Factor Shares, Redistribution and Growth in a Captured Democracy

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Abstract

A model of endogenous growth is presented, based on productive public expenditures, but featuring some degree of income inequality, and polarization in policy preferences. The main innovation lays in the political process determining capital taxation that relies, both on voting and on "influence activities," exploited by the capitalist elite in order to capture some political power at the expenses of the median voter. In particular, investments in lobbying activities allow the rich to obtain lower capital taxes, to the benefit of both themselves and economic growth. The model's equilibrium dynamics features variable taxes and lobbying. In addition, it is established the existence of a transitional dynamics featuring convergence to a balanced growth path, characterized by constant taxes and lobbying (and a constant growth rate of consumption and capital). Capital accumulation leads, along the transitional path, to more and more lobbying, that asymptotically cause taxation to reach precisely the tax rate preferred by the capitalists (induced by a very large political pressure on the government). Specifically, the (unique) balanced growth equilibrium features the maximization of the net interest rate, as well as the economy's growth rate and capitalists' welfare. On the transitional path, lobbying does reduce the workers' political weight (and their consumption), and therefore makes fiscal policy relatively more and more capitalists friendly. Policy polarization (loosely speaking reflecting inequality) has somewhat interesting effects along the transitional path to balanced growth. Hereby, actual taxes do become more capitalists friendly relatively to the Meltzer and Richard's (1981) standard median voter tax benchmark, *mutatis mutandis*. In the end, full convergence is established, from a pure democracy ruled through the *de jure* power only, to a political-economic realm totally dominated *de facto* by the few capitalists, i.e. to an "oligarchic technocracy," possibly dominated by the "top 1%" of the population or so.

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A hundred men acting uniformly in concert, with a common understanding, will triumph over a thousand men who are not in accord and can therefore be dealt with one by one. Meanwhile it will be easier for the former to act in concert and have a mutual understanding simply because they are a hundred and not a thousand. It follows that the larger the political community, the smaller the will the proportion of the governing minority to the governing majority will be, the more difficult will it be for the majority to organize for reaction against the minority.

— Gaetano Mosca (1939)

The major contributions who fund political campaign are, by definition, rich (poor people cannot afford to do so), and they are not interested in trowing money away. To believe that the rich do not use their money to buy influence and promote policies they like is not simply to be naïve. Such a stance contradicts the basic principles of economics as well as the ways in which the rich people have amassed their wealth— surely not throwing it around it while expecting no return on it.

— Branko Milanovic (2016)

1 Introduction

There is now a wide consensus about the political institutions being of paramount importance for both political and economic outcomes. Formal models indicate that changes in political institutions should be expected to have important consequences in patterns ranging from income taxation and public good provisions, to the rights of recently enfranchised minorities. Income inequality often importantly interacts with significant transformations of the macro-political environment.[1] The available empirical evidence in this respect is mixed, though. For example, Rodrik (1999) empirically demonstrates, that democracies do pay higher wages, compared to non-democracies. However, Acemoglu, Ticchi and Vindigni (2011), demonstrate that fiscal redistribution in a democracy can sometimes be relatively low, even in presence of high inequality, when the government is controlled by an anti-redistribution coalition involving the rich and the bureaucrats. Such coalition supports the creation of an organization apparatus state with weak fiscal capacity, but generating rents for itself. Also, in an important paper, Perotti (1996) casts doubt on the empirical relevance of the Meltzer and Richard's (1981) celebrated positive theory of redistribution, that has been (and still is) an important benchmark in political economy.

[1] See Bertola, Foellmi, and Zweimüller (2014) for a rigorous and rather comprehensive discussion of the income distribution's role in macroeconomics and in political economy.

More recently, Acemoglu, Naidu, Restrepo and Robinson (2019) show, that democracies, despite their heterogeneity, tend to outperform on average non-democracies in terms of economic growth, because of their superior capability of creating a playing field for entrepreneurship. However, Barro (1997, 1999), and Mulligan, Gil and Sala-i-Martin (2004) find no substantial difference in policy outcomes and performances between different political regimes, a finding that casts doubt on the economic importance of political institutions.

The broad picture emerging from some of these studies is, thus, that democracies and non-democracies may not be so different in some important dimensions, including the workings of some of their economic institutions (e.g. the structure of their labor market), the programs about income redistribution and public goods provision implemented by the state, and, ultimately, their economic growth rate.

Acemoglu and Robinson (2006, 2008a and 2008b) attempt to reconcile some of these puzzling findings, by observing that political regimes, including democracy, are characterized by a potential de-coupling between the *de jure* political power and the *de facto* political power. *De jure* political power is determined by the existing formal political institutions; the *de facto* political power is instead acquired with material means by a small subset of the citizenry, that can afford such "investment." The de-coupling of these two forms of power can lead to the potential "capturing" by a population's (usually affluent) minority of a formally democratic political realm. Therefore, while a "captured democracy" possesses formal political institutions, somewhat similarly to a non-captured or "constitutional democracy,"[2] as we opt to refer to captured democracy as a realm where *de jure* power is significantly overwhelmed by *de facto* power. And its pseudo-democratic economic institutions and other economic outcomes may turn out to be highly distorted, and very different from those that would be quite naturally expected in an environment where *de jure* power would by far and large prevail, that is in a constitutional democracy.

Understanding the political logic of captured democracies, requires bearing in mind that formal political institutions are usually defined by Constitutions,[3] that carefully allocate different forms of *de jure* political power to different state's bodies and branches. They therefore also create checks' and balances' system, that prevents an excessive concentration of power in any particular articulation of the state apparatus. Separation of powers, in turn, credibly

[2]The use of the locution "constitutional democracy," as opposed to "captured democracy," is not entirely appropriate, since the latter also usually relies on a Constitution. But Constitutions' working tends to be highly distorted, if not totally subverted, by major investments in the *de facto* political power by the rich elite.

[3]The Constitution can also include highly consolidated political practises. Though not part of any written document, such practices and related traditions are a potential source of *de jure* political power.

ensures the existence of a level playing field for fair economic competition, and represents the political foundation of sound economic institutions. This is the case to a much lesser degree in many non-democracies, as well as in captured democracies. Hereby the endogenously limited importance of the *de jure* power combined with the predominant importance of *de facto* political power,[4] creates a highly biased environment for economic and political activity, that causes the emergence and persistence of potentially highly distorted and dysfunctional economic institutions (e.g. monopsonistic labor markets, in absence of appropriate regulation). Or, else, it may cause the emergence, especially in democracies that are either captured, or potentially feature some formal political institutions strongly biased in favor of the rich (e.g. Bénabou, 2000) of very conservative "social contracts." Such arrangements feature very limited redistribution, that may or may or may not be harmful for economic efficiency and growth per sé, but tends to increase inequality, and therefore, may potentially undermine the stability of democracy itself in the long run (e.g. Acemoglu and Robinson, 2005).[5]

In this paper, we study how the economic growth process of a constitutional democracy,[6] can be affected by investments' influence activities by the rich elite. It is assumed, that this democracy features a highly polarized society, that is divided between a large mass of workers and a small minority of capitalists.

The *de jure* political institutions are basic: people decide by majority voting on the base of the one man-one vote principle, and (since all the relevant standard assumptions apply), a "government of the median voter" is elected.[7] We don't model in any particular detail the workings of representative democratic government obtaining power, and, in particular, the preferences and behavior of politicians in office. Rather, we make the simplifying assumption,

[4] Yet, perhaps surprisingly, some form of Constitutionalism exists also in dictatorships. See Ginsburg and Simpser (2013) on this interesting but probably under-researched topic.

[5] Bénabou (2000) shows that, depending on the balance between distortions and efficiency gains both caused by redistribution, the "American" social contract, featuring high inequality and low redistribution may or may not conduct to more economic growth than "European" social contract, with just the opposite features.

[6] We are agnostic regarding the origin of the *status quo* constitutional democracy. It may have been in place for a short time (and therefore represents a newly created realm), following a political transition from some sort of non-democratic regime previously in place. Or, else, it may have been in power for a while, but experiencing, for some reason, some relative stagnation. Such inertial situation ended with the start of the process, that eventually set in motion the development of the economy. But also, at the same time, triggered the "class struggle," that induced the rich elite to acquire *de facto* power.

[7] For instance, electoral competition (not explicitly modelled) may involve two Downsian political parties, solely concerned with winning office, and both committing to implement the same "most popular" tax policy, if elected. It therefore doesn't matter, which one of the two parties will be actually elected. None of the two parties has any *ex post* commitment problems due to their policy agnosticism. See also Alesina and Rosenthal (1995) for a discussion of political competition with and without policy motivated politicians.

that the preferred tax policy by the median voter can be distorted downwards by the rich elite investing in lobbying activities, according to an "influence function." Such function intuitively features relatively standard properties properties, including decreasing marginal returns to the volume of influence activities. Lobbying spending will affect and track the accumulation of capital, and it will gradually turn the original constitutional democracy into a regime where power is fully captured by the capitalist elite, resembling a "quasi-capitalist plutocracy."[8]

The economic environment is a simple generalization of Barro (1990),[9] and especially of Alesina and Rodrik (1994), featuring the potential provision of a productive public good. This provision is financed with proportional taxation of capital, that allows growth to be potentially endogenous. The small capitalist elite is in favor of some positive, but relatively limited, capital taxation, maximizing at one time the economy's growth rate, and their own welfare. The large mass of workers, instead, supports a much higher taxation, in order to redistribute some factor income in their favor, but not as high to shoot down growth. This is essentially in line with Meltzer and Richard's canonical positive theory of redistribution in a democracy. According to this theory, the workers have all the *de jure* political power in a pure constitutional democracy, based on majority voting only; therefore, in absence of some form of "activation" by the rich minority, they would impose their own preferred tax rate.

In our model, instead, we allow the rich to invest in influence activities, or lobbying, on the democratic government, in order to acquire some *de facto* political power, and to tilt fiscal policy in their favor. Following the spirit of a long tradition in the social sciences (including Mosca, 1939; Olson, 1965; Becker, 1983), we assume that only the capitalists, are able to solve the canonical collective action problem (and related crucial *free rider* issue faced by any social group) because of their very small number. Therefore they are able to get organized and form a pressure group of their own. Nevertheless, coordination is only partial, and each capitalist takes as given the amount of resources invested in political influence activities by their peers.

Because ours is fundamentally a growth theory, we are chiefly interested in understanding how the process of (endogenous) growth and the de-coupling between different forms of political power affect each other; and, ultimately, how their interaction shapes the economy's dynamic performance, both in terms of development and of income distribution's of the pat-

[8] We use the locution "quasi" because the fiscal instrument chosen by the government by assumption to raise taxes, proportional taxation of capital, is not necessarily the one preferred by the capitalists.

[9] It is worth noticing that Barro (1990) continued the endogenous growth revolution started by Romer (1986 and 1990). A result of all of these papers, is to let the marginal productivity of capital remains strictly bounded from above from the rate of time preference. This allows the economy not to fall in the usual neoclassical steady state. Barro obtains this result with the clever and elegant assumption that the aggregate production function depends on productive public expenditures, that potentially create a strong economic role for the state.

tern. Our main result is the following: in a model of endogenous growth based on private capital accumulation, and relying on a non-accumulable factor of production and a productive public good, growth stimulates lobbying activities, and vice versa.[10] Economic growth, and lobbying by the rich elite are linked by the degree of economic polarization in the society. In a richer and more unequal country, the relatively few capitalists have potentially much more to lose to the workers if these politically prevail, and therefore invest more in the *de facto* power, to prevent their expropriation as much as possible. At the same time, the relatively low taxes induced by capitalists' influence activities on the government, stimulate capital accumulation (by raising the interest rate), at the expense of the equalization of the *disposable* factor income distribution.[11] This pattern features a potential complementary between factor-income inequality and redistributive fiscal policy. In addition, it is broadly consistent with the evidence presented by Barro (2000), who show that inequality encourages growth in developed countries. Forbes (2000) also re-assesses the relation between inequality and growth, using a methodology based on panel estimation, allowing to control for time-invariant country-specific effects, that eliminates a potential source of omitted-variable bias. As Forbes writes (2000,. p. 869). "Results suggest that in the short and medium term, an increase in a country's level of income inequality has a significant positive relationship with subsequent economic growth."

Interestingly, the model (unlike Bertola's 1993, and Alesina and Rodrik's 1994) reaches its balanced growth path by firstly going through a transitional dynamics, where the variables grow at different and time-varying rates. Taxes, in particular, are not constant, but *decrease* over time (together with the implicit redistribution in favor of workers, that they generate), until they reach the "technocratic" rate preferred by the rich elite, that is attained in the "quasi-plutocratic" balanced growth path). As a result, the post-tax distribution of income becomes more polarized (i.e. the total income accruing to the workers constantly declines as a share of the total post-tax income of both social classes).

In addition, the model can help shedding light on the important phenomenon known as "raise of fiscal conservatism" (e.g. Saint-Paul, 2001). By this expression we mean a progressive

[10] As explained in greater detail below, this result is, however, not enitely general across different classes of growth models. For example in a Schumpeterian model of growth à la Aghion and Howitt (see Aghion and Howitt, 1998, or Acemoglu, 2009, for an introduction to the Schumeterian framework), lobbying may be harmful for growth. It may be specifically so by slowing down the process of creative destruction. See also the discusssion below on this point.

[11] Evidence broadly consistent with this pattern is provided by Acemoglu, Naidu, Restrepo, and Robinson (2015), who argue that the expectation that democracy should lead to a reduction of income inequality is not met when power is captured by a rich elite. Furthermore, the transition to democracy is not necessarily associated to an uniform reduction of inequality, but can lead to changes in patterns of public spending, fiscal redistribution and economic structures, that have ambiguous effects on inequality.

retrenchment of the welfare state, and a parallel reduction of the labor share (e.g. Karabarbounis and Neiman, 2014), observed with variable degree of intensity in the last decades or so, in many industrialized democracies (mostly in the U.S. and in the U.K., less so in Continental Europe). This process has been explained with variable degree of success by other prominent theories.[12] Our model potentially helps explaining the same broad pattern, i.e. the generalized rise of fiscal conservatism, and crisis of the traditional, mid XX century, form of welfare state, as well as the more recent rise of the "top 1%," i.e. the unparalleled rise of economic fortunes enjoyed by the richest 1% of the population in the last few decades. It does so by emphasizing the interaction of economics and politics in a relatively developed and unequal society, and in particular the greater incentives faced by the rich elite to invest in the $de\ facto$ political power. Additional eevidence consistent with this claim has been recently provided by Aghion, Antonin and Bunel (2021, and especially, Ch. 5),[13] who document an especially striking fact: the share of national income accruing to the top 1% of the population has increased significantly with the intensity of lobbying between 1998 and 2008. They conclude (2021, Ch. 5, p. 89) that: "[...] This outcome confirms that lobbying is indeed an other source, distinct from innovation, of inequality at the top)." In their Schumpeterian framework, lobbying indeed enables incumbent firms maintaining their market power and their rents, protecting their sector from competition, but also allows them to have easier access to credit and to pay less taxes.

It should be noticed, however, that in a Schumpeterian framework lobbying is likely to be harmful for growth for at least two reasons (Aghion, Antonin and Bunel, Ch. 5, p. 92). First: firms destine resources to lobbying at the expense of innovation. Second: lobbying slows down the process of creative destruction, that is the essence of growth in any Schumpeterian framework.[14] An excessive political empowerment of the rich may also be harmful for growth in models featuring the scope for efficient redistribution in favor of the poor (e.g. Bénabou, 2000). For example, by allowing them to partially overcome market failures such as credit market imperfections, that tend to inhibit their investments in human capital, as well as the acquisition of insurance against idiosyncratic labor income shocks We therefore remark that our result that lobbying unambiguously stimulates growth by reducing capital income taxes,

[12] Notable example include skilled-biased technical change (e.g. Autor, Katz and Krueger, 1998; Acemoglu and Restrepo, 2020) and trade with developing countries (e.g. Autor, Dorn, Hanson, and Song, 2014; Adão, Carrillo, Costinot, Donaldson, and Pomeranz, 2022). Both explanations posit, for different reasons, a sharp reduction of the demand of unskilled labor. Furthermore the price, commanded by it in a competitive labor market, falls in relative and *absolute* terms. This fact in turn potentially explains the observed raising inequality patterns usually noticed within many developed countries.

[13] On the relation between inequality and lobbying, see also Aghion, Akcigit, Bergeaud, Blundell, and Hemous (2019).

[14] See also Akcigit, Baslandze and Lotti (2023) on this topic.

and thereby stimulating capital accumulation, does help us isolating one potentially important effect of (capitalist) lobbying, but cannot be regarded as general, since it hinges on assumptions that are too much model-specific.[15]

It is also interesting to observe that in our framework, the overall allocation of political power changes quite substantially, even though the model features no abrupt change in political institutions, triggered for example by revolution, a military coup or a civil war. This occurs as the endogenous relative political weight of the two classes changes considerably over time, in favor of the small minority of capitalists, and in parallel with capital accumulation.

The (pro-rich) "peaceful revolution," going on during the growth process, implies that the nature of the political regime also evolves accordingly, from the initial constitutional democracy to an oligarchic technocracy. The constitutional democracy features very little investment in the *de facto* political power by the elite, and is ruled by the median voter, whereas the regime eventually emerging at relatively high levels of economic development, is mostly or entirely controlled by the capitalist minority, and is therefore also referred to a capitalist plutocracy.

Our paper is related to a number of bodies of literatures, stressing the importance of the interaction between inequality and democratic politics in various guises. Firstly, a relatively large set of contributions appeared though the 1990s, and emphasized the complex links existing between economic growth, politics and the distribution of income in non-representative agent setups, such as Bertola (1993, 1996), Perotti (1993), Saint-Paul and Verdier (1993, 1996), Alesina and Rodrik (1994), Persson and Tabellini (1994), Bénabou (1996, 2000), Bourguignon and Verdier (2000). The more recent, influential work of Piketty (2014),[16] is also clearly related to this paper. Piketty argues, that an exceptional concentration of economic and political power in a small elite (the "top 1%" or so) has been generally observed in the last few decades in the most important economies of the world, as an almost "natural" consequence of capitalist development.[17] The already mentioned growing importance on money invested by the rich and super-rich in electoral politics, especially but not only in the US, and particularly at the Federal government level (see for instance the studies of Bartels, 2010, Gilens 2012, and Gilens and Page, 2014, Page and Gilens, 2020), is indeed staggering. Gilens (2012) shows, in

[15] The concluding Section of the paper, provides an additional discussion of this point. It also briefly mentions potential extensions of our basic framework, that may lead to a more general characterization of the process of political lobbying and of its effects on economic growth.

[16] See also Boushey, Delong, and Marshall Steinbaum (eds.) (2017), for an extensive critical discussion of Piketty (2014).

[17] Our paper, however, does not hinges on Piketty's famous "$r > g$" condition in order to explain the political-economic dynamics, that he documents in his important work.

particular, that the legislators' responsiveness to people in the 90th percentile of the income distribution smoothly increases as the issue becomes more relevant to the rich elite. This sharply contrasts with legislators' responsiveness to the issues of concern for the poor (i.e. the bottom 10th percentile of income's distribution) and the middle class (the 50% percentile of income's distribution), that is essentially a flat line, indicating an almost total lack of concern for the issues that are more salient for the lower social classes. Furthermore, as pro-rich policies increase the income of rich, the rich are almost alone in making relevant contributions to politicians, and therefore obtain disproportionate attention form them. Ultimately, the one-person one vote system is replaced by the one-dollar one vote rule, which "is nothing else that the projection on the political plane of the existing distribution of income," (see Milanovic, 2016, p. 190). Finally, in a very recent contributions, Page and Gilens (2020, Ch. 4, p. 114) add that: "As best as we can tell from their contributions to political candidates, most American billionaires tend to be conservative on economic issues. Most of them favor limited social spending, relatively low taxes on upper-income people, and only modest (if any) government regulation of the economy."

Furthermore, we must mention the early 2000s literature regarding the persistence of institutions in presence of reallocation of political power. These works include the seminal papers of Acemoglu and Robinson (2006, 2008a and 2008b), showing that drastic changes in political institutions don't need to take place along radical transformations in economic institutions. This is, essentially, because in equilibrium the economic elite buy enough $de\ facto$ power to offset the negative (for them) shock to the augmentation of the $de\ jure$ power of the masses, triggered by a transition to formal democracy. Acemoglu, Ticchi and Vindigni (2011) expand on this topic by showing, that the strategic creation by the elite of a state apparatus with limited (inefficiently low) fiscal capacity leads to under-provisioning of public goods.

But this allows the rich elite to preserve much of its power, even after a major political transition,[18] by forming a pro *status quo* "perverse coalition" with the state's bureaucrats. None of the papers mentioned on the persistence of power across different political institutions, though, addresses the question of how economic development affects and is affected by the endogenous dynamics of political power and related smooth power struggle (i.e. the absence of political regimes transitions), within a full-fledged model of endogenous growth, as we do

[18] The paper in question also contributes to the growing literature explaining how state underdevelopment and failure is connected to the elite's aspiration to both preserve some power (mainly $de\ facto$), after a transition to formal democracy has occurred, and to the persistence of relatively high inequality (due to low redistribution) in the post-democratic transition period.

hereby.

Also worth to be mentioned, is the literature on lobbying theory, beginning with Becker's classic paper (1983), and including the later contributions of Austen-Smith (1987), Baron (1994), Besley and Coate (2001), and Grossman and Helpman (2002). Relative to all of these important papers, our model relies on a much simpler lobbying process, that is nevertheless applied to the dynamic environment of an infinite-horizon economy growing endogenously, in presence of a fundamental political-institutional conflict.

2 The Model: Foundations

We consider a model of endogenous growth, that is partially similar to the one presented in Alesina and Rodrik's (1994) seminal paper on inequality and growth. There is an infinite-horizon economy in continuous time, populated by a finite number of individuals, with identical preferences represented by

$$\int_0^\infty e^{-\rho t} \ln(c_t^i) dt, \tag{1}$$

where ρ represents the time discount factor and c_t^i is the consumption at time t of a generic individual i at time t.

Firms operate with an "extended" neoclassical production function. As in Barro's (1990) paper, the technology available in our model relies *inter alia* on the provision of a productive public good by the government. Specifically, the aggregate production function has the following form

$$y = Ak^\alpha g^{1-\alpha} \ell^{1-\alpha}, \tag{2}$$

with $\alpha \in (0, 1)$. In this expression, k stands for the accumulable factor of production, including physical, but also human, capital;[19] g indicates the stock of productive public spending supplied by the government; finally ℓ stands for the total supply of non-accumulable factor of production, that is to say, raw labor.[20]

[19] Since the notion of "capital" must be broadly interpreted, the model will potentially account for the fact part of the top earners (i.e. bankers, top mangers), will have themselves a dual role of "capitalists" as well as "workers." This fact makes contemporary "globalization capitalism," partly different from the XIX century "patrimonial capitalism," or "classical" or the "Belle Époque capitalism." The latter form of capitalism featured a very high correlation between ownership of capital and high incomes, and was thereby largely dominated by pure reinters only (Milanovic, 2014, p. 527-528).

[20] We remark that, unlike private capital, the productive public good g isn't a model's state-variable, but a control variable, linked with taxes and the government (static) budget constraint. See equation (3) reported below. Futagami, Morita, and Shibata (1993), consider instead an interesting version of Barro's (1990) model, where g corresponds to a state-variable (public capital), and find that, unlike in Barro's (1990), growth maximization is not equivalent to the maximization of the welfare of the representative agent.

Productive public expenditures are financed with proportional capital's taxation, and the government budget constraint is assumed to be always balanced, so that, at each time, we have that the following equation holds[21]

$$g = \tau k. \tag{3}$$

Combining the last two equations, one gets a new form of the production function, namely

$$y = A\tau^{1-\alpha}\ell^{1-\alpha}k. \tag{4}$$

The crucial feature of this last equation is to be linear in the accumulable factor of production, so that, in principle, it can potentially allow the (net) interest rate and marginal productivity of capital not to fall below the rate of time preference (at least if taxes do not increase too much).[22]

We follow Alesina and Rodrik (1994) in considering a generalized version of Barro's model, where the "representative agent" setup is replaced by the assumption, that people are indeed heterogeneous, in the sense of having a different initial (i.e. at time $t = 0$)relative endowment of capital and labor income. Specifically, citizens differ in their initial *relative* ownership share of the aggregate raw labor stock vs. their relative ownership share of the aggregate capital stock; therefore, for a generic individual i, the following formula applies,

$$\sigma_0^i = \frac{\ell^i/1}{k_0^i/k_0} \in [0, \infty), \tag{5}$$

a formula naturally assuming the normalization to 1 of the aggregate stock of unskilled labor. The parameter σ will shape individual's preferences on the tax rate τ, which generates some factor-income redistribution. Therefore, such preferences will depend on the relative endowment of non accumulable vs. accumulable factors of production. Note that while the numerator of formula (5) is, obviously, always constant, the denominator may, in principle, change over time, with individual i potentially getting richer or poorer in terms of relative endowment of capital income.[23] In our framework, though, the initial distribution of income takes a particularly simple form, as we assume the existence of two "social classes" only. A

[21] Note that, even if taxes are proportional, more capital-rich individuals contribute more, for any given tax rate, simply because they have more to give. Furthermore, the tax policy in question tends, *inter alia*, to redistribute factor-income from capital to labor by boosting wages (for given capital stock), as demonstrated below.

[22] If, instead, the opposite event happens, growth is well-known to end, due to the evaporation of the individual incentives to save, and the economy ends up in a stationary state.

[23] The output of formula (5) is a *datum* of history, reflecting the initial conditions of the economy, that could be any. However, in principle, it may be that σ_t^i becomes different, as times goes by, from σ_0^i, for some t. As explained later, however, this will never occur in equilibrium.

small minority, of finite size λ^k, of the population is formed by capitalists,[24] owning *in equal proportion* all the capital, and doing all the savings. They have initially no political power whatsoever, but they alone invest in lobbying activities over time (see below), gaining *de facto* some political voice at the expense of the workers. The set of capitalists is denoted as $\mathcal{K}$; all capitalists are alike, and therefore (in the symmetric equilibrium we shall focus on), behave like a single "representative capitalist," denoted as i. A large, but of finite size λ^ℓ, mass of individuals is formed by workers, who have no capital and *do not save*, in line with the classic Kaldor-Pasinetti assumption. Workers also do not engage in any lobbying activity, since, as already mentioned, their class is too large to get organized and solve the relevant free-rider problem.[25] They therefore only work, supplying *individually* a quantity of labor equal to $1/\lambda^\ell$ (and therefore a *total* quantity of labor equal to 1, as mentioned above), and consume entirely their wage income at each point in time. The set of workers is denoted by L. We assume that the total number of capitalists and workers has a size normalized to 1, i.e. it is the case that $\lambda^k + \lambda^\ell = 1$. It follows from this normalization that equation (2) represents as the same time the aggregate and the per-capita production function.

Notice, that the somewhat extreme assumptions made on the initial value of the distributional parameter σ, imply that $\sigma_0 \in \{0, \infty\}$. Each worker has no capital income and therefore $\sigma_0^i = \infty$, $\forall\ i \in \mathcal{L}$; each capitalist has no labor income, and therefore $\sigma_0^i = 0$, $\forall\ i \in \mathcal{K}$. Because workers don't save by assumption, and capitalists don't have (and never acquire) any labor income, also by assumption, σ_0^i remains constant over time, for any $i \in \mathcal{L} \cup \mathcal{K}$. The existence of only two types of individuals at each point in time implies, that, as discussed in greater detail below, only two constant taxes are always preferred by the the two subsets of *citizens* ($\mathcal{L}$ and $\mathcal{K}$), over any other potential arrangement. However, because of lobbying, the political process will generally deliver, along the transition to the balanced growth path, a "compromise" taxation (varying over time), as also explained below.

Before proceeding, it will be useful to progress to describe the economic environment, beginning with the computation of the factor rental rates (capital and labor) faced by the individuals as a function of the taxes. The individuals act as price-takers in competitive

[24] The capitalists elite may be though of as representing, in particular, the so-called "top 1%" of the distribution of income in society, when $\lambda^k \downarrow 0$. The assumption is consistent with the observation that in most industrialized countries the distribution of capital income has been extremely unequal, at least over the last fifty years or so. In particular, the corresponding Gini coefficient for capital income has been often around 90% in most industrial countries countries, since the early 1980s. On the contrary, the corresponding Gini coefficient for labor income has been remarkably lower, by a factor of 50% or so (see Milanovic, 2023, Ch. 7, pp. 272-273).

[25] See the already quoted seminal works of Mosca (1939), Olson (1965), and Becker (1983) on this point.

markets.[26] Using the Cobb-Douglas specification, assumed for the production function (and omitting here for simplicity all time subscripts), we have that the post-tax gross and net rental rate of capital read, respectively,

$$r \equiv \frac{\partial y}{\partial k} = \alpha A \tau^{1-\alpha} \equiv r \overbrace{(\tau)}^{+}, \text{ and } r^{k}(\tau) = [r(\tau) - \tau]. \tag{6}$$

In addition, the post-tax rental rate of labor reads

$$w \equiv \frac{\partial y}{\partial \ell} = (1-\alpha) A \tau^{1-\alpha} k \equiv \omega \overbrace{(\tau)}^{+} k, \text{ and } r^{\ell}(\tau) = \omega(\tau) k. \tag{7}$$

Both formulas (reflecting the normalization to 1 of the aggregate labor supply) obviously apply, since both factor markets are perfectly competitive, and the neoclassical functional theory of income distribution is thus relevant in this setup; therefore, each factor obtains a gross reward equal to its marginal productivity. It is worth to remember, that only capital is taxed, at rate τ, so that its *net* marginal reward is *not* equal to $r(\tau)$ but to $[r(\tau) - \tau]$, and it will turn out to be a non-monotonic function of taxes. The wage rate, instead, increases monotonically with τ (for any given accumulated k). Intuitively, this is the reason, why "capitalists" will prefer *less* taxation than "workers:" they better internalize its cost, including the potentially harmful consequences of too much taxation on economic growth, as well as on their own welfare.[27] It is appropriate to specify, at this juncture, that the overall net income of any capitalist $i = k$ reads, in the symmetric equilibrium that we will consider,

$$y^{k} = \left(\alpha A \tau^{1-\alpha} - \tau\right) \frac{k}{\lambda^{k}}. \tag{8}$$

This expression clearly reflects that the aggregate capital stock k is evenly split among the λ^{k} equal capitalists, i.e. the stock of capital owned by a generic capitalist $i \in K$, reads $k^{i} = \lambda^{k}\ k$.

In addition, the overall net income of a generic worker ℓ reads,

$$y^{\ell} = \frac{(1-\alpha) A \tau^{1-\alpha} k}{\lambda^{\ell}}. \tag{9}$$

[26] It may be useful to remind, that since the neoclassical theory of income distribution obviously applies, the total factor income accruing to an agent, from any factor of production, is simply equal to the marginal productivity of that factor of production, times its personal endowment of that same factor. Also, because of Euler's theorem, all output is exhausted by rewarding all the factors of production, that are priced according to their marginal productivity (i.e. there is no left-over income to deal with).

[27] Notice that, while taxes are in principle unrestricted (i.e. they can potentially go all the way up to 100%), equation (6) makes clear that, in concrete, this is not the case. In particular, the interest rate can't be negative, of course (otherwise nobody would hold any capital), and that implies that $\tau \leq (\alpha A)^{1/\alpha} \equiv \tau^{*}$. This equation potentially introduces an endogenous "state capacity" constraint into the model; but it is not so relevant hereby, as it will never bite in practice.

This expression reflects that each worker supplies individually $1/\lambda^{\ell}$ units of labor.[28]

Of particular importance, among the menu of feasible taxes that the government can levy, is the (constant) tax rate, defined as τ^{k}, that maximizes the net interest rate (or net marginal productivity capital), as moreover (as we will demonstrate later), also the welfare of the capitalists in a hypothetical oligarchy where this class is fully in control. Such tax reads

$$\tau^{k}=\left[\alpha\left(1-\alpha\right)A\right]^{\frac{1}{\alpha}}. \tag{10}$$

Also important is the tax rate τ^{ℓ}, being constant as well, that maximizes the welfare of the (pure) workers (indicated with the superscript ℓ in the following), in absence of *any* lobbying activity by the capitalists. That is, in a political environment where workers, who are the absolute majority of the population (including the median voter) are always in power (and implement their very preferred taxation), due to the absence of any political manipulation by the capitalist minority. Hereby, therefore, the MVT applies trivially, and the tax τ^{ℓ} in question is implicitly defined by the equation[29]

$$\tau^{\ell}-\alpha A\left(1-\alpha\right)A\tau^{\ell^{1-\alpha}}=\rho\left(1-\alpha\right). \tag{11}$$

Intuitively, at $\tau=\tau^{\ell}$ the marginal gain of increasing taxation in terms of boosting the current wage rate, is just offset by the corresponding marginal loss in terms of reducing the net interest rate, and therefore rate of growth of the aggregate capital stock (and of the future wages), from the point of view of a pure worker. That is, an agent who owns no capital, does no lobbying and consumes at each time its whole wage income by assumption. Importantly, τ^{ℓ} represents the

[28] These formulas immediately reveal that the income of a capitalist is much higher than the income of a worker, as λ^{ℓ} as assumed to be much larger than λ^{k}, that, as already mention, may tend to 0 in the limit. Hence, referring to the capitalists as "rich" and to the workers as "poor," is fully justified. The functional distribution of income, and the very different size of the two classes, are the base of such justification.

[29] Formally, equation (11) emerges as the first order condition of the following problem

$$\max_{\tau}\int_{0}^{\infty}e^{-\rho t}\ln(c_{t}^{i})dt=\int_{0}^{\infty}e^{-\rho t}\left[\left(\alpha A\tau^{1-\alpha}-\tau-\rho\right)t+\ln((1-\alpha)A\tau^{1-\alpha}k_{0})\right]dt$$

$$=\frac{\alpha A\tau^{1-\alpha}-\tau-\rho}{\rho^{2}}+\frac{\ln\left((1-\alpha)A\tau^{1-\alpha}k_{0}\right)}{\rho}.$$

An expression that obviously reflects that

$$c_{t}^{i}=\ln((1-\alpha)A\tau^{1-\alpha}k_{0}e^{\left(\alpha A\tau^{1-\alpha}-\tau-\rho\right)t}).$$

It is straightforward to verify that the first order condition for the last equation corresponding to the integral, with respect to τ, or

$$\tau\left[1-\left(\alpha\left(1-\alpha\right)A\right)\tau^{-\alpha}\right]=\rho\left(1-\alpha\right).$$

is equivalent to equation (11) with $\tau=\tau^{\ell}$. It can be verified, in addition, the relevant second order condition for a maximum point is satisfied.

highest level of redistributive taxation supported by the political-economic system, as it reflects the pure fiscal policy preferences of the virtual median voter; relatively to such benchmark, policy can only be distorted downwards, by the exercise of political pressure by the capitalist elite on the government.[30]

2.1 Political Process and Lobbying Technology

As anticipated in the Introduction, we consider as basic political framework, a democracy originally based on the *de jure* power only, but eventually turning in a captured democracy, due to the ongoing growth of the *de facto* power of the rich elite. Therefore, the political process is only partially based on majority voting, and the government in office (not explicitly modeled), represents imperfectly a virtual median voter. Democracy potentially evolves according to the lobbying activities performed by the rich elite on the government (tending to increase over time), that shift the balance of overall power towards the latter class. The rich elite gradually acquire more *de facto* political power, whereas formal institutions don't change.

The representative government sets the capital tax rate τ, whose revenues are used to finance the provision of productive public good g.[31] Such policy has different effects: first and foremost, it allows the economy to grow endogenously, by making the production function linear in k (see equation (4)). In addition, it affects in a non-trivial way, the functional distribution of income and factor shares: the rate of reward of capital income may increase or decrease with

[30] Equation (11) represents the special case of a more general equation reported in Alesina and Rodrik (1994, equation (15), p. 474), obtaining for $\sigma_0^i = \infty$, and defining implicitly the preferred tax policy of the generic individual i with $\sigma^i \in [0, \infty)$. The equation in question in Alesina and Rodrik's reads

$$\tau^i \left[1 - \alpha A (1-\alpha) \tau^{i^{-\alpha}}\right] = \theta^i \left(\tau^i\right) \rho (1-\alpha),$$

with

$$\theta^i \left(\tau^i\right) \equiv \frac{\omega (\tau^i) \sigma^i}{\omega (\tau^i) \sigma^i + \rho}.$$

It can be demonstrated that, in agreement with Meltzer and Richard's (1981), the tax rate τ^i increases with the distance between the income of median and the mean voter, where i represents the median voter.

Our equation (11) reported in the main text can be regarded as a special case of the Alesina and Rodrik's (1994) equation reported above, obtaining for $\sigma^i \to \infty$ (i.e. workers have no capital income's endowment at all). Notice that, in this case $\theta^i \left(\tau^i\right)$ converges to 1 for any τ^i. Interestingly, it can be demonstrated that the preferred tax rate of a pure worker (or the tax rate implemented by what we may call a "left-wing populist" government in absence of lobbying), also leads to positive long run growth. This specific voter and its own government rationally understand that: wages (like gross interest rates) depend positively on taxes, but wages (unlike gross and interest rates) also depend positively on capital. Therefore, a pure worker uses taxes to both boost its own static wage income, and to promote capital accumulation, in order to increase its future path of labor income, depending on the future path of k. This is also the reason why expropriating entirely the capitalists, a policy that would obviously stop growth altogether, is not a desirable policy, even for people owning no capital whatsoever.

[31] This occurs at balanced budget; see equation (3).

it (since it is "humped shaped" in the tax rate), whereas the wage rate is always increasing in τ, for any given k. These different effects generate a fundamental distributive conflict between capitalists and workers. Just as in Alesina and Rodrik (1994), the former would like taxes to be set at the level, that just maximizes the economic growth rate (i.e. the net interest rate). Workers instead, would like taxes to be set at a higher level, and are prepared to trade-off some growth with a static expansion of their wage income.

This conflict is resolved by the postulated political process, reflecting an *ad hoc* "generalized" democratic decision-rule, (as opposed to a full-fledged dynamic political game), that gives weight both to the preferences of the mass of workers and to the small rich elite. Crucially, such weight is endogenous, and depends on the overall political influence effort of the rich elite.

Specifically, we assume the existence of a "tax function," $\tau(\cdot) : \mathbb{R}_+ \to [0,1]$, depending on the total pressure, $P_t^{\mathcal{K}}$, exercised at each point by the capitalist class (and defined more formally below), that has the following properties. It is a smooth, everywhere strictly decreasing function, featuring diminishing returns to scale, i.e. $\tau'(\cdot) < 0$ and $\tau''(\cdot) > 0$. In addition, the following "initial condition" and "limit condition" at the boundary of its domain are satisfied,

$$\tau\left(P_t^{\mathcal{K}} = 0\right) = \tau^{\ell}, \tag{12}$$

and

$$\lim_{P_t^{\mathcal{K}} \to \infty} \tau\left(P_t^{\mathcal{K}}\right) = \left[\alpha\left(1-\alpha\right)A\right]^{\frac{1}{\alpha}} \equiv \tau^{k}. \tag{13}$$

Some comments are necessary here to explain the assumptions made above. Because, unlike in Alesina and Rodrik's (1994), we are now in a partially captured democracy, where the median voter theorem, henceforth MVT,[32] does not apply anymore, together with the one man-one vote principle, underpinning it. Rather, fiscal policy reflects a "compromise" between the ideal policy of the (pure) workers, and of the (pure) capitalists. Workers trivially include the median voter, since they are all identical and make up for more 50% of the "electorate." Capitalists, instead, are exclusively concerned with the maximization of the economy's growth rate, equivalent to the maximization of their own welfare (just as in Alesina and Rodrik's paper). Such compromise reflects the relative political power of the two social classes in question. Crucially, this is *endogenous* due to the potential influence activity on the government exercised by the "capitalist class." In particular, as equation (12) highlights, the political process implements, in absence of any lobbying activity by the capitalists, the very preferred policy of

[32] See Austen-Smith and Banks (2005) for an excellent introduction to social choice theory, and discussion of the median voter theorem.

the workers, τ^{ℓ}. This policy simply corresponds to the *virtual* median voter equilibrium of the two-classes society we are considering. However, as the capitalists keep accumulating wealth, they will also invest more and more in lobbying, in order to reduce the rate of capital taxation. The model flexibly accommodates the corresponding increment in their *de facto* political power, at the expenses of the *de jure* political power of the workers. In other words, political institutions endogenously change in a peculiar sense. While formal political institutions (i.e. democracy) don't change, the real overall political power's allocation features a smooth transition from the one obtaining in a pure, or constitutional, democracy, to the one emerging in a partially captured democracy. Such regime, featuring a time-varying mixture of *de jure* and *de facto* political power of the two social classes, will eventually become entirely hegemonized by the rich elite in the very long run, due to the dynamics of their political pressure (that will grow without bound).

In particular, in the long run, the political process will implement a rate of capital taxation equal to the very preferred tax rate of the capitalist, τ^{k}, i.e. the one maximizing the net marginal productivity of capital (see equation (6)). This policy emerges in what we may term a pure capitalists–dominated technocratic regime. Or, alternatively, an oligarchic technocracy, i.e. a political regime solely concerned with economic growth maximization or, equivalently, with the maximization of the intertemporal welfare of the pure capitalists. In order words, the captured democracy effectively evolves in the long run into the government of the capitalists only. This occurs hen the capitalists become rich enough to obtain the full control of the political system. We remark that in our setup, the rich do achieve this goal by using their own means only, i.e. money and other financial resources, as opposed to any kind of violent activity (exercised, for example, by forming a coalition with the army or paramilitary troops[33]).

It is convenient to introduce here the *particular* specification for the tax function, that we shall adopt at some point. The functional form assumed entails no substantial loss of the generality of any of the results, considerably simplifying the model's solution,[34] and can be written as

$$\tau\left(p_t^i + P_t^{\mathcal{K}\setminus\{i\}}\right) = \tau^k + \left(\tau^{\ell} - \tau^k\right)\exp\left(-P_t^{\mathcal{K}\setminus\{i\}} - p_t^i\right) = \tau^k + \Delta\exp\left(-P_t^{\mathcal{K}\setminus\{i\}} - p_t^i\right). \quad (14)$$

[33] On civili-military politics and political transitions see, for example, Acemoglu, Ticchi and Vindigni (2010).

[34] In particular, the specification in expression (14) will be useful at some point to control the ratio $\tau''\left(p_t^i\right)/\tau'\left(p_t^i\right)$, thereby avoiding a potentially troublesome form of indetermination in computing an important limit. Any other functional form, achieving the same result, is essentially equivalent, in terms of the model's solution.

where p_t^i denotes the lobbying spending of capitalist i at time t, and

$$P_t^{\mathcal{K}\setminus\{i\}} \equiv \sum_{j\in\mathcal{K}\setminus\{i\}} p_t^j, \tag{15}$$

denotes the total pressure exercised at time t by the whole capitalist class, with the *exception* of the representative capitalist i, that takes the former as *given*.[35] The *total* political pressure (already introduced informally above in equation (13)) exercised by the capitalists class as whole at time t, and additive in all of its components, will instead be denoted as

$$P_t^{\mathcal{K}} = \sum_{j\in\mathcal{K}} p_t^j. \tag{16}$$

We will naturally focus on a symmetric equilibrium, where all the (identical) capitalists make at each time the same decisions. Such symmetry assumption implies, in particular, that each and all of λ^k capitalists present in the economy, will choose the same lobbying effort, equal to p_t^i. Therefore we can write that, in equilibrium, we have that

$$P_t^{\mathcal{K}} = p_t^i + P_t^{\mathcal{K}\setminus\{i\}} = \lambda^k p_t^i, \tag{17}$$

Henceforth, we will refer to taxation as $\tau\left(p_t^i\right)$, rather than as $\tau\left(P_t^{\mathcal{K}}\right)$, whenever that causes no confusion, in order to make the notation less heavy.

Equation (14) comprises in addition the term Δ, which is defined as

$$\Delta \equiv \tau^\ell - \tau^k, \tag{18}$$

and reflects the extent of potential policy *polarization*, i.e. the difference between the "ideal" taxes of the workers (τ^ℓ), and of the capitalists (τ^k) respectively.[36]

Expression (14) has an interesting interpretation as a weighted average of the preferred tax of the pure capitalists, and of the pure workers. Such taxes are endogenously weighted by one factor, $\exp\left(-P_t^{\mathcal{K}\setminus\{i\}} - p_t^i\right)$, reflecting the endogenous degree of political pressure put by the economic elite on the government. So that, as already mentioned above, in absence of any lobbying activity whatsoever by the rich, the MVT applies, as it always does in the model of fully consolidated, constitutional democracy proposed by Alesina and Rodrik (1994). As political pressure increases, equilibrium taxes decrease. It is possible to demonstrate that

[35] Notice that the use of the summation symbol reflects that the number of capitalists is finite.

[36] We remind that τ^ℓ and τ^k represent the taxes chosen in a pure democracy (i.e. in absence of any lobbying by the capitalist) by the median voter, and in an oligarchy where the capitalists have all power. Both happens to be constant (see Alesina and Rodrik, 1994).

they converge to the preferred taxes of the rich, once the economy reaches its balanced growth path, as lobbying activity eventually becomes infinite.[37]

It is interesting to remark that, according to expression (12), higher inequality leads to higher taxation, for any *given* level of political pressure. This is because the preferred tax τ^{ℓ} of the virtual median voter increases with inequality, according to the standard Meltzer and Richard's (1981) logic. On the contrary, the preferred tax rate of the capitalists τ^k depends only on technological parameters. In particular, it is independent from the preferences parameters such as ρ, as well as on inequality index), due to its purely technocratic nature, i.e. growth maximizing. Therefore, again for any given (finite) p_t^i, inequality unambiguously increases taxation in expression (14). Nevertheless, higher inequality induces the rich to lobby more to protect their wealth, leading to an *a priori* ambiguous overall impact of inequality on taxes. As a result, the impact of inequality on economic growth, is also ambiguous off the balanced growth path.

It can be verified that expression (12) satisfies all the assumptions made, concerning the tax function $\tau(\cdot)$. In particular, $\tau(\cdot)$ decreases with p^i because $\tau^{\ell} > \tau^k$; in addition, $\tau_p'(\cdot)$ decreases with p^i, but at an increasing rate, since, the following formulas apply

$$\tau_p'\left(p_t^i\right) = -\Delta \exp\left(-P_t^{\mathcal{K}\setminus\{i\}} - p_t^i\right) < 0, \tag{19}$$

and

$$\tau_{pp}''\left(p_t^i\right) = \Delta \exp\left(-P_t^{\mathcal{K}\setminus\{i\}} - p_t^i\right) > 0. \tag{20}$$

3 The Political-Economic Optimization Program of Capitalists

The program, that capitalist i solves, consists in maximizing its discounted lifetime utility, given the static constraint reflecting its present income, and the dynamic constraint representing the evolution of its wealth. At each point in time, this reflects its endowment of capital, its consumption decision, its *lobbying effort* and the government's policy, that now depends both on the preferences of the mass of workers, and on the preferences of the capitalist class, together with their lobbying activity. As we already know, the government's policy consists in the tax rate τ_t levied on capital income at time t, in order to finance the provision of the productive public good g_t. However, because taxes depend *inter alia*, on lobbying by the capitalists, they are not taken as given anymore by the individuals (as they are in Alesina and Rodrik, 1994);

[37] However, investment in lobbying stops increasing in balanced growth, and therefore becomes negligible with respect to the growing variables, such as capital and consumption.

they will vary over time (hence the use of the time subscript in the notation), reflecting the potential variation of the intensity of the lobbying activity.[38] Assuming, as already mentioned (recall equation (1)), logarithmic preferences, and a discount rate ρ of future welfare, the generic capitalist i solves the following problem

$$\max_{\{c_t^i, p_t^i\}} U_0^i\left(\{c_t^i\}\right) = \int_0^\infty e^{-\rho t} \ln(c_t^i) dt, \tag{21}$$

subject to the static and dynamic budget constraint of the same individual, that read, respectively,

$$y_t^i = \left[r\left(\tau_t\right) - \tau_t\right] k_t^i. \tag{22}$$

and

$$\frac{dk_t^i}{dt} = \left[r\left(\tau_t\right) - \tau_t\right] k^i - c_t^i - p_t^i. \tag{23}$$

Equation (23) is the differential equation describing the evolution of the capital stock owned by a capitalist. We remind that all of them are initially equal, and remain equal in the symmetric equilibrium we will focus on, just like the workers; furthermore all of them own only capital income. Its right-hand-side includes its income, that is simply equal to its post tax capital income, net of its consumption, and net of the lobbying expenditures p^i incurred to influence government's fiscal policy and the rate of capital taxation, in particular. Using equation (6), we write equation (23) in the growth rate form

$$\frac{\dot{k}_t^i}{k_t^i} = \left[aA\tau^{1-\alpha}\left(p_t^i\right) - \tau\left(p_t^i\right)\right] - \frac{c_t^i}{k_t^i} - \frac{p_t^i}{k_t^i}. \tag{24}$$

This is one of the main innovations of our model, compared, for instance, to Alesina and Rodrik (1994): taxes *are not* anymore taken as given and perceived to be constant by the individuals, but reflect the (time-varying) lobbying activity done by the rich elite.

The description of the capitalists program is completed by the writing of the usual transversality condition, establishing that the shadow value of k_t^i must by asymptotically nil, or

$$\lim_{t\to\infty} \mu_t k_t^i = 0,$$

Finally, we assume that the stock of initial capital is given, and equally distributed among capitalists, i.e. we have that

[38] As we shall see, taxes will be constant in the balanced growth path, eventually reached by the economy, after experiencing a process of transitional dynamics. When the economy is off the balanced growth state, consumption and capital grow at a different, and time-changing, rate. Taxes, as already mentioned, are also not constant, and public spending and lobbying (both as a share of capital) aren't.

$$k_0 = \lambda^k k_0^i > 0, \text{ with } k_0^i = k_0^j, \forall\ i \text{ and } j \in \mathcal{K}, \text{ given.}$$

Moving forward, by standard arguments (i.e. Pontryagin's Maximum principle[39]), the capitalists solve their dynamic optimization problem by maximizing the following Hamiltonian function

$$\mathcal{H} = e^{-\rho t} \ln(c_t^i) + \mu_t \left\{ \left[aA\tau^{1-\alpha}\left(p_t^i\right) - \tau\left(p_t^i\right) \right] k_t^i - c_t^i - p_t^i \right\} \ . \tag{25}$$

The standard conditions leading to the maximization of the Hamiltonian function above, include the first order condition for consumption, or

$$\frac{\partial \mathcal{H}}{\partial c_t^i} = e^{-\rho t} \frac{1}{c_t^i} - \mu_t = 0, \tag{26}$$

a condition leading to the law of motion of consumption itself, depending on the dynamics of the Hamiltonian multiplier μ, so that

$$\frac{\dot{c}_t^i}{c_t^i} = -\frac{\dot{\mu}_t}{\mu_t} - \rho. \tag{27}$$

In addition, we have a (novel) first order condition, regarding the new control variable, represented by the political pressure or intensity of the lobbying activity exercised by the capitalists on the government, and reading

$$\frac{\partial \mathcal{H}}{\partial p_t^i} = \mu_t \left\{ \left[\alpha\left(1-\alpha\right) A\tau^{-\alpha}\left(p_t^i\right) - 1 \right] \tau_p'\left(p_t^i\right) k_t^i - 1 \right\} = 0. \tag{28}$$

Furthermore, the solution of the dynamic program in question requires that the co-state variable μ satisfies the following differential equation

$$-\dot{\mu}_t = \mathcal{H}_{k^i}, \tag{29}$$

an equation leading to the following differential equation for the Hamiltonian multiplier μ,

$$-\frac{\dot{\mu}_t}{\mu_t} = \left[aA\tau^{1-\alpha}\left(p_t^i\right) - \tau\left(p_t^i\right) \right]. \tag{30}$$

3.1 Towards the Full Solution of the Capitalists' Dynamic Optimization Problem

As, obviously, $\mu_t \neq 0$, equation (28) implies that,

$$\left\{ \left[\alpha\left(1-\alpha\right) A\tau^{-\alpha}\left(p_t^i\right) - 1 \right] \tau_p'\left(p_t^i\right) \right\} k_t^i = 1. \tag{31}$$

[39] See Liberzon (2012), for an excellent introduction to the calculus of variations and to optimal control theory.

The interpretation of this condition is straightforward: at equilibrium, the marginal gain from lobbying for a capitalist, in terms of reduction of the fiscal burden on its income (the net interest rate times the stock of capital accumulated), equals to its corresponding marginal cost of lobbying, equal to 1.

Equation (31) is of utmost importance, since it defines the political pressure schedule $p_t^i = p\left(k_t^i\right)$, as an *implicit function* of the stock of capital k_t^i; p_t^i also depends parametrically on the distance Δ (defined by equation (18)) between the tax rate preferred by the workers and by capitalists.[40] The parameter Δ, as we know, reflects the redistribution potential of a constitutional democracy (where the *de jure* political power alone always matters) vs. a fully captured democracy (where the *de facto* political power alone always matters). We further proceed to characterize some of equation's (31) most important properties. Notice firstly that, by assumption, taxes are decreasing in political pressure, i.e. $\tau_p'(\cdot) < 0$. This fact, and from equation (31) that

$$\left[\alpha(1-\alpha) A\tau^{-\alpha}\left(p_t^i\right)\right] < 1,$$

implying that, for any $p_t \in \mathbb{R}_+$, the following inequality holds

$$\tau\left(p_t^i\right) > \left[\alpha(1-\alpha) A\right]^{\frac{1}{\alpha}} \equiv \tau^k. \tag{32}$$

That is, taxation, under any finite level of lobbying, is strictly *greater* than the growth maximizing tax rate τ^k, but it is locally declining whereas the net interest rate (corresponding to the term in curly brackets in equation (31)) is locally increasing. In other words, lobbying helps aligning equilibrium taxes and net interest rates to the ideal fiscal policy of the pure capitalists, but a gap keeps existing, reflecting the (partial) persistence of the *de jure* political power of the workers, as long as the economy does not reach its balanced growth path. When this event happens, instead, it features the complete erosion of any residual formal political power of the lower class, due to the overwhelming political pressures, exercised by the rich capitalists. To such a volume of pressure, the government in office responds by implementing exactly the capitalists's ideal tax policy.

[40] We remark that equation (31) only applies to an interior solution for the lobbying effort (i.e. $p_t^i > 0$). An interior solution is, indeed, not guaranteed to always exist, as the tax function $\tau(\cdot)$ does not satisfy all of the Inada conditions. Nevertheless, it is straightforward to show that an interior solution for political pressure always obtains when k_t^i is above some threshold $\hat{k}$. By definition, at $\hat{k}$ the marginal gain from lobbying at $p_t^i = 0$, equals to the marginal cost (equal to 1). We assume that the initial capital stock k_0 is high enough to insure that political pressure is always positive, i.e. that $k_0 > \lambda^k \hat{k}$. We also remark that, if this condition is not satisfied, the economy would experience an initial period of growth where the capitalists have no de facto power at all, and the preferred policy of the policy of the median voter τ^ℓ is always implemented by the political process. All of this happens until the capital stock becomes high enough to trigger some positive investment in lobbying, according to equation (31).

We can now demonstrate two noteworthy results, respectively connecting lobbying with both capital accumulation, and the potential redistribution cleavage Δ, allowed for *across* the two extreme political environments featuring in our model.

Remark 1 *The level of political pressure exercised by the representative capitalist is a smooth function $p_t^i = p(\cdot)$ of its capital stock. The function increases with the representative capitalist's own endowment of capital, i.e. $p_k'\left(k_t^i\right) > 0$, and therefore, as economic growth progresses. Also, in the limit, it is the case that $p\left(k_t^i\right) \to \infty$, as $k_t^i \to \infty$.*

Proof. See Appendix. ■

Remark 2 *Political pressure by the representative capitalist also positively depends on Δ, i.e. the parameter reflecting the relative redistribution potential in a constitutional democracy vs. a technocracy. Therefore, we have that $p_t^i = p(\cdot;\Delta)$, with $p_\Delta'(\cdot;\Delta) > 0$.*

Proof. See Appendix. ■

An immediate consequence of Remark 1 is, that the overall political pressure put by the capitalists increases as the economy grows. Hence, richer economies experience *higher* investments in political influence by the economic elites, that lead to an expansion of the relative political power of the capitalists.[41] A *lower* degree of fiscal redistribution (hereby in the form of wage subsidization), is what ultimately follows. However, it should be added that the lower classes also potentially gain from more economic growth, since wages are a linear function of the aggregate capital stock, as we already know.[42]

Remark 2 reflects the "defensive" role of lobbying for the capitalists' economic interests: as the scope Δ of potential expropriation of the rich in a constitutional democracy vs. a

[41] Incidentally we remark that, economic growth needs not make democratic insitutions stronger. This is because, as we have just shown, growth tends to shift the balance between the *de jure* political power of the workers, and the *de facto* political power of the capitalists, reducing the relevance of the formal institutions that are the foundations of a constitutitonal democracy. This result appear not too square very well with the celebrated "modernization hypothesis," (e.g. Lipset, 1959), according to which economic growth leads to the emergence, or to the consolidation of democracy. It is worth mentioning here that the influential study of Acemoglu, Robinson, Johnson and Yared (2008), is unable to find evidence of a causal relation linking economic growth ro democratization over a relatively long period of time. They interpret this finding by arguing that the many previous studies on this matter, mistakely failed to control for country fixed effects, that can be collerated with both growth and democracy.

[42] In a large class of endogenous growth models (including ours of course), wages are linear in the aggregate capital stock. Therefore, a rapid accumulation of capital (at the expense of a lower wage lower subsidization by means of high capital taxes) is partly beneficial for the workers themselves. As equation (7) shows, the wage rate grows at the same rate of the capital stock (at least at constant taxes, that emerge in balanced growth). Therefore, a faster capital growth rate tends to enrich the workers as well, since capital accumulation by the rich, in some sense "trickles-down" on the poor themselves (see Aghion and Bolton, 1997).

technocratic oligarchy increases, the capitalists may attempt to defend their wealth by lobbying more. Indeed, it is possible to show, that the *difference* between the tax preferred by the virtual median voter, τ^{ℓ}, and the rate of overall taxation delivered by the political process, $\tau\left(p_t^i;\Delta\right)$ may increase in Δ. The delivered policy (unlike τ^{ℓ}) depends on the policy polarization parameter Δ both directly and indirectly, as in equilibrium p_t^i is a function of Δ (and $P_t^{\mathcal{K}} = \lambda^k p_t^i$). Letting,

$$\tau^{\ell} - \tau\left(\lambda^k p_t^i;\Delta\right) = \Delta - \Delta \exp\left(-\lambda^k p_t^i\right),$$

an expression resulting from a straightforward transformation of the tax function (14), and recalling that $\tau\left(p_t^i;\Delta\right) = \tau\left(\lambda^k p_t^i\left(k_t^i;\Delta\right);\Delta\right)$, we have that,

$$\frac{\partial\left[\tau^{\ell} - \tau\left(\lambda^k p_t^i\left(k_t^i;\Delta\right);\Delta\right)\right]}{\partial\Delta} = 1 + \Delta \exp\left(-\lambda^k p_t^i\right)\lambda^k \frac{\partial p_t^i}{\partial\Delta} > 0. \tag{33}$$

This result represents, in some sense, a reversal on Meltzer and Richard's (1981) canonical logic, as the tax rate actually implemented by the political process, decreases *relative* to tax rate ideally preferred by the workers (and by the virtual median voter in particular), as the policy polarization parameter Δ increases.[43] The parameter Δ may reflect, in some broad sense, the extent of inequality existing in the society.[44]

We can additionally demonstrate an important result concerning the limit behavior of political pressure, as $k_t^i \to \infty$. This result will be specifically useful in the characterization of the model's balanced growth path.

Remark 3 *It is the case, that the following limit result holds,*

$$\lim_{k_t^i\to\infty} \frac{\partial p_t^i\left(k_t^i\right)}{\partial k_t^i} = \lim_{k_t^i\to\infty} \frac{\partial p\left(k_t^i\right)}{\partial k_t^i} = 0. \tag{34}$$

In addition, we have the straightforward consequence that

$$\lim_{k_t^i\to\infty} \frac{p_t^i\left(k_t^i\right)}{k_t^i} = \lim_{k_t^i\to\infty} \frac{\partial p_t^i\left(k_t^i\right)}{\partial k_t^i} = 0. \tag{35}$$

Proof. See Appendix. ■

[43] Notice that, as τ^{ℓ} does not depend on Δ, $\tau\left(p_t^i\left(k_t^i;\Delta\right);\Delta\right)$ must necessarily fall. This is to make sure that the left-hand-side of equation (33) is positive, just as its right-had-side.

[44] This statement is not literally precise, since the tax rate τ^{ℓ} preferred by the pure workers, corresponds to an *infinite* value of $\sigma^m - 1$. Such parameter expresses the distance between the median and the mean income, i.e. the indicator of income inequality used in Alesina and Rodrik's (1994). It cannot, obviously, increase beyond infinity, that reflects the maximum possible level of inequality. On the other, the tax rate τ^k preferred by the pure capitalists, depends only on technological parameters, and doesn't depend on any inequality index.

Combining equation (27), and equation (30), we can obtain at this point the full characterization of the law of motion of the consumption of the representative capitalist, that, taking advantage of Remark 1, reads,

$$\frac{\dot{c}_t^i}{c_t^i} = \left[aA\tau^{1-\alpha}\left(p_t^i\right) - \tau\left(p_t^i\right)\right] - \rho = \left[aA\tau^{1-\alpha}\left(p\left(k_t^i\right)\right) - \tau\left(p\left(k_t^i\right)\right)\right] - \rho. \tag{36}$$

Notice that equation (36) reflects the preliminary fact, that taxes are a function of political pressure (see (14)), but political pressure is obviously also endogenous, and determined in model's dynamic equilibrium. In addition (see equation (24), we have that the dynamics of k_t^i follows the dynamic rule

$$\frac{\dot{k}_t^i}{k_t^i} = \left[aA\tau^{1-\alpha}\left(p_t^i\right) - \tau\left(p_t^i\right)\right] - \frac{c_t^i}{k_t^i} - \frac{p_t^i}{k_t^i} \tag{37}$$

$$= \left[aA\tau^{1-\alpha}\left(p\left(k_t^i\right)\right) - \tau\left(p\left(k_t^i\right)\right)\right] - \frac{c_t^i}{k_t^i} - \frac{p\left(k_t^i\right)}{k_t^i}.$$

At this point, we have characterized the dynamic evolution of capitalists' consumption and investment (equations (36) and (37)), as well as the level of lobbying activities performed by each of the capitalists, and the corresponding taxes implemented by the government (as well as the level of productive public good provision), as a function of accumulated capital stock by capitalists.

In the present setup, however, unlike in Alesina and Rodrik (1994) and other classic models of endogenous growth (e.g. Romer 1986 and 1990, Barro 1990), the economy doesn't immediately reach its balanced growth path, but experiences a transitional dynamics, reflecting, that taxes, as well as lobbying activity, change over time.

To make further progress in the solution of the model, we therefore need to carefully study the behavior of the dynamical system for consumption- and capital-path describing equations (36) and (37). Such analysis is performed in the following Section.

Before doing so, a parametric restriction needs to be introduced, is order to make sure that the net interest rate remain strictly bounded from above the subject rate of time preferences, making endogenous growth possible. Because, as we have seen before, the net interest rate depends on time-varying taxes, that in turn depend on the capital stock of the economy, we could introduce such crucial restriction only at this juncture of the paper.

Condition 1 *We assume that the following condition is satisfied:*

$$R\left(k_0\right) \equiv aA\tau^{1-\alpha}\left(p\left(k_0\right)\right) - \tau\left(p\left(k_0\right)\right) > \rho. \tag{38}$$

Condition 1 states the net marginal productivity of capital strictly exceeds the rate of time preference at time $t = 0$, when $k = k_0$. Therefore, growth is possible initially (i.e. at time $t = 0$), and, due to the smoothness (that is straightforward to prove) of the function $R(\cdot)$, for values of k sufficiently close to k_0.

Importantly, the same condition turns our to hold for any k over the range $[k_0, \infty)$.

Remark 4 *The function $R = R(k)$, defined in expression (38), and representing the net interest rate, is strictly increasing in k over the range $[k_0, \infty)$. It follows that Condition 1 is always satisfied $\forall\ k \in [k_0, \infty)$ as well.*

Proof. See Appendix. ■

Naturally, this result will play a key role in making sure that endogenous growth will be possible in our model, by keeping the net interest rate always strictly bounded away from the rate of subjective time preference.

3.2 Transitional Dynamics and Model's Balanced Growth Path

Notice that, as reflected in our notation, we have at this point obtained a system of differential equations, describing the simultaneous evolution of consumption and capital, i.e. one of the two control variables, and the model's state variable. Equation (31) defines p_t^i as an implicit function of k_t^i. This means, that the model's solution leads to the equilibrium expression of the capitalist's political pressure, in the form of the function $p_t^i = p\left(k_t^i\right)$. This further implies that the pair of differential equations (36) and (37) correspond to a dynamical system in *two* variables, c_t^i and k_t^i.

Because we are dealing with an endogenous growth model, we can't look for a "steady state" in the conventional way (it doesn't exist), but we must appropriately re-normalize the system, introducing what Barro and Sala-i-Martin (2003) define "control-like" and "state-like" variables. Because both of these variables will be constant along the normalized path of balanced endogenous growth of the economy, we can look for their steady state values, and proceed, to linearize the dynamical system around its (normalized) steady state (as standard in many models of exogenous growth). This linearization enables us to ascertain the nature of the steady state, and therefore to determine the qualitative behavior of the dynamical system at hand, in a neighborhood of its rest point.

To study the system's transitional dynamics, we introduce a control-like, and a state-like variable. Specifically, we define the control-like variable x as the ratio of the consumption to the capital of the representative capitalist. A log-differentiation of x straightforwardly generates its law of motion, that reads,

$$x \equiv \frac{c_t^i}{k_t^i} \Rightarrow \frac{\dot{x}}{x} = -\rho + x + y \Rightarrow \dot{x} = (x + y - \rho)\, x = x^2 + (y - \rho)\, x. \tag{39}$$

In addition, we proceed to define the state-like variable y as the ratio of the equilibrium political pressure exercised by capitalist i, and its own capital stock. Again, a log-differentiation of y generates its law of motion, that reads,

$$y \equiv \frac{p\left(k_t^i\right)}{k_t^i} \Rightarrow \dot{y} = \frac{\dot{k}_t^i}{k_t^i}\left[p_k'\left(k_t^i\right) - y\right]. \tag{40}$$

Both equations (39) and (40) will be linearized around their steady state in the Appendix, where the saddle-path's equation, taking the economy to its steady state will also be computed.

The economy's dynamic evolution is mainly characterized by the next two Propositions.

Proposition 1 *For every initial condition, the economy converges to an unique balanced growth path, following a saddle-path, converging to the (normalized) steady state* $\{x^* = \rho, y^* = 0\}$. *In balanced growth, both the economy's growth rate and the capitalists's welfare are maximized, by taxing at the (constant) rate (10). Political pressure remains positive and infinite, but stops growing and therefore becomes negligible as compared to the accumulated stock of capital, whereas the accumulated stock of capital keeps growing at the constant equilibrium rate. This is equal to the growth rate of consumption.*

Proof. See Appendix. ■

Proposition 1 leads to the next Proposition, providing additional characterization of the economy's dynamic behavior in balanced growth.

Proposition 2 *Along the balanced growth path, the economy's stock of capital, and the consumption of all individuals (workers and capitalists alike) all grow at the constant rate*

$$\gamma^\infty = \frac{\alpha}{1-\alpha}\left[\alpha\,(1-\alpha)\,A\right]^{\frac{1}{\alpha}} - \rho > 0. \tag{41}$$

In addition, all capitalists consume a constant fraction ρ *of their wealth, their only source of income, and all workers consume entirely their income, deriving from labor* ***only****.*

Proof. Immediate consequence of the previous Proposition. ■

Notice that the positivity of γ^∞ is ensured by Condition 1, that guarantees that initial net interest rate (i.e. the one initially applying, in correspondence of $k = k_0$, or $R(k_0)$), is higher than the subjective rate of time preference ρ. Since the net interest applying in balanced growth, and reported on the left-hand-side of inequality (41), is strictly greater than $R(k_0)$, Condition 1 clearly implies that inequality (41) holds $a\ fortiori$.[45]

The main result, conveyed by the two just stated Propositions, is thus that the dynamic equilibrium, obtained along the balanced growth path, almost entirely coincides with the equilibrium, obtained in an economy always ruled by a technocratic-oligarchic regime (at no cost). Under this regime the capitalist elite have all political power, captured by lobbying (i.e. the pivotal "voter" has $\sigma^i = 0$), and can therefore implement their preferred fiscal policy. So, for example, on the transitional path the consumption of the representative capitalist (as a share of its wealth) is equal to $c_t^i/k_t^i = \rho - p\left(k_t^i\right)/k_t^i$, and this implies that part of the cost of lobbying is absorbed though a reduction of consumption (and the remaining part through a reduction of investment, of course). This expression eventually leads to the equality $c_t^i/k_t^i = \rho$, obtaining as the balanced growth path is reached, corresponding to the consumption function of any pure rich in Alesina and Rodrik (1994).[46]

3.3 Capital Income Share and Inequality Dynamics

Inspired by Piketty (2014), Milanovic (2014, 2016, 2023), and Bengtsson and Waldenström (2018), we characterize the evolution of the ratio θ_t^k between the (post-tax) income of the capitalist class as a whole, and the total (post-tax and public spending) income of workers and capitalists combined (or the post-tax income share of capital), that reads[47]

$$\theta_t^k = \frac{\left(\alpha A \tau_t^{1-\alpha} - \tau_t\right) k_t}{\left[\left(\alpha A \tau_t^{1-\alpha} - \tau_t k_t\right) + \tau_t k_t + \frac{\lambda^\ell (1-\alpha) A \tau_t^{1-\alpha} k_t}{\lambda^\ell}\right]} = \frac{\alpha A \tau_t^{1-\alpha} - \tau_t}{A \tau_t^{1-\alpha}} = \alpha - \frac{\tau_t^\alpha}{A}. \tag{42}$$

Proposition 3 *The ratio θ^k between the (post-tax) income of the representative capitalist, and the total (post-tax) income of the economy, expressed by equation 42, increases during the transitional phase until the balanced growth path is reached, where it becomes constant.*

Proof. See Appendix. ■

[45] We remind that the tax rate obtaining as lobbying goes to infinity (and preferred by the capitalists over any other tax), maximizes the net interest rate. This follows from the assumption stated in (13), and it explains the form of the net interest rate reported in (41), as well as why inequality (41) holds, making endogenous growth possible.

[46] This result follows from the equalization of the consumption and wealth growth rate along the balanced growth path, that is ultimately a consequence of Uzawa's theorem. See Acemoglu (2009) on this matter.

[47] Notice that the denominator of this fraction reflects that the term $\tau_t k_t$ appears twice: as a tax on capital income, and as "reward" of the factor of production g.

This result, that highlights the fact that factor shares can change over time, depending on the dynamics of taxes on and off the balanced growth path. Off the balanced growth path, taxes are decreasing because of the increasing political pressure put by the capitalists on the government; thereby the (net) interest is increasing, and so are the incentives to save. This is because the increasing political pressure of the capitalists causes their social weight to increase, making fiscal policy becomes more and more conservative; this raises, in turn, the share of (post-tax) income accruing to capital. This finding is significantly related to some recent literature (e.g. Bengtsson and Waldenström, 2018, in particular), which documents the existence, over the long run, of robust co-movements between the capital income share and income inequality (whether measure by top income shares or by the GINI coefficients). Bengtsson and Waldenström conclude (p. 741, 2018) that, "With our newly compiled long-run dataset, we have shown that capital shares and income inequality are correlated, even if this relationship varies by region as well as between different time periods. Overall, the results yield support to assertions that the capital-labor split is an important determinant of inequality." While, as the authors acknowledge, the mechanism explaining this association needs to be further investigated, our paper suggests that the politics fiscal policy may be part of the story. In particular, our model suggests the existence of a potential *complementarity* between factor-income inequality and lobbying. Democratic societies where capital income is more unevenly distributed, are likely to experience greater effort of manipulation of the political process by the capitalist elite, that may further exacerbate inequality, producing a potentially dangerous loop of capital income concentration, as well as political power, at the top of the society.

In addition, Elsby, Hobijn, and Sahin (2013), show that between 1980 and 2013, the capital income share of net income has increased from 35 % to 40%. Interestingly, the period in question coincides with the time of major increment in interpersonal inequality observed in the U.S. While a higher capital income share does not, in principle, necessarily lead to a higher interpersonal inequality, it does so where, like in most modern capitalists societies (as well as in our model), the property of capital is strongly concentrated in the hands of a few rich capitalists (see also Milanovic, 2016).

On the balanced growth path, instead, taxes are constant, at the specific level preferred by the capitalists. This result reflects that the political pressure exercised by the capitalists, is now constant, and equal to zero as a share of the capital stock, that is instead ever growing. Therefore, the capitalists' social weight becomes permanently constant as well, at the maximum possible level (i.e. the *de jure* political power of the workers becomes virtually irrelevant). Importantly, the Appendix shows that, as workers become gradually politically less irrelevant, the income share accruing to labor remains constant over time, positive and equals to $\theta^{\ell}_{\infty} = (1-\alpha)$. Instead, as also shown in the Appendix, the income share corresponding to the public good g, declines over time, but converges to $\theta^{g}_{\infty} = \alpha(1-\alpha)$. Most importantly, we show in the Appendix that[48]

$$\theta^{k}_{\infty} \equiv \lim_{t\to\infty} \theta^{k}_{t} = \alpha^2. \tag{43}$$

This result is especially important since it implies that capital accumulation does not eventually swallow-up all output. Such outcome would occur, for instance, in Piketty's basic framework, in absence of major exogenous shocks such as wars, or of a drastic redistribution of income triggered by a potential "revolution threat" eventually posed to the capitalists by the workers. In our theory, taxes reduces (after some point), the interest rate, and growth itself, that are clearly jointly endogenous, and cannot grow without bound. This mechanism keeps in check the power of capitalists, which cannot become "excessive," unless limited by some traumatic exogenous event, shocking the economy[49]

4 Conclusion and Directions for Future Work

We have presented an endogenous growth with initial political institutions corresponding to a constitutional democracy. Hereby, power's nature and origin is mostly *de jure*, and the majority of the citizenry (i.e. the median voter) is fully in control.[50] However, the balance between the *de jure* and the *de facto* political power changes endogenously, along the equilibrium growth path, as economic development gives to the capitalist elite the incentive to invest more and more in the *de facto* political power. This lobbying activity is implemented in order to prevent both the expropriation of capitalist elite's wealth and the related factor-income redistribution in favor of wages. In the end, the rich minority ends up being fully in control of the polity, and implements a technocratic policy, featuring limited redistribution to the workers. In this context the main holder of the *de jure* power (i.e. the median voter)

[48] Obviously, we then also have that the sum of the income shares related to the three factors of production used, equals to one in balanced growth, since we have that

$$\theta^{k}_{\infty} + \theta^{\ell}_{\infty} + \theta^{g}_{\infty} = 1.$$

[49] See Milanovic (2023, Epilogue, pp. 292-293) for a discussion of this issue.

[50] Furthermore, democratic institutions are "fully consolidated," in the sense that they are not threatened by any form of potential drastic change (e.g. a revolution or a military *coup d'etat*).

politically becomes almost irrelevant.

The presented model has a number of limitations and shortcuts, that could be addressed in potential future research. Firstly and foremost, the assumption of one-sided lobbying by the rich elite only could generalized. I regard this assumption as potentially plausible in certain environments, where for example the working class is disporportionaly large, uneducated, and with limited "class consciousness," combined with poor leadership.[51] Elsewhere, the assumption in question may prove less appealing, and a generalization of the model allowing for two-sided lobbying may appear more reasonable.

The model adopts the Kaldor-Pasinetti type of assumption: according to this assumption workers never save. Again, while some arguments may be made to justify such assumption in our context and even more generally (see Gomes, 2000, and the references cited therein), its generalization might be desirable in future some work, treating workers and capitalists more symmetrically. Indeed, our result that economic growth leads to a smooth empowerment of a small rich elite, should be taken with some caution, despite being $prima\ facie$ consistent with the important patterns of inequality dynamics and redistribution observed in many democracies in the last few decades (e.g. the raise of "fiscal conservatism" or the "raise of the top 1%"). Hereby, we have treated workers as a purely passive subject, whereas a prospective model's extension, allowing for instance for bilateral lobbying, might partially generalize this result, giving to the workers some more political voice. In addition, the progressive empowerment of the capitalists increases the interest rate and boosts the incentive to save and economic growth. As already mentioned in the Introduction, I do not regard this result as very general, since an excessive empowerment of the capitalists (or of part of them) may be detrimental for growth in different setups. For instance, inequality can excessively empower incumbent innovators in a Schumpeterian model of growth, thereby slowing down the process of creating destruction (e.g. Aghion and Howitt, 1992). In addition, inequality can make politically more difficult to implement efficiency enhancing redistributive policies (e.g. Bénabou, 2000). In potential future work, it would be interesting, not just to generalize the one-sided lobbying assumption formulated in our benchmark setup, but allowing for more dynamic decisions by workers, for instance regarding saving and human capital accumulation.

Finally, it would be interesting to understand how a state's fiscal capacity constraint, potentially binding for workers (if they had full political power), would affect the lobbying, redistribution and growth patters in the model. This constraint could therefore lead to a

[51] All these elements contribute to the explanation of why the working class is unable get organized in a lobby as the capitalists are.

reduction of the tax rate demanded by the virtual median voter, as well as the overall fiscal revenues collected by the government.

Presumably, if the capitalists faced a lower fiscal redistribution potential threat by the workers,[52] their incentive to invest in the *de facto* political power would to be lower, allowing the virtual median voter and raise its voice in front of the government. Yet, taxes and (indirect) redistribution through higher wages (for given k), may not increase beyond a certain point, due to the relative state's fiscal weakness. Growth, on the other hand, could potentially *increase* to some non-negligible extent, as a result: capitalists would *at one time*, invest less in wasteful *de facto* power acquisition (see also Barro, 2000), *and* experience lower taxation.[53] If this conjecture was correct, the conclusion would follow: that a weaker state, compared to a stronger state, might lead to a *higher* economic growth rate. In a strong state realm, the virtual median voter may not be able to commit to demand a lower tax rate than its (relatively high) preferred excise, thereby forcing the elite to engage in wasteful influence expenditures. On the other hand, one should bear in mind that, in principle, the state capacity constraint could actually be too stringent. Just as it is in a failed or quasi-failed state, located outside the "narrow corridor of liberty" (see Acemoglu and Robinson, 2019), whereby the failure of the state to provide valuable public goods, can drastically limit, or even almost inhibit economic growth at all. A tentative conclusion in this regard might therefore be, that an intermediate level of fiscal capacity, might be preferable, in some circumstances, to both a very high and a very low one.

[52] Due to the limited ability of the state to tax.

[53] Acemoglu (2010) develops a somewhat related point, illustrating some potential disadvantages for the society of a too much strong state, in terms of overinvestment of resources for the purpose of state capture.

5 Appendix

5.1 Proof of Remark 1

To make progress in the proof, and to simplify the exposition, let us define the following expression, which refers to the inequality (31) reported in the main text, evaluated in the symmetric equilibrium we are considering[54]

$$B\left(\lambda^k p_t^i\right) \equiv \left[\alpha\left(1-\alpha\right) A\tau^{-\alpha}\left(\lambda^k p_t^i\right) - 1\right] < 0, \tag{44}$$

which highlights that, *in our symmetric equilibrium*, total political pressure at each time is $\lambda^k p_t^i$.

For future reference, we remark here that, around the equilibrium, we have that

$$B'\left(\lambda^k p_t^i\right) = -\alpha^2\left(1-\alpha\right) A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right)\lambda^k > 0. \tag{45}$$

The Implicit Function theorem, and in particular the implicit differentiation of equation (31) with respect to k_t^i, around the equilibrium point, imply that

$$B\left(\lambda^k p_t\right)\left[\tau'\left(\lambda^k p_t^i\right)\right]^2 \frac{\partial p_t^i}{\partial k_t^i}\lambda^k k_t^i + \left[-\alpha^2\left(1-\alpha\right) A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right) - 1\right]\tau''\left(\lambda^k p_t^i\right)\lambda^k \frac{\partial p_t^i}{\partial k_t^i} k_t^i$$
$$+ B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right)\lambda^k = 0,$$

an expression which implies that political pressure increases with capital, or[55]

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right)}{\left\{B\left(\lambda_t^k p_t^i\right)\left[\tau'\left(\lambda^k p_t^i\right)\right]^2 + \left[-\alpha^2\left(1-\alpha\right)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right) - 1\right]\tau''\left(\lambda^k p_t^i\right)\right\} k_t^i} > 0. \tag{46}$$

It is convenient to re-write expression (46), dividing numerator and denominator by $\tau'\left(\lambda^k p_t^i\right)$, as

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)}{\left\{B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right) + \left[-\alpha^2\left(1-\alpha\right)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right) - 1\right]\tau''\left(\lambda^k p_t^i\right)/\tau'\left(\lambda^k p_t^i\right)\right\} k_t^i}. \tag{47}$$

Using the equation (14) introduced earlier, it is easy to verify that, around the equilibrium we have that

$$\tau'\left(\lambda^k p_t^i\right) = \left(\tau^k - \tau^\ell\right)\exp\left(-\lambda^k p_t^i\right)\lambda^k,$$

---

[54] The sign of B is obviously negative since $\tau\left(p_t^i\right)$ is greater than τ^k, B is (see below) decreasing in τ, and, finally B is equal to zero when $\tau = \tau^k$.

[55] Notice that both the numerator and the denominator of the following expression are negative. Also, the terms λ^k reported outside the parenthesis all cancel out.

and that

$$\tau''\left(\lambda^k p_t^i\right) = -\left(\tau^k - \tau^\ell\right)\exp\left(-\lambda^k p_t^i\right)\left(\lambda^k\right)^2.$$

It follows that $\forall\ p_t^i$, we have that $\tau''\left(\lambda^k p_t^i\right)/\tau'\left(\lambda^k p_t^i\right) = -\lambda^k$, and equation (47) assumes the simpler form

$$\frac{\partial p_t^i}{\partial k_t^i} = \frac{-B\left(\lambda^k p_t^i\right)}{\left\{B\left(\lambda^k p_t^i\right)\tau'\left(\lambda^k p_t^i\right) + \left[\alpha^2\left(1-\alpha\right)A\tau^{-\alpha-1}\left(\lambda^k p_t^i\right)+1\right]\lambda^k\right\}k_t^i} > 0. \tag{48}$$

Equation (48) reflects a noteworthy result: the capitalists invest in lobbying in order to alleviate the potential fiscal pressure exercised on the them by the virtual median voter, a pressure that is increasing the richer (and the economy as whole) are of the accumulable factor of production (i.e. the higher is k_t^i), by basic Meltzer and Richard's (1981) logic. In other words, lobbying increases as the "representative" capitalist becomes richer; therefore, together the growth of its wealth, its willingness to protect it as much as possible from the government also increases.

5.2 Proof of Remark 2

We seek to compute the expression of $\partial p_t^i/\partial\Delta$.

For convenience we report again equation (31), defining implicitly the symmetric equilibrium level of taxation, $\tau\left(\lambda^k p_t^i;\Delta\right)$, as a function of the pressure exercised by capitalist i, and depending also on the exogenous "fiscal exploitation" parameter Δ or

$$\underbrace{\left[\alpha\left(1-\alpha\right)A\tau^{-\alpha}\left(\lambda^k p_t^i;\Delta\right)-1\right]}_{(-)}\underbrace{\tau'\left(\lambda^k p_t^i;\Delta\right)}_{(-)} = \frac{1}{k_t^i}. \tag{49}$$

The notation used above highlights the fact τ depends on the variable p_t^i, but also, parametrically, on Δ.

Also for convenience, we recall the equilibrium expression for $\tau\left(\lambda^k p_t^i;\Delta\right)$, or

$$\tau\left(\lambda^k p_t^i;\Delta\right) = \tau^k + \Delta\exp\left(-\lambda^k p_t^i\right). \tag{50}$$

Additionally, we remark that equation (14) implies that

$$\partial\tau\left(\lambda^k p_t^i;\Delta\right)/\partial\Delta = \exp\left(-\lambda^k p_t^i\right) > 0,\ \partial\tau\left(\lambda^k p_t^i;\Delta\right)/\partial p_t^i = -\Delta\exp\left(-\lambda^k p_t^i\right)\lambda^k < 0,$$

and that

$$\partial^2\tau\left(p_t^i;\Delta\right)/\partial p_t^{i2} = \Delta\exp\left(-\lambda^k p_t^i\right)\left(\lambda^k\right)^2 > 0,\ \partial^2\tau\left(p_t^i;\Delta\right)/\partial p_t^i\partial\Delta = -\exp\left(-\lambda^k p_t^i\right)\lambda^k < 0.$$

Differentiating implicitly expression (49) with respect to Δ, we obtain that

$$\underbrace{-\alpha^2 (1-\alpha) A\tau^{-\alpha-1}\left(\lambda^k p_t^i;\Delta\right)\left[\tau_p'\left(\lambda^k p_t^i;\Delta\right)\right]^2}_{(-)}\lambda^k\frac{\partial p_t^i}{\partial\Delta}$$

$$+\underbrace{\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i;\Delta\right)-1\right]}_{(-)}\underbrace{\tau_{pp}''\left(\lambda^k p_t^i;\Delta\right)}_{(+)}\lambda^k\frac{\partial p_t^i}{\partial\Delta}$$

$$=\underbrace{\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i;\Delta\right)}_{(+)}\underbrace{\tau_\Delta'\left(\lambda^k p_t^i;\Delta\right)\tau_p'\left(\lambda^k p_t^i;\Delta\right)}_{(-)}$$

$$-\underbrace{\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i;\Delta\right)-1\right]}_{(-)}\underbrace{\tau_{p\Delta}''\left(\lambda^k p_t^i;\Delta\right)}_{(-)},$$

which means that

$$\frac{\partial p_t^i}{\partial\Delta}=\frac{1}{\lambda^k}\frac{\left\{\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i;\Delta\right)\tau_\Delta'\left(\lambda^k p_t^i;\Delta\right)\tau_p'\left(\lambda^k p_t^i;\Delta\right)+\right.}{-\alpha^2(1-\alpha)A\tau^{-\alpha-1}\left(\lambda^k p_t^i;\Delta\right)\left[\tau_p'\left(\lambda^k p_t^i;\Delta\right)\right]^2+\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i;\Delta\right)-1\right]\tau_{pp}''\left(\lambda^k p_t^i;\Delta\right)} \tag{51}$$

$$\left.-\left[\alpha(1-\alpha)A\tau^{-\alpha}\left(\lambda^k p_t^i;\Delta\right)-1\right]\right\}\ \tau_{p\Delta}''\left(\lambda^k p_t^i;\Delta\right).$$

The sign of this expression is positive, i.e. $\partial p_t^i/\partial\Delta > 0$, as both the numerator and the denominator of this fraction are negative. We conclude that a higher value of Δ, which as we know reflects a higher potential scope of expropriation of the rich by the poor, induces the the former to invest more in influencing the government.

Next, we can attempt to determine the effect of Δ on the equilibrium difference between the tax rate potentially implemented by the virtual median voter vs. the tax rate emerging from the actual political process of a partially capture democracy. We know that

$$\tau\left(\lambda^k p_t^i;\Delta\right)=\tau^k+\Delta\exp\left(-\lambda^k p_t^i\right).$$

Subtracting τ^ℓ from both members, we obtain that

$$\tau\left(\lambda^k p_t^i;\Delta\right)-\tau^\ell=\tau^k-\tau^\ell+\Delta\exp\left(-\lambda^k p_t^i\right),$$

or, equivalently,

$$\tau^\ell-\tau\left(\lambda^k p_t^i;\Delta\right)=\Delta-\Delta\exp\left(-\lambda^k p_t^i\right),$$

an expression that implies the following result

$$\frac{\partial\left[\tau^{\ell}-\tau\left(\lambda^{k}p_{t}^{i};\Delta\right)\right]}{\partial\Delta}=1+\Delta\exp\left(-\lambda^{k}p_{t}^{i}\right)\lambda^{k}\frac{\partial p_{t}^{i}}{\partial\Delta}>0,$$

obviously equivalent to expression (33) reported in the main text.

We conclude that an increment in policy polarization index Δ, *magnifies* the difference between the virtual median voter's preferred taxes, and the actual taxes implemented by the political system under political pressure, i.e. it increases in some sense the scope and effectiveness of lobbying.

5.3 Proof of Remark 3

For convenience, we again report below equation (31), or

$$\left[\alpha\left(1-\alpha\right)A\tau^{-\alpha}\left(\lambda^{k}p_{t}^{i}\right)-1\right]\tau'\left(\lambda^{k}p_{t}^{i}\right)k_{t}^{i}=1, \tag{52}$$

as well as equation (48),

$$\frac{\partial p_{t}^{i}}{\partial k_{t}^{i}}=\frac{-B\left(\lambda^{k}p_{t}^{i}\right)}{\left\{B\left(\lambda^{k}p_{t}\right)\tau'\left(\lambda^{k}p_{t}^{i}\right)+\left[a^{2}\left(1-\alpha\right)A\tau^{-\alpha-1}\left(p_{t}^{i}\right)+1\right]\lambda^{k}\right\}k_{t}^{i}}>0. \tag{53}$$

Because both the numerator of this expression is positive, and so are each terms of the denominator, we can divide both the numerator and the denominator by $-B\left(\lambda^{k}p_{t}^{i}\right)$, and re-write the whole fraction as

$$\frac{\partial p_{t}^{i}}{\partial k_{t}^{i}}=\frac{1}{-\tau'\left(\lambda^{k}p_{t}^{i}\right)k_{t}^{i}+\frac{\left[\alpha^{2}(1-\alpha)A\tau^{-\alpha-1}\left(p_{t}^{i}\right)+1\right]\lambda^{k}}{-B\left(\lambda^{k}p_{t}^{i}\right)}k_{t}^{i}}. \tag{54}$$

Moreover, from equation (52), we can write that

$$\frac{1}{-\tau'\left(\lambda^{k}p_{t}^{i}\right)k_{t}^{i}}=\left[\alpha\left(1-\alpha\right)A\tau^{-\alpha}\left(\lambda^{k}p_{t}^{i}\right)-1\right].$$

Recall now that, by the assumption concerning the limit behavior of the tax function, we have that

$$\lim_{P_{t}^{\mathcal{K}}\to\infty}\tau\left(P_{t}^{\mathcal{K}}\right)=\lim_{p_{t}^{i}\to\infty}\tau\left(\lambda^{k}p_{t}^{i}\right)=\left[\alpha\left(1-\alpha\right)A\right]^{\frac{1}{\alpha}}\equiv\tau^{k}, \tag{55}$$

a result which also implies that

$$\lim_{p_{t}^{i}\to\infty}\left[\alpha\left(1-\alpha\right)A\tau^{-\alpha}\left(\lambda^{k}p_{t}^{i}\right)-1\right]=\lim_{p_{t}^{i}\to\infty}B\left(\lambda^{k}p_{t}^{i}\right)=0. \tag{56}$$

But, since

$$\lim_{p_{t}^{i}\to\infty}\tau'\left(\lambda^{k}p_{t}^{i}\right)=\lim_{p_{t}^{i}\to\infty}\tau'\left(\lambda^{k}p_{t}^{i}\right)=\left(\tau^{k}-\tau^{\ell}\right)\exp\left(-\lambda^{k}p_{t}^{i}\right)=0,$$

it must then be the case that the following limit result applies

$$\lim_{k_i^i \to \infty} \left[-\tau' \left(\lambda^k p_t^i \right) k_t^i \right] = \lim_{k_i^i \to \infty} \left\{ -\tau' \left[\lambda^k p \left(k_t^i \right) \right] k_t^i \right\} = \infty,$$

for otherwise equation (52) would fail to hold. Such result, in turn, implies that

$$\lim_{k_i^i \to \infty} \frac{\partial p_t^i}{\partial k_t^i} = \lim_{k_i^i \to \infty} \frac{1}{-\tau' \left(\lambda^k p_t^i \right) k_t^i + \frac{\left[\alpha^2 (1-\alpha) A \tau^{-\alpha-1} \left(p_t^i \right) + 1 \right] \lambda^k}{-B\left(p_t^i \right)} k_t^i} = 0.$$

This is the case since

$$\lim_{k_i^i \to \infty} \frac{\left[\alpha^2 (1-\alpha) A \tau^{-\alpha-1} \left(\lambda^k p_t^i \right) + 1 \right] \lambda^k}{-B \left(\lambda^k p_t^i \right)} = \infty,$$

as the denominator of this fraction tends to zero as k_t^i, and therefore p_t^i (see expression (56)). tends to infinity, whereas the numerator clearly tends clearly to a finite number.

This is what had to be demonstrated (see equation (34)).

In addition, using de l'Hospital theorem, we can prove the additional result, that will be useful in the following,

$$\lim_{k_t^i \to \infty} \frac{p_t^i \left(k_t^i \right)}{k_t^i} = \lim_{k_i^i \to \infty} \frac{\partial p_t^i}{\partial k_t^i} = 0. \tag{57}$$

5.4 Proof of Remark 4

Recall that the function $R(\cdot)$ is as smooth function of k, defined as

$$R(k) = aA\tau^{1-\alpha} \left(p(k) \right) - \tau \left(p(k) \right),$$

and it is such that $R(k_0) > \rho$. We have that

$$R'(k) = a (1-\alpha) A \tau^{-\alpha} \left(p(k) \right) \tau_p' \left(p(k) \right) p_k'(k) - \tau_p' \left(p(k) \right) p_k'(k).$$

This expression is equal to

$$R'(k) = \underbrace{\left[a (1-\alpha) A \tau^{-\alpha} \left(p(k) \right) - 1 \right]}_{(-)} \underbrace{\tau_p' \left(p(k) \right)}_{(-)} \underbrace{p_k'(k)}_{(k)} > 0,$$

where the sign of the term in squared brackets is negative since it is equivalent to the term (44), which is negative for any finite k, as we already know.

We conclude that

$$\gamma(k) \equiv aA\tau^{1-\alpha} \left(p(k) \right) - \tau \left(p(k) \right) - \rho > 0,$$

for any $k \in [k_0, \infty)$, as claimed.

5.5 Proof of Proposition 1

We seek to determine the steady state of the normalized system. We begin with the first dynamic equation, and obtain that

$$\dot{x} = 0 \Rightarrow x + y = \rho \Rightarrow x^* = \rho - y^*$$

The second equation instead has the following steady-state, $y^* = 0$, an immediate consequence of the equation (57) above.

We can now linearize the system around its steady-state, $\{x^* = \rho, y^* = 0\}$. The first linearized equation, involving x, reads (see equation (39)),

$$\dot{x} = (2x^* + y^* - \rho)(x - x^*) + x^*(y - y^*) = \rho(x - \rho) + \rho(y - 0). \tag{58}$$

In addition, we remind that the second equation of our dynamical system, involving y, reads, (see equation (40))

$$\dot{y} = \left\{ \left[aA\tau^{1-\alpha}\left(p\left(k_t^i\right)\right) - \tau\left(p\left(k_t^i\right)\right)\right] - x - y \right\} \left[p'_k\left(k_t^i\right) - y\right]. \tag{59}$$

We can then linearize this differential equation around its steady state, keeping in mind (see equation (57)) that

$$p'_k\left(k_t^i\right) \to p^\infty = 0,$$

and that (recall the assumption stated in (13))

$$\left[aA\tau^{1-\alpha}\left(p\left(k_t^i\right)\right) - \tau\left(p\left(k_t^i\right)\right)\right] \to R^\infty = \frac{\alpha}{1-\alpha}\left[\alpha(1-\alpha)A\right]^{\frac{1}{\alpha}},$$

as $k_t^i \to \infty$, so that

$$\dot{y} = -(R^\infty - \rho - 0)(y - 0) = -(R^\infty - \rho)\, y. \tag{60}$$

The determinant of the matrix J of the linearized system reads: $\det J = -\rho(R^\infty - \rho) < 0$. This means that the unique rest point of the normalized dynamical system is $\{x^* = \rho, y^* = 0\}$ and is a saddle-point. For any initial conditions of the system, there exists one and only path leading the economy to the balanced growth state.

The dynamic analysis of the model can be completed by computing the equation of saddle path, i.e. the linear manifold leading to steady state.

To obtain such last equation, we need to solve the system of differential equations just computed, namely the pair of functional equation given by equations (58) and (60). Some simple algebra shows that

$$x_t - \rho = -\frac{\rho}{\gamma^\infty + \rho} y_0 e^{-\gamma^\infty t},$$

and

$$y_t = y_0 e^{-\gamma^\infty t}.$$

Dividing member-by-member the general integrals of the two differential equations from which we departed, we finally obtain the equation of the saddle path, that reads

$$x = -\frac{\rho}{\gamma^\infty + \rho} y + \rho. \tag{61}$$

Equation (61) represents a straight line in the Cartesian space. It is easy to see that its *slope*, measuring the speed of convergence to the statedly state is decreasing in ρ and increasing in γ^∞. Both results are not surprising, as higher rate of temporal impatience clearly makes people prefer current. as opposed to future consumption. Similarly, a higher asymptotic growth rate, γ^∞, will instead speed up convergence, for the opposite reason. The economy moves along this line until the steady state $\{x^* = \rho, y^* = 0\}$ is finally reached.

5.6 Proof of Proposition 3

We remind that the income of the representative capitalist at time $t < \infty$ reads $r^k(\tau_t) k_t^i = [r(\tau_t) - \tau_t] k_t^i$. This expression reflects the factor income of capitalist i owning k_t^i units of capital at time t, when capital income is taxed at the rate τ_t, delivering a net interest rate (the rate of reward of each unit of capital) equal to $r(\tau_t) - \tau_t$. This expression is, as we know, a strictly concave function of τ_t, maximized at $\tau^k \equiv [\alpha(1-\alpha) A]^{1/\alpha}$.

also remind that the income of the representative worker at time $t < \infty$ reads $r^\ell(\tau_t) k_t = \omega(\tau_t) k_t$. This expression reflects the factor income of worker i owning 1 unit of labor at time t, when labor income (which is a linear function of the aggregate capital stock k_t) is taxed at the rate τ_t, delivering a wage rate (the rate of reward of each unit of labor) equal to $\omega(\tau_t) k_t$. This expression is, as we also already know, strictly increasing in τ_t.

Defining the ratio of the total income of capitalist class vs. the sum of total income of capitalists and workers and of the provision of the public good at time t, as θ_t^k, we have that

$$\theta_t^k = \frac{\lambda^k r^k(\tau_t) k_t^i}{\lambda^k r^k(\tau_t) k_t^i + \tau_t k_t + \lambda^\ell r^\ell(\tau_t) k_t} = \frac{\left(\alpha A \tau_t^{1-\alpha} - \tau_t\right) k_t}{\left[\left(\alpha A \tau_t^{1-\alpha} - \tau_t\right) k + \tau_t k_t + \frac{\lambda^\ell (1-\alpha) A \tau^{1-\alpha} k_t}{\lambda^\ell}\right]} = \alpha - \frac{\tau_t^\alpha}{A}.$$

In addition, we have that θ_t^ℓ and θ_t^g, that are similarly defined, read, respectively

$$\theta_t^\ell = \frac{\lambda^\ell r^\ell(\tau_t) k_t}{\lambda^k r^k(\tau_t) k_t^i + \tau_t k_t + \lambda^\ell r^\ell(\tau_t) k_t} = \frac{\frac{\lambda^\ell (1-\alpha) A \tau_t^{1-\alpha} k_t}{\lambda^\ell}}{\left[\left(\alpha A \tau_t^{1-\alpha} - \tau_t\right) k + \tau_t k_t + \frac{\lambda^\ell (1-\alpha) A \tau^{1-\alpha} k_t}{\lambda^\ell}\right]} = (1-\alpha),$$

$\forall\ t \in \mathbb{R}_+$, and

$$\theta_t^g = \frac{\tau_t k_t}{\lambda^k r^k(\tau_t) k_t^i + \tau_t k_t + \lambda^\ell r^\ell(\tau_t) k_t} = \frac{\tau_t k_t}{\left[\left(\alpha A \tau_t^{1-\alpha} - \tau_t\right) k + \tau_t k_t + \frac{\lambda^\ell (1-\alpha) A \tau_t^{1-\alpha} k_t}{\lambda^\ell}\right]} = \frac{\tau_t^\alpha}{A}.$$

Taking the time-derivative of the there expressions reported above, we obtain that

$$\dot{\theta}_t^k = -\alpha \frac{\tau_t^{\alpha-1}}{A} \dot{\tau}_t,\ \dot{\theta}_t^\ell = 0,\ ,\dot{\theta}_t^g = \frac{\tau_t^{\alpha-1}}{A} \dot{\tau}_t. \tag{62}$$

Lastly, we know that along the transitional path (see equation (13)), we have that

$$\dot{\tau}_t = \dot{\tau}\left(P_t^{\mathcal{K}}\right) = \tau_{P^{\mathcal{K}}}\left(P_t^{\mathcal{K}}\right) \dot{P}_t^{\mathcal{K}} = \tau_{P^{\mathcal{K}}}\left(P_t^{\mathcal{K}}\right) \lambda^k \dot{p}_t^i < 0.$$

This result obtains since $\tau_{P^{\mathcal{K}}}\left(P_t^{\mathcal{K}}\right) < 0$ (see equation (14)), and since $\dot{p}_t^i = p_k\left(k_i^i\right) \dot{k}_t^i > 0$ and $\dot{k}_t^i > 0$, on and off-the balanced growth path. We conclude that the right-hand-side of equation (62) is positive. Therefore, the post-tax capital income share is increasing along the transitional path to balanced growth (i.e. $\dot{\theta}_t^k > 0$), and the income share corresponding to the productive public good is decreasing (i.e. $\dot{\theta}_t^g < 0$), along the same path.

Furthermore, because

$$\lim_{t\to\infty} \tau_t = \lim_{t\to\infty} \tau\left(P_t^{\mathcal{K}}\right) = \tau^k \equiv \left[\alpha\left(1-\alpha\right) A\right]^{1/\alpha},$$

since $\lim_{t\to\infty} P_t^{\mathcal{K}} = \infty$. We also have that, as reported in the main text,

$$\theta_\infty^k \equiv \lim_{t\to\infty} \left(\alpha - \frac{\tau_t^\alpha}{A}\right) = \alpha^2,$$

a result which implies that the share of total income accruing (post-tax) to capital, remains strictly bounded away from 1 asymptotically, even if it constantly increases over time. In addition, we also have that

$$\theta_\infty^g \equiv \lim_{t\to\infty} \frac{\tau_t^\alpha}{A} = \alpha\left(1-\alpha\right).$$

On the balanced growth path instead, we have that $\dot{\tau}_t = 0$, since $\dot{P}_t^{\mathcal{K}} = \lambda^k \dot{p}_t^i = 0$, i.e. the total political pressure exercised by the capitalist class is constant over time. This result obtains since in balanced growth, we have that,

$$\dot{p}_t^i = \lim_{k_i^i\to\infty} \frac{\partial p_t^i}{\partial k_t^i} \dot{k}_t^i = \lim_{k_i^i\to\infty} \left(\frac{\partial p_t^i}{\partial k_t^i}\right) \dot{k}_t^i = 0,$$

which is the case as expression (57) implies that

$$\lim_{k_i^i\to\infty} \frac{\partial p_t^i}{\partial k_t^i} = 0.$$

It follows that in balanced growth taxes are constant, and therefore that income inequality is also constant (i.e. $\dot{\theta}_t^k = 0$). This reflects that once the economy has reached its balanced growth state, the level of taxation permanently corresponds to the technocratic (i.e. growth maximizing), fiscal policy preferred by the capitalists.

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