.

If no claims occur, the surplus meets the barrier at point $\hat{t}$, whose coordinates are $\left(\frac{b-u}{p},b\right)$.

Let $H(u,b)=\frac{b-u}{p}$, and $\mathfrak{D}(u,b,t)$ the expected present value, at time 0, of the dividends paid in the time interval $\left[H(u,b),t\right)$.

We define the random variable

$$\mathcal{V}(u,b,W_1,X_1),\ 0\le u\le b,$$

such that:

$$\begin{cases} \mathrm{e}^{-\delta t}V(u+pt-x,b) & \text{if } 0\le t\le H(u,b) \text{ and } x\le u+pt \\ 0 & \text{if } 0\le t\le H(u,b) \text{ and } x>u+pt \end{cases}$$

and

$$\begin{cases} \mathrm{e}^{-\delta t}V(b-x,b)+\mathfrak{D}(u,b,t) & \text{if } t>H(u,b) \text{ and } x\le b \\ \mathfrak{D}(u,b,t) & \text{if } t>H(u,b) \text{ and } x>b \end{cases}$$

For the purpose of determining $\mathfrak{D}(u,b,t)$, we observe that, by assumption, the first claim occurs at time t, with $t>H(u,b)$. In the time interval $\left[H(u,b),t\right)$ the insurance company receives the premium flow p, where $p>\lambda\mu$, and immediately allocates part of it to the payment of dividends, resulting in:

$$U(\tau)=D(\tau),\ \forall\tau\in\left[H(u,b),t\right).$$

The expected present value, at time 0, of the dividends paid in the time interval $\left[H(u,b),t\right)$ is

$$\mathfrak{D}(u,b,t)=\int_{H(u,b)}^{t}p\mathrm{e}^{-\delta s}\mathrm{d}s=\frac{p}{\delta}\left[\mathrm{e}^{-\delta H(u,b)}-\mathrm{e}^{-\delta t}\right].$$

Since $V(u,b)=E\left[\mathcal{V}(u,b),W_1,X_1\right]$ for the positions made and for the previous one, we have:

$$\begin{aligned} V(u,b)=&\int_0^{H(u,b)}\int_0^{u+pt}\mathrm{e}^{-\delta t}V(u+pt-x,b)f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t \\ &+\int_{H(u,b)}^{+\infty}\int_0^{b}\left[\mathrm{e}^{-\delta t}V(b-x,b)+\mathfrak{D}(u,b,t)\right]f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t \\ &+\int_{H(u,b)}^{+\infty}\mathfrak{D}(u,b,t)f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t. \end{aligned}$$

By collecting appropriately, previous equation becomes:

$$\begin{aligned} V(u,b)=&\int_0^{H(u,b)}\int_0^{u+pt}\mathrm{e}^{-\delta t}V(u+pt-x,b)f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t \\ &+\int_{H(u,b)}^{+\infty}\int_0^{b}\mathrm{e}^{-\delta t}V(b-x,b)f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t \\ &+\int_{H(u,b)}^{+\infty}\int_0^{b}\mathfrak{D}(u,b,t)f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t \\ &+\int_{H(u,b)}^{+\infty}\int_b^{+\infty}\mathfrak{D}(u,b,t)f_{X,W}(x,t)\mathrm{d}x\mathrm{d}t. \end{aligned} \tag{6}$$

We observe that the last two terms in Equation (6) are equal to:

$$\begin{aligned}&\int_{H(u,b)}^{+\infty}\int_0^{+\infty}\mathfrak{D}(u,b,t)f_{X,W}(x,t)\,\mathrm{d}x\mathrm{d}t\\&=\int_{H(u,b)}^{+\infty}\int_0^{+\infty}\mathfrak{D}(u,b,t)\frac{\partial^2F_{X,W}(x,t)}{\partial x\partial t}\mathrm{d}x\mathrm{d}t\\&=\int_{H(u,b)}^{+\infty}\mathfrak{D}(u,b,t)\int_0^{+\infty}\frac{\partial}{\partial x}\frac{F_{X,W}(x,t)}{\partial t}\mathrm{d}x\mathrm{d}t.\end{aligned}\tag{7}$$

From Equation (3) it results:

$$\begin{aligned}\frac{\partial F_{X,W}(x,t)}{\partial t}=&F_X(x)F_W'(t)+\theta(F_X(x))^a(1-F_X(x))^b\\&\cdot\Big[c(F_W(t))^{c-1}F_W'(t)(1-F_W(t))^d\\&+d(1-F_W(t))^{d-1}(-F_W'(t))(F_W(t))^c\Big].\end{aligned}\tag{8}$$

Within (7), because of (8), it follows:

$$\begin{aligned}&\int_0^{+\infty}\frac{\partial}{\partial x}\Big\{F_X(x)F_W'(t)+\theta(F_X(x))^a(1-F_X(x))^b\\&\cdot\Big[c(F_W(t))^{c-1}F_W'(t)(1-F_W(t))^d\\&+d(1-F_W(t))^{d-1}(-F_W'(t))(F_W(t))^c\Big]\Big\}\mathrm{d}x=F_W'(t);\end{aligned}$$

where we recall that, by assumption,

$$F_W'(t)=f_W(t)=\lambda\mathrm{e}^{-\lambda t}$$

finally, substituting, it therefore results:

$$\begin{aligned}V(u,b)=&\int_0^{H(u,b)}\int_0^{u+pt}\mathrm{e}^{-\delta t}V(u+pt-x,b)f_{X,W}(x,t)\,\mathrm{d}x\mathrm{d}t\\&+\int_{H(u,b)}^{+\infty}\int_0^{b}\mathrm{e}^{-\delta t}V(b-x,b)f_{X,W}(x,t)\,\mathrm{d}x\mathrm{d}t\\&+\int_{H(u,b)}^{+\infty}\lambda\mathrm{e}^{-\lambda t}\mathfrak{D}(u,b,t)\,\mathrm{d}t.\end{aligned}\tag{9}$$

Calling

$$\int_{H(u,b)}^{+\infty}\lambda\mathrm{e}^{-\lambda t}\mathfrak{D}(u,b,t)\,\mathrm{d}t=\frac{p}{\lambda+\delta}\mathrm{e}^{-(\lambda+\delta)H(u,b)}=\Phi(u,b).\tag{10}$$

Equation (9), for (5) and (10), results:

$$\begin{aligned}V(u,b)=&\int_0^{H(u,b)}\int_0^{u+pt}\mathrm{e}^{-\delta t}V(u+pt-x,b)\big[f_X(x)f_W(t)+\theta K_X(x)K_W(t)\big]\mathrm{d}x\mathrm{d}t\\&+\int_{H(u,b)}^{+\infty}\int_0^{b}\mathrm{e}^{-\delta t}V(b-x,b)\big[f_X(x)f_W(t)+\theta K_X(x)K_W(t)\big]\mathrm{d}x\mathrm{d}t+\Phi(u,b).\end{aligned}\tag{11}$$

Since $f_w(t)=\lambda\mathrm{e}^{-\lambda t}$, Equation (11) becomes:

$$\begin{aligned}V(u,b)=&\lambda\int_0^{H(u,b)}\int_0^{u+pt}\mathrm{e}^{-(\delta+\lambda)t}V(u+pt-x,b)f_X(x)\,\mathrm{d}x\mathrm{d}t\\&+\theta\int_0^{H(u,b)}\int_0^{u+pt}\mathrm{e}^{-\delta t}V(u+pt-x,b)\cdot K_X(x)K_W(t)\,\mathrm{d}x\mathrm{d}t\\&+\lambda\int_{H(u,b)}^{+\infty}\int_0^{b}\mathrm{e}^{-(\delta+\lambda)t}V(b-x,b)f_X(x)\,\mathrm{d}x\mathrm{d}t\\&+\theta\int_{H(u,b)}^{+\infty}\int_0^{b}\mathrm{e}^{-\delta t}V(b-x,b)K_X(x)K_W(t)\,\mathrm{d}x\mathrm{d}t+\Phi(u,b),\end{aligned}$$

it follows that,

$$
\begin{aligned}
V(u,b) = {} & \lambda\int_0^{H(u,b)} \mathrm{e}^{-(\delta+\lambda)t}\mathrm{d}t\int_0^{u+pt} V(u+pt-x,b) f_X(x)\mathrm{d}x \\
& + \theta\int_0^{H(u,b)} \mathrm{e}^{-\delta t} k_W(t)\mathrm{d}t\int_0^{u+pt} V(u+pt-x,b)\cdot k_X(x)\mathrm{d}x \\
& + \lambda\int_{H(u,b)}^{+\infty} \mathrm{e}^{-(\delta+\lambda)t}\mathrm{d}t\int_0^{b} V(b-x,b) f_X(x)\mathrm{d}x \\
& + \theta\int_{H(u,b)}^{+\infty} \mathrm{e}^{-\delta t} K_W(t)\mathrm{d}t\int_0^{b} V(b-x,b) K_X(x)\mathrm{d}x + \Phi(u,b).
\end{aligned} \tag{12}
$$

We denote by $u+pt=z$, $\sigma_1(z)=\int_0^z V(z-x,b) f_X(x)\mathrm{d}x$, $\sigma_2(z)=\int_0^z V(z-x,b) K_X(x)\mathrm{d}x$, $\gamma=\frac{\delta+\lambda}{p}$, and $\gamma_i=\frac{\delta+\lambda_i}{p}$, then Equation (12) can be written as:

$$
\begin{aligned}
V(u,b) = {} & \frac{\lambda}{p}\int_u^b \mathrm{e}^{-\gamma(z-u)}\sigma_1(z)\mathrm{d}z \\
& + \frac{\theta}{p}\int_u^b \mathrm{e}^{-\delta\frac{z-u}{p}}\sigma_2(z) K_W\left(\frac{z-u}{p}\right)\mathrm{d}z \\
& + \frac{\lambda}{p}\int_b^{+\infty} \mathrm{e}^{-\gamma(z-u)}\sigma_1(b)\mathrm{d}z \\
& + \frac{\theta}{p}\int_b^{+\infty} \mathrm{e}^{-\delta\frac{z-u}{p}}\sigma_2(b) K_W\left(\frac{z-u}{p}\right)\mathrm{d}z + \Phi(u,b).
\end{aligned} \tag{13}
$$

In Cossette et al., is observed that condition (4), that is,

$$K_W(t) = f_W(t) g'(F_W(t))$$

can be written in the following way:

$$K_W(t) = \sum_{i=1}^{c+1} a_i \mathrm{e}^{-\lambda_i t} \tag{14}$$

where

$$\lambda_i = \lambda(d+i-1),\ i=1,2,\cdots,c+1.$$

and

$$a_i = \frac{c!\lambda^c(-\lambda_i)}{\prod_{j=1,j\neq i}^{c+1}(-\lambda_i+\lambda_j)},$$

with the following initial condition (see Cossette et al. (2008, 2011) and Li and Garrido (2004)):

$$\sum_{i=1}^{c+1} a_i = K_W(0) = 0 \tag{15}$$

Substituting (14) into (13) it follows that:

$$
\begin{aligned}
V(u,b) = {} & \frac{\lambda}{p}\int_u^b \mathrm{e}^{-\gamma(z-u)}\sigma_1(z)\mathrm{d}z \\
& + \frac{\theta}{p}\int_u^b \mathrm{e}^{-\delta\frac{z-u}{p}}\sigma_2(z)\left[\sum_{i=1}^{c+1} a_i \mathrm{e}^{-\lambda_i\left(\frac{z-u}{p}\right)}\right]\mathrm{d}z
\end{aligned}
$$

$$+\frac{\lambda}{p}\int_{b}^{+\infty}\mathrm{e}^{-\gamma(z-u)}\sigma_1(b)\mathrm{d}z$$
$$+\frac{\theta}{p}\int_{b}^{+\infty}\mathrm{e}^{-\delta\frac{z-u}{p}}\sigma_2(b)\left[\sum_{i=1}^{c+1}a_i\mathrm{e}^{-\lambda_i\left(\frac{z-u}{p}\right)}\right]\mathrm{d}z+\Phi(u,b). \tag{16}$$

We observe that (16) can be written in the following way:

$$\begin{aligned}V(u,b)=&\frac{\lambda}{p}\int_{u}^{b}\mathrm{e}^{-\gamma(z-u)}\sigma_1(z)\mathrm{d}z\\&+\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\int_{u}^{b}\mathrm{e}^{-\gamma_i(z-u)}\sigma_2(z)\mathrm{d}z\\&+\frac{\lambda}{p}\int_{b}^{+\infty}\mathrm{e}^{-\gamma(z-u)}\sigma_1(b)\mathrm{d}z\\&+\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\int_{b}^{+\infty}\mathrm{e}^{-\gamma_i(z-u)}\sigma_2(b)\mathrm{d}z+\Phi(u,b),\end{aligned}$$

that is, recalling (10),

$$\begin{aligned}V(u,b)=&\frac{\lambda}{p}\int_{u}^{b}\mathrm{e}^{-\gamma(z-u)}\sigma_1(z)\mathrm{d}z+\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\int_{u}^{b}\mathrm{e}^{-\gamma_i(z-u)}\sigma_2(z)\mathrm{d}z\\&+\frac{\lambda}{p\gamma}\mathrm{e}^{-\gamma(b-u)}\sigma_1(b)+\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\frac{\mathrm{e}^{-\gamma_i(b-u)}}{\gamma_i}\sigma_2(b)+\frac{1}{\gamma}\mathrm{e}^{-\gamma(b-u)}.\end{aligned} \tag{17}$$

Deriving (17) respect to u, we obtain:

$$\begin{aligned}\frac{\partial V(u,b)}{\partial u}=&\frac{\lambda}{p}\gamma\int_{u}^{b}\mathrm{e}^{-\gamma(z-u)}\sigma_1(z)\mathrm{d}z-\frac{\lambda}{p}\sigma_1(u)\\&+\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\gamma_i\int_{u}^{b}\mathrm{e}^{-\gamma_i(z-u)}\sigma_2(z)\mathrm{d}z+\frac{\lambda}{p}\mathrm{e}^{-\gamma(b-u)}\sigma_1(b)\\&+\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\mathrm{e}^{-\gamma_i(b-u)}\sigma_2(b)+\mathrm{e}^{-\gamma(b-u)}.\end{aligned} \tag{18}$$

Finally, multiplying both terms of (17) by γ and subtracting Equation (18) from this product, while recalling Equation (15), we obtain the following integro-differential equation, satisfied by the expected present value of dividends:

$$\begin{aligned}\gamma V(u,b)-\frac{\partial V(u,b)}{\partial u}=&\frac{\lambda}{p}\sigma_1(u)+\frac{\theta}{p}\gamma\sum_{i=1}^{c+1}a_i\int_{u}^{b}\mathrm{e}^{-\gamma_i(z-u)}\sigma_2(z)\mathrm{d}z\\&-\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\gamma_i\int_{u}^{b}\mathrm{e}^{-\gamma_i(z-u)}\sigma_2(z)\mathrm{d}z\\&+\frac{\theta}{p}\gamma\sum_{i=1}^{c+1}a_i\frac{\mathrm{e}^{-\gamma_i(b-u)}}{\gamma_i}\sigma_2(b)\\&-\frac{\theta}{p}\sum_{i=1}^{c+1}a_i\mathrm{e}^{-\gamma_i(b-u)}\sigma_2(b),\end{aligned} \tag{19}$$

with the following boundary conditions:

$$V(b,b)=\frac{\lambda}{p\gamma}\sigma_1(b)+\frac{\theta}{p}\sum_{i=1}^{c+1}\frac{a_i}{\gamma_i}\sigma_2(b)+\frac{1}{\gamma}$$
$$\left.\frac{\partial V(u,b)}{\partial u}\right|_{u=b}=1,$$

and

$$\left.\frac{\partial V^n(u,b)}{\partial u}\right|_{u=b} = -\frac{\lambda}{p}\sum_{j=1}^{n-1}\gamma^{n-j-1}\sigma_1^j(b)+\gamma^{n-1}, n=2,\cdots,c.$$

5. Concluding Remarks

In this paper, we present a risk model that includes a linear upper barrier and assumes a dependence structure between the amount of payments related to each claim and the time elapsed between two subsequent claims. The introduction of an upper barrier ensures a dividend payment flow as soon as the surplus reaches the barrier; the difference between the surplus and the barrier is paid by the insurance company as dividends.

The dependence structure is defined by a generalized Farlie-Gumbel-Morgenstern copula. We obtained the integral and integro-differential equations for the discounted value of dividend payments, deriving some boundary conditions.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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