30th International Congress on Sound and Vibration
ICSV30 AMSTERDAM 8-11 JULY 2024

REVISITING THE LATTICE GAS AUTOMATA: APPLICATIONS IN ACOUSTICS

Johan Augusto Bocanegra and Davide Borelli
University of Genoa, Genoa, Italy
email: augusto.bocanegra@unige.it

Lattice Gas Automata (LGA) has emerged as a numerical technique based on the propagation and interaction of particles in a lattice. In the Computational Fluid Dynamics field, the Lattice Gas Automata is considered the precursor of the popular Lattice Boltzmann Method. Recognized for its parallelization capabilities and original mesoscopic formulation, the Lattice Gas Automata simulates the small-scale phenomena, making it applicable to various fluid dynamic problems in the time domain. This method indirectly addresses the Navier-Stokes equations within a weakly compressible limit conducive to acoustic wave propagation. This literature review explores the model's basis and some applications in acoustics. The bibliography is collected and presented, identifying the key topics of each study. Beyond offering a retrospective of past research, this work provides insight into the evolution of the lattice models and their potential in the acoustical domain.

Keywords: LGA, LBM, lattice Boltzmann, cellular automata.

1. Introduction

Lattice Gas Automata (LGA) is a computational method used primarily in fluid dynamics and related fields. It operates on a lattice grid where particles move from one lattice site to another according to simple rules, resembling the behaviour of gas molecules in motion. Each lattice site can be occupied by one or more particles, and collisions between particles are governed by local collision rules. The significance of numerical methodologies in acoustics cannot be overstated. Various techniques span from boundary and finite element methods to statistical energy analysis and particle tracing methods. Among these, Lattice Gas Automata (LGA) has garnered attention within Computational Fluid Dynamics (CFD) circles owing to its parallelization capabilities and unique mesoscopic approach based on simulation of molecular interactions.

Diverging from conventional CFD methodologies like the Finite Volume Method (FVM), where the Navier-Stokes equations are directly solved within a meshed domain, LGA adopts a mesoscopic perspective drawing from kinetic theory. It employs a digital representation of particles to discretize the material medium across time and space, enabling its application to a spectrum of fluid dynamic quandaries. Utilizing a lattice model, typically structured in one, two, or three dimensions with intricate geometries, LGA initiates from local interactions and evolves through discrete-time simulations.

Originating in the 1980s, LGA introduced a novel approach to fluid mechanics simulation, which culminated in the development of the Lattice Boltzmann Method (LBM). Employing an interaction rule among discrete molecules within a lattice, LGA revolutionized fluid dynamics simulations. The LGA solves fluid dynamics by applying an interaction rule between a discrete number of molecules that move from node to node in the lattice (or grid), with a restricted set of possible velocities. This set (lattice, particles, velocity base, and interaction rule) defines the Lattice Gas Automata structure. Wells and Renaut's vision from 1997 foretold the rise of alternative methods like LGA for simulating wave propagation, signalling a paradigm shift in numerical methodologies within acoustics, [1].

The interaction rule represents a collision operator, this operator is locally calculated in both LGA and LBM, allowing the possibility to apply parallel computing techniques in an almost straightforward way. The evolution from LGA to LBM was necessary to overcome drawbacks such as statistical noise or Galilean invariance, and among other details, the LBM considers a floating-point variable instead of the Boolean variables considered in LGA. The efficacy of lattice-based methods in acoustics has been proved, e.g., LBM in acoustics is discussed comprehensively by [2] in diverse domains ranging from porous acoustic materials to aeroacoustics.

This review aims to survey pertinent research pertaining to the application of Lattice Gas Automata in acoustical contexts, spanning publications and conference proceedings over the past four decades. It elucidates the specific acoustical challenges addressed using LGA, delineates adaptations of the method for these challenges, and discusses validation methodologies for obtained results. Additionally, this work aims to discern contemporary trends and developments in the utilization of LGA within acoustical realms, offering both retrospective insights and prospective trajectories for this numerical approach.

2. Methods

This Literature Review approaches the relevant literature in a systematic way that includes the definition of relevant Research Questions (RQ) according to the research objectives, Inclusion Criteria (IC) and Exclusion Criteria (EC) to filter the items found, a search strategy (SE) with relevant related keywords to be searched in the databases, a data extraction (DE) procedure to present the results coherently, and finally the conclusions as referred by [3] and [4]. The methodology encompasses several key components outlined below:

Research Questions (RQ): Four primary research questions (RQ) guide the investigation:

RQ 1: What specific acoustical problems have been addressed using LGA?

RQ 2: How has LGA been adapted to address acoustical challenges?

RQ 3: What techniques are utilized to validate the results obtained through LGA in the acoustical domain?

RQ 4: What are the prevailing research trends concerning the application of LGA in acoustics?

Inclusion Criteria (IC): To ensure relevance and consistency, the following inclusion criteria were established:

IC 1: Literature items must be in the English language to facilitate comprehension and analysis.

IC 2: Literature items must pertain specifically to acoustics, aligning with the focus of the review.

IC 3: Accepted publication formats include articles, conference papers, reviews, and book chapters, ensuring diverse and comprehensive coverage of the topic.

Exclusion Criteria (EC):

Preprints are excluded from consideration to maintain the integrity and reliability of the review process.

Search Strategy (SE):

A comprehensive search strategy was devised to identify relevant literature:

SE 1: Direct searches were conducted in reputable databases utilizing selected keywords relevant to acoustics and LGA.

SE 2: Manual searches were conducted to identify additional relevant items through cross-referencing.

Search String:

The search string employed was as follows:

("acoustic" OR "sound" OR "noise") AND ("Lattice Gas" OR "Lattice Gas Automata" OR "LGA")

Data Extraction (DE): Data extraction was performed systematically to capture pertinent information from the identified literature:

DE 1: Manual filtering was conducted to select literature items meeting the inclusion criteria.

DE 2: Acoustic problems addressed using LGA were identified and categorized.

DE 3: Adaptations of LGA specific to acoustical applications, including grid types and collision models, were documented.

DE 4: Validation techniques employed for assessing LGA results in acoustics were identified and categorized.

DE 5: Literature was sorted chronologically to discern research trends over time.

3. Lattice Gas Automata: basic formulation

The LGA is a type of Cellular Automata in which the state of an element (cell) in a discretized space is determined by the state of the neighbour elements. In this Cellular Automata the state of each cell represents some particles (molecules) of a gas traveling in a specific discrete direction with a velocity c_i. For each cell η_i represents a particle traveling with a velocity c_i to the neighbor cell and only can take the value of 0 -absence or 1 – presence of the particle. The physical velocity of the particles by the discretization steps Δx and Δt for space and time takes the value given by $c_{ip} = (\Delta x/\Delta t)c_i$.

The particles or *"digital-molecules"* representing the fluid in this model are points or dots with an equal mass that travel in a grid; the method presents an evolutionary representation of the fluid for each discrete timestep Δt, the particles travel to the neighbor position in the grid and therefore a simple collision rule is applied to maintain the mass and energy balance. From this description, it is evident that in LGA all the particles have the same velocity and can only travel from one node to the neighbor in each timestep. This propagation step is known as *streaming* in the absence of a collision, a simple equation, Eq. (1), describes the streaming step:

$$\eta_i(\boldsymbol{r} + \boldsymbol{c_i}\Delta t, t + \Delta t) = \eta_i(\boldsymbol{r}, t) \tag{1}$$

The position of the cell in the array is represented by r and t is the computational time (an integer). When the collision takes place, the collision equation, Eq. (2) can be formulated as:

$$\eta_i(\boldsymbol{r} + \boldsymbol{c_i}\Delta t, t + \Delta t) = \eta_i(\boldsymbol{r}, t) - Q(\eta) \tag{2}$$

Where the collision operator $Q(\eta)$ is a Boolean operator that describes the collision following the rules of conservation of mass, energy, and momentum. Two collision operators or rules related to the lattice topology were proposed, both rules have conservation of mass and momentum: HPP for the square lattice and FHP for the hexagonal lattice, see Fig. 1.

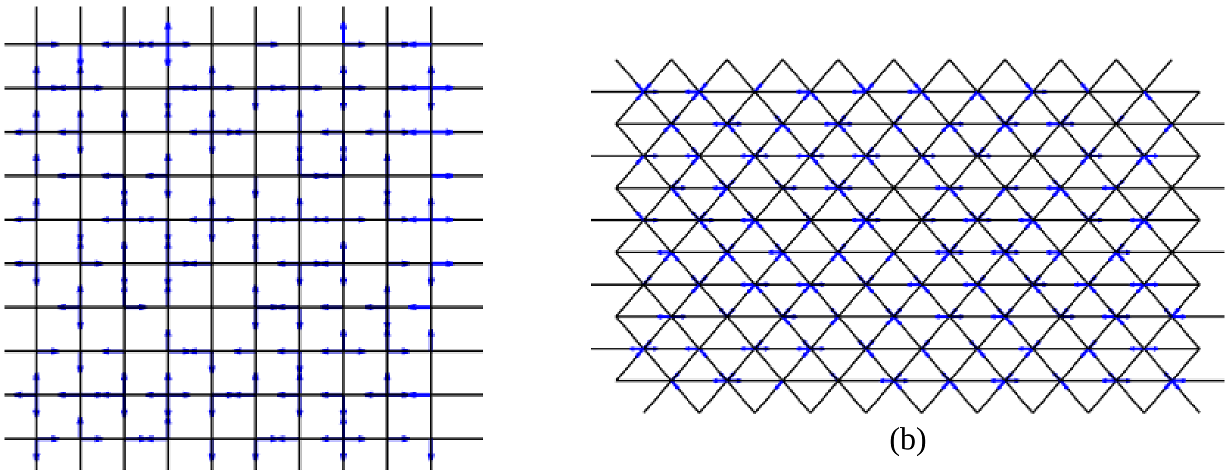

Figure 1: Schematic of the common lattice tipologies used in LGA. (a) Square lattice (HPP); (b) Hexagonal lattice (FHP).

3.1 collision rules

The HPP proposed in 1973 by J. Hardy, Y. Pomeau, and O. de Pazzis, [5], [6] can be summarized as:

- If two particles have a frontal collision, the results are two particles propagating in orthogonal directions.
- If the collision in a node is from orthogonal incoming velocities, no changes in the outgoing particles are made.
- If 3 or 4 particles have a collision, no changes in the outgoing particles are made.

It is easy to see that in the three cases:

a. the number of incoming and outcoming particles is equal (mass conservation)

b. the sum of the momentum before and after the collision is conserved (momentum conservation).

This model has the property of having time-reversal invariance. Some advantages of this method are related to not having round errors by using Boolean values for each η_i, which implies only 4 bits per node (low memory usage), the collision is local and easy to implement and points to a direct parallelization of the calculation. Some drawbacks are that non-equilibrium state can be reached, and the inherent anisotropy of the lattice; this scheme fails to achieve rotational invariance, and for this reason it cannot reproduce the NS equations. This problem can be solved by the use of a hexagonal grid and the FHP collision model (Frisch, Hasslacher and Pomeau [7]) uses 6 bits of information per node and can recover the NS equations. The collision rule leads to a non-deterministic scheme that breaks the time-reversal property of the HPP model.

- If two particles have a frontal collision, there are two possible results. The results are chosen randomly after each collision.
- If 3 particles have a collision, the outgoing particles bounce back.

By adding a rest particle in each node, the related viscosity of the system changes, but the model becomes more accurate.

3.2 Macroscopic properties

The macroscopic properties of the gas are obtained by summation over the direction indices on each node of interest on the grid, the density as the sum of particles in each grid position and the velocity by a momentum summation. Basic physical properties can emerge from this representation of a gas, and some works related to wave propagation and acoustics were developed in the past using LGA, but the inherent drawbacks of this method were effectively overcome by the most sophisticated Lattice Boltzmann Method using a higher-order quantization for the grid and a collision scheme emerging directly from the Boltzmann equation.

The computational calculation only requires a summation over the y degrees of freedom in each position i.e., for local density $\rho(\boldsymbol{r},t)$ the Eq. (3):

$$\rho(\boldsymbol{r},t) = \sum_{i=1}^{y} \eta_i(\boldsymbol{r},t) \tag{3}$$

The fluid velocity can be obtained by the Eq. (4):

$$\boldsymbol{u}(\boldsymbol{r},t) = \frac{1}{\rho} \sum_{i=1}^{y} \boldsymbol{c_i} \eta_i(\boldsymbol{r},t) \tag{4}$$

The calculation of the macroscopic properties using those equations in a single node include large amount of noise, and it is necessary to average the summation on a bigger region of space including several nodes.

4. Results

Several notable attempts to simulate acoustic waves using the Lattice Gas Automata (LGA) model have been documented. Here is a summary of some of these works:

- **Chen et al. (1988):** Presented an LGA model based on momentum and energy conservation laws to solve a linear wave equation. Their model effectively simulates wave phenomena such as pulse propagation in 1D and double-slit interference patterns in 2D. It demonstrates computational efficiency and applicability to complex geometries, [8].
- **Huang et al. (1988):** Simulated wave propagation in non-homogeneous media, specifically addressing reflected and transmitted waves across a simple interface. The classical HPP rule was adapted to incorporate a planar interface, demonstrating accurate simulation of wave behavior including Snell's laws and amplitude polarities, [9].
- **Krutar et al. (1991):** Investigated sound scattering and propagation in heterogeneous media, particularly in the ocean. Their LGA model, known as the Los Alamos model, statistically treated particles' identity and introduced modifications to particle movement probabilities. The model successfully reproduced various wave propagation phenomena, showcasing its versatility across different topologies and scenarios, [10].
- **Mora (1992):** Developed the Phononics Lattice Solid (PLS) method to simulate seismic p-waves in heterogeneous solids, incorporating a scattering term and variable sound speed. The method utilizes acoustic "phonons" and a finite differences scheme, demonstrating convergence and accurate simulation of wave behavior, [11].
- **Sudo and Sparrow (1995):** Presented a comprehensive review of sound wave propagation in LGA, developing 1D and 2D wave models. Their model considers lattice particles as information carriers and utilizes variable velocity sets for discretization, demonstrating second-order accuracy and anisotropic dispersion, [12].
- **Jianxing et al. (1996):** Compared FD and LGA models for simulating seismic waves, employing the LS lattice solid model. Their model successfully handled complex media and geometries, showcasing similar wave formation to FD schemes, [13].
- **Stansell and Greated (1997):** Implemented LGA for acoustic streaming in a 2D pipe, demonstrating good agreement with theoretical predictions and showcasing potential in solving the Navier-Stokes equations, [14].

- **Sudo (2001):** Presented a time-domain LGA model for wave propagation with low computational consumption, showcasing minimal dispersion and effective propagation characteristics, [15].
- **Chen et al. (2004):** Simulated a thermoacoustic engine using LGA, optimizing engine geometry and validating results against experimental data, [16].
- **Zhang et al., (2005):** Simulated thermoacoustic waves obtaining Self-excited oscillation and non-linear effects [17]
- **Korus (2007):** Explored alternative LGA models for wave propagation, highlighting the method's capabilities and comparing them with FD schemes, [18].
- **Chen et al. (2010):** Investigated thermoacoustic phenomena using LGA, identifying two distinct amplitude vibrations with two steady states for a given temperature gradient, [19].
- **Ota et al. (2010):** Utilized LGA for real-time acoustic rendering, enabling interaction with virtual sound fields in large-scale environments, [20].

For a comprehensive overview of the works discussed in this section, please refer to Table 1.

Table 1: Summary of the collected literature about LGA in acoustics.

Author and year	Grid	Collision model	Special features	Acoustic problem	Validation
Chen et al., 1988 [8]	D1 and D2		E, p conservation	Wave propagation- pulse, double-slit interference	Analytical
Huang et al., 1988 [9]	D2Q4	HPP	Grid size for each domain	Propagation of a wave between two different media with a planar interface	Analytical
Krutar et al., 1991 [10]	D2 and D3		Different grids	Wave propagation model	Analytical
Mora, 1992 [11]	D2Q6	HPP	LB Phononic Solid Lattice P-SL	Propagation of P-waves in an inhomogeneous medium, 2 layers, transmitted reflected, and refracted waves.	Analytical/ Numerical FD
Sudo and Sparrow, 1995 [12]	1D, 2DQ4	HPP	Multiple velocity state	Wave propagation model	Analytical / Numerical FD
Jianxing et al., 1996 [13]		FHP	LS	Seismic waves in inhomogeneous media	Numerical FD
Stansell and Greated, 1997 [14]	D2Q6	FHP III		Acoustic streaming in a two-dimensional pipe	Analytical/ experimental
Sudo, 2001 [15]	1D and D2Q6		Two components	Acoustic wave propagation in 2d	Analytical/numerical FD
Chen et al., 2004 [16]	D2Q9	FHP		Thermoacoustic waves	Analytical
Zhang et al., 2005 [17]	D2Q9	FHP		Thermoacoustic waves	Analytical
Korus, 2007 [18]	D2Q8	FHP		Pulse propagation	Numerical FDM
Chen et al., 2010 [19]	D2Q9	FHP		Thermoacoustic waves	Analytical
Ota et al. 2010 [20]	D2Q6	FHP		Real-time acoustic rendering	Analytical

5. Discussion

The early works on Lattice Gas Automata (LGA) underscore its inherent capability to address complex geometries through voxel or cell representation, leveraging a local collision rule that facilitates substantial parallelization of the numerical scheme. Operating with Boolean variables and operators, LGA optimizes memory usage without inducing round errors, unlike some other methods. Skordos (1995) posed a pertinent query regarding memory utilization in Lattice Boltzmann Method (LBM) compared to LGA, highlighting the quest for an optimal distribution of bit-information to physical degrees of freedom, [21].

Simulation with LGA span from fundamental wave propagation to intricate thermoacoustic effects. Challenges such as anisotropy or statistical noise diminish as the method evolves into LBM. A synopsis of the literature on LGA reveals a diverse array of acoustic problems faced, including wave propagation, isotropy, dissipation, diffraction, and thermoacoustics. Validation predominantly was based on analytical approaches.

Revisiting lattice gas automata underscores the multifaceted nature of lattice-based methods and their possible role in tackling complex acoustical phenomena. Moreover, this revisitation sheds light on the distinct challenges that underscore the need for versatile computational approaches, such as the Lattice Boltzmann Method. Further research and refinement of the methods in the mesoscale simulation could drive to advancements in our comprehension of acoustic phenomena across diverse domains.

6. CONCLUSIONS

This study offers a comprehensive overview of the Lattice Gas Automata (LGA) method and its utility in the field of acoustics. Through an extensive literature review spanning from 1988 to 2010, this paper has examined the application of LGA in addressing various acoustic challenges. A curated collection of research items has been analyzed and summarized, providing insights into the evolution and versatility of LGA in simulating acoustic phenomena.

In the initial stages of LGA development, validation of results primarily relied on analytical assessments, with comparisons frequently made against outcomes derived from alternative numerical methods. This practice underscores the rigorous approach taken by researchers to ensure the accuracy and reliability of LGA simulations.

The synthesis of literature items and their descriptions serves as a valuable resource for understanding the coverage of acoustic problems faced using LGA methodologies.

REFERENCES

[1] Wells, V. L. and Renaut, R. A. Computing Aerodynamically Generated Noise, *Annu. Rev. Fluid. Mech* **29** 161–99, (1997).

[2] Bocanegra, J. A., Misale, M., and Borelli, D. A systematic literature review on Lattice Boltzmann Method applied to acoustics, *Engineering Analysis with Boundary Elements*, **158,** 405–29, (2024). https://doi.org/10.1016/j.enganabound.2023.11.007

[3] Ferreras-Fernández, T., Martín-Rodero, H., García-Peñalvo, F. J. and Merlo-Vega, J. A. The systematic review of literature in LIS: an approach, *Proceedings of the Fourth International Conference on Technological Ecosystems for Enhancing Multiculturality - TEEM '16* the Fourth International Conference, Salamanca, Spain, 2-4 November, (2016).

[4] Torres-Carrion, P. V., Gonzalez-Gonzalez, C. S., Aciar, S. and Rodriguez-Morales, G. Methodology for systematic literature review applied to engineering and education, *2018 IEEE Global Engineering Education Conference (EDUCON)* , Santa Cruz de Tenerife, Canary Islands, Spain, 17-20 April, 1364–73, (2018).

[5] Hardy, J., Pomeau, Y. and de Pazzis, O. Time Evolution of a Two-Dimensional Classical Lattice System, *Physical Review Letters*, **31,** 276 (1973).

[6] Hardy, J., Pomeau, Y. and de Pazzis, O. Time evolution of a two-dimensional model system. I. Invariant states and time correlation functions, *Journal of Mathematical Physics* **14** 1746, (1973)

[7] Frisch, U., Hasslacher, B. and Pomeau, Y. Lattice-Gas Automata for the Navier-Stokes Equation, *Phys. Rev. Lett.*, **56** 1505–8, (1986)

[8] Chen, H., Chen, S., Doolen, G. and Lee, Y. C. Simple Lattice Gas Models for Waves, *Complex Systems*, **2,** 259–67, (1988).

[9] Huang, J.I., Chu, Y.H. and Yin, C.S. Lattice-gas automata for modeling acoustic wave propagation in inhomogeneous media, *Geophys. Res. Lett.*, **15**, 1239–41, (1988).

[10] Krutar, R. A., Numrich, S. K., Squier, R. K., Pearson, J., and Doolen, G. Computation Of Acoustic Field Behavior Using A Lattice Gas Model, *OCEANS 91 Proceedings*, Honolulu, Hawaii, USA, 446–52, (1991).

[11] Mora, P. The lattice Boltzmann phononic lattice solid, *J Stat Phys* **68** 591–609, (1992).

[12] Sudo, Y. and Sparrow, V. W. Sound propagation simulations using lattice gas methods, *AIAA Journal*, **33,** 1582–9, (1995).

[13] Jianxing, H., Youming, L., and Qinxue, L. Modelling research for simulating wave propagation in solid by lattice gas automata, *SEG Technical Program Expanded Abstracts 1996* S, 650–3, (1996).

[14] Stansell, P. and Greated, C. A. Lattice gas automaton simulation of acoustic streaming in a two-dimensional pipe, *Physics of Fluids*, **9,** 3288–99, (1997).

[15] Sudo, Y. Lattice Gas Time Domain Methods for Acoustics with Reduced Computer Memory Requirements, *Journal of Computational Acoustics*, **9,** 1239–58, (2001).

[16] Chen, Y., Liu, X., Zhang, X. Q., Yan, L., Yang, C. X. and Li, Z. C. Thermoacoustic simulation with lattice gas automata, *Journal of Applied Physics*, **95,** 4497–9, (2004).

[17] Zhang, X. Q., Chen, Y. and Liu, X. Lattice gas modeling of thermoacoustic oscillations, *Proceedings of the Twentieth International Cryogenic Engineering Conference (ICEC20)*, 305–8, (2005).

[18] Korus, L. Alternative Methods of Wave Motion Modelling, *11th International Conference, KES 2007 XVII Italian Workshop on Neural Networks* Lecture Notes in Artificial Intelligence Knowledge-Based Intelligent Information and Engineering Systems: KES 2007 - WIRN 2007, Vietri sul Mare, Italy, 335–45, (2007).

[19] Chen, Y., Fang, W. and Chen, Z. Lattice gas simulation of thermoacoustic engine, *International Congress on Sound and Vibration ICSV17*, Cairo, Egypt, 6 (2010)

[20] Ota, M., Takeuchi, T., Iwaya, Y., Matsuoka, H. and Suzuki, Y. Sound-field lattice gas cellular automaton for real-time acoustic rendering, *2010 4th International Universal Communication Symposium*, Beijing, China, 229–32, (2010).

[21] Skordos, P. A. *Modeling flue pipes: subsonic flow, lattice Boltzmann, and parallel distributed computers* Ph.D. Massachusetts, USA, Massachusetts Institute of Technology, (1995).