# Market-Period Adjustments, Short-Run and Long-Run Dynamics: A Keynesian Theory of Real Business Cycles<sup>\*</sup>

Marco Guerrazzi

## Abstract

In this paper, relying on the seminal Keynesian analysis of the market for goods, I aim at integrating the market-period adjustments of relative-price expectations with the short-run dynamics generated by the occurrence of demand shocks and the long-run dynamics implied by capital accumulation and supply shocks. Specifically, I build a dynamic setting in which output, (un)employment and real wages tend to converge towards a nondeterministic short-run equilibrium pinned down by long-run entrepreneurial expectations. Furthermore, calibrating the model economy by taking as reference the US economy, I show that the cyclical properties of the resulting theoretical framework are consistent with a number of observed business cycle regularities.

Keywords: Market-period and short-run equilibria, Demand and supply shocks, Shifting equilibrium model, Animal spirits, Real business cycles.

JEL Classification: E12, E24.

# I. Introduction

The traits of the Keynesian theory of capital accumulation encapsulated in the Harrod-Domar growth model retrieves some seminal implications of the principle of effective demand to determine the warranted growth rate of income that allows the economy to walk along a path of full utilization of its installed productive capacity (cf. Harrod,

<sup>&</sup>lt;sup>\*</sup> I would like to thank the editorial committee and two anonymous referees for their comments and suggestions. The usual disclaimer applies.

1939). By contrast, the corresponding natural growth rate of income that would allow the economy to maintain the full employment of available labour is determined by exogenous demographic and technological factors. Obviously, when the two mentioned critical rates happen to be equal, the economy can easily proceed on a path in which capital and labour are fully employed over time (cf. Domar, 1946). On the contrary, when the two differ, the actual degree of substitution between labour and capital implied by the existing production technology becomes crucial to ensure the required adjustment of quantities and prices towards a long-run balanced growth path in which the utilization rates of two production factors grow at the same rate (cf. Solow, 1956).

The arguments briefly recalled above lend themselves to a couple of critical considerations. On the one side, they emphasise the sharp distinction between long-run and short-run dynamics inherited by modern macroeconomic theory in which demandled growth usually appears to be stringently conditioned by an accommodating supply (cf. Smith, 2012; Fazzari et al. 2020). On the other side, they suggest that the role of capital accumulation in the functioning of the principle of effective demand underlying the short-run determination of equilibrium employment remains largely unexplored (cf. Robbins, 1968, Chapter 3; Davidson, 1978, Chapter 5; Silverberg, 1987). A further corroboration of the last statement is found in the existing analytical literature on effective demand in which the stock of employed capital – or the material input vector – is usually taken as given and only variations in its degree of utilization are seriously taken into consideration (cf. Vandenborre, 1958; Casarosa, 1981; 1984; Kurz, 1990; Shiozawa, 2021).

In this paper, relying on the seminal Keynesian analysis of the market for goods, I aim at filling the gaps pointed out above by integrating the market-period adjustments of relative price expectations with the short-run dynamics generated by the occurrence of demand shocks and the long-run dynamics driven by capital accumulation and supply shocks. Specifically, avoiding to rely on forward-looking optimization by acknowledging that the relevant economic decisions of firms are usually taken in a situation of deep uncertainty in which only the near past is known with certainty, I build a dynamic model with capital accumulation where the level of output, the (un)employment rate, and the real wage prevailing in the market period tend to converge towards a shifting short-run equilibrium allocation pinned down by long-run entrepreneurial expectations (cf. Kregel, 1976; Dutt, 1991-1992).

From a theoretical point of view, I consider an intertemporal model economy in which competitive entrepreneurs are assumed to form and revise two distinct types of expectations. The former is given by market-period expectations of relative prices which are assumed to drive their employment decisions according to the principle of effective demand (cf. Keynes, 1936, Chapters 5 and 20). In each of the mentioned market period, depending on the extent of actual expenditure, the value of these predetermined expectations may be different from the relative prices implied by actual market transactions and such a divergence is assumed to trigger an adaptive process of revision conditioned on realized outcomes (cf. Guerrazzi, 2023). By contrast, the latter

type of expectations is given by self-fulfilling long-run-period expectations that drive investment decisions – relevant for capital accumulation – whose revision process is assumed to be completely unrelated to realized outcomes because of the animal spirits of competing entrepreneurs (cf. Farmer, 2008; Guerrazzi, 2011, 2012; Guerrazzi and Gelain, 2015). In general, the way in which these two types of expectations are revised in each market period together with the law of motion of the capital stock and the one of productivity shocks underlying the evolution of total factor productivity (TFP) are able to define the actual market dynamics of output, (un)employment, and real wages. Consequently, as opposed to what happens in conventional real-business-cycle (RBC) models in which macroeconomic fluctuations are driven only by changes in the fundamentals of the economy, i.e., variations of preferences, technology and/or endowments, in the model economy developed in this paper there is also room for cyclical movements triggered by extrinsic uncertainty (cf. Cass and Shell, 1983).

In addition, I provide evidence that the backward-looking setting described above is interesting not only from a theoretical perspective but also on the empirical ground. Specifically, relying on the numerical techniques typical of RBC contributions, I show that calibrating the model economy by taking as reference the US economy over the last 50 years, the theoretical framework developed in this paper is able to replicate a number of observed business cycle regularities such as the stickiness displayed by real wages as well as the strong volatility of unemployment rates (cf. Shimer, 2005; Ravn and Simonelli, 2007).

The paper is arranged as follows. Section 2 develops the theoretical framework. Section 3 explores its numerical properties. Finally, Section 4 concludes.

#### **II. Theoretical Framework**

Sweeping any aggregation/disaggregation problems and omitting to deal with financial markets, I consider a dynamic model economy without any government activity and foreign trade in which time is measured on a discrete scale and where the output of the representative firm is given by a stochastic Cobb-Douglas combination of the current stock of capital goods and the flow of employed labour services. Specifically, using the subscript  $t \in \mathbb{N}$  to denote the single unit of time, I assume that the production – or the utilization – function that summarizes the technology available to the single firm is given by the following expression:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where  $Y_t$  is the current flow of the final output,  $A_t$  is a measure of TFP,  $K_t$  is the current capital stock,  $L_t$  is the number of employed workers, whereas  $\alpha \in (0,1)$  is the output elasticity with respect to the installed capital.

On the right-hand-side of eq. (1), the first two variables, namely  $A_t$  and  $K_t$ , are predetermined whereas the third, namely  $L_t$ , is determined period by period on the basis of the firm's expectation of relative prices. On the one hand, as in standard RBC models, I make the hypothesis that TFP moves over time according to an erratic AR(1) process (cf. Kydland and Prescott, 1982). Hence,

$$\ln A_t = \mu + \xi \ln A_{t-1} + \eta_t \tag{2}$$

where  $\mu > 0$  is a positive drift,  $\xi \in (0,1)$  is a measure of the persistence of productivity shocks, whereas  $\eta_t$  is a stochastic disturbance that is assumed to be normally distributed with a mean equal to zero and a constant variance equal to  $\sigma_{\eta}^2$ .

On the other hand, capital dynamics is described by the standard accumulation rule according to which the productive capacity of the firm tends to wear out at a constant rate with its utilization, but it can be likewise fuelled by the decision to purchase new capital goods (cf. Nerlove and Arrow, 1962). Formally speaking, this leads to the following expression:

$$K_t = I_{t-1} + (1 - \delta)K_{t-1} \tag{3}$$

where  $I_{t-1}$  is the real investment expenditure carried out by the firm in the previous period, whereas  $\delta \in (0,1)$  is the depreciation rate of capital.

In the model economy under scrutiny, the current number of workers actually employed by the firm is determined according to the principle of effective demand introduced by Keynes (1936) in his *General Theory* (cf. Keynes, 1936, Chapter 3). On the basis of the analytical formalization put forward by Casarosa (1981, 1984), the actual operation of such a principle is assumed to rely on the interaction of two distinct schedules, namely, the supply and the expected demand functions, both defined in terms of proceeds – or revenues – collected by the firm and measured in nominal wage units (cf. Keynes, 1936, Chapter 5). In other words, aiming at pinning down the level of employment desired by the firm, the marginal cost of labour as well as nominal sales proceeds are assumed to be deflated relying on the nominal wage rate instead of a price index (cf. Farmer, 2010).

On the one side, the supply function – say  $Z_t$  – is assumed to be given by the current value of output in wage units for which the representative productive unit endowed with the production function in eq. (1) finds profitable to employ exactly  $L_t$  workers (cf. Vandenborre, 1958). In a purely competitive economy where such a firm decides period by period how many workers to employ by maximizing its current gross profits, such a value of output measured in units of the nominal wage rate is given by the following expression:

$$Z_t = \frac{1}{1-\alpha} L_t \tag{4}$$

As suggested by Farmer (2010), the value of output implied by eq. (4) follows immediately from the first-order condition for the level of employment that maximizes period by period the gross profit of the firm, and it is aimed at describing the market behaviour of atomistic entrepreneurs that compete with one another for the production factors by means of adjustments of relative prices (cf. Guerrazzi, 2012).<sup>1</sup> Contrary to what Davidson (1978, Chapter 5) hypothesized, in this version of the Keynesian supply function the value of sales proceeds measured in wage units conveyed by eq. (4) is not indexed to the stock of capital of the firm but depends only on its level of employment.

On the other side, in each period, the expected demand function – say  $D_t^e$  – is given by the value of sale proceeds measured in wage units that the representative firm expects to receive by employing  $L_t$  workers because of its predetermined expectations of the price level and the nominal wage rate. Specifically, the former is the expectation of what the firm expects to collect for each unit of output that is sold in the market for goods, whereas the latter is the expectation of what it expects to spend for each unit of labour that is purchased in the corresponding market so that their combination returns the predetermined expectation of the real wage rate per worker. Formally speaking, considering the production function in eq. (1), such a value of output measured in nominal wage units will be given by the following expression:

$$D_t^e = \frac{A_t K_t^{\alpha} L_t^{1-\alpha}}{W_t^e} \tag{5}$$

where  $W_t^e > 0$  is the expected real wage rate at time t or, equivalently, the inverse of the expected price-wage ratio (cf. Guerrazzi, 2023).

As illustrated in Figure 1, in each period, given the predetermined values of  $W_t^e$ ,  $A_t$  and  $K_t$ , the equilibrium level of employment – say  $L_t^*$  – is the one such that  $Z_t$  is equal to  $D_t^e$ . According to the expressions in eq.s (4) and (5), its analytical expression is the following:

$$L_t^* = K_t \left( (1 - \alpha) \frac{A_t}{W_t^e} \right)^{\frac{1}{\alpha}}$$
(6)

<sup>&</sup>lt;sup>1</sup> As is it well know, the first-order condition for gross profit maximization implies that the marginal productivity of employed labour has to be equal to the real wage rate (cf. Dutt, 1991-1992, p. 209). It is worth noticing that such a condition remains exactly the same also in a dynamic *q*-model of investment with adjustment costs in which the representative firm optimizes over an infinite horizon (cf. Lucas, 1967, p. 325).



Figure 1: Market-period equilibrium employment

The equilibrium level of employment at time *t* conveyed by eq. (6) is the one such that the expected revenues of the representative firm are exactly equal to the revenues deriving from selling the corresponding amount of produced output, and it corresponds to the equilibrium employment for the "day" or for the Marshallian market period.<sup>2</sup> Such a value of equilibrium employment linearly depends on the current value of the stock of capital, and it is not necessarily associated to full employment. On the contrary, both in "daily" and in short-run equilibria, the actual value of  $L_t^*$  may be well below the amount of labour services that workers are willing to supply at the current wage so that it could be well associated to the presence of some involuntary unemployment, a possibility that is not contemplated by traditional RBC models because they are usually built on the equality between labour supply and labour demand that follows from utility and profit maximization over an infinite horizon (cf. Kydland and Prescott, 1982; Long and Plosser, 1983). In more general terms, in the model economy under scrutiny unemployment will amount to the following difference:

$$U_t^* = L_t^S - L_t^*$$
 (7)

where  $L_t^S$  is the value of labour supply in period t.

The employment function in eq. (6) implies that the common value of  $Z_t$  and  $D_t^e$  achieved when  $L_t$  is equal to  $L_t^*$  – denoted by  $\overline{D}_t^e$  in Figure 1 – corresponds to the effective demand measured in wage units whose amount is definitely consistent with the

<sup>&</sup>lt;sup>2</sup> Keynes (1936, Chapter 5, p. 47) defined the "day" as "the shortest interval after which the firm is free to revise its decision as to how much employment to offer. It is, so to speak, the minimum effective unit of economic time."

profit maximization of the representative firm (cf. Keynes, 1936, Chapter 20; Vandenborre, 1958; Casarosa, 1981, 1984; Guerrazzi, 2023). Furthermore, given the prevailing values of  $A_t$ ,  $K_t$  and  $W_t^e$ , the equilibrium level of employment implied by eq. (6) pins down – through eq. (1) – the equilibrium level of output actually produced by the firm in period t whose analytical expression can be written as

$$Y_t^* = A_t^{\frac{1}{\alpha}} K_t \left(\frac{1-\alpha}{W_t^e}\right)^{\frac{1-\alpha}{\alpha}}$$
(8)

In each period, the level of final output implied by eq. (8) and the corresponding amount of labour conveyed by eq. (6) are not necessarily sold and purchased at the relative prices expected by the firm whose expression appears at the denominator of the expected demand function in eq. (5). Along the lines traced out by Casarosa (1981, 1984), I assume that actual relative prices can be inferred from a third relationship, i.e., an expenditure function that conveys the terms of market transactions for each level of employment. Formally speaking, such a relationship is given by a linear combination of the supply function in eq. (4) such as the following expression:

$$D_t = \hat{l}_t + cZ_t \tag{9}$$

where  $D_t$  is the value of the expenditure in consumption and investment goods measured in wage units,  $c \in (0,1)$  is a measure the of the reactivity of these expenditures with respect to the proceeds collected by the firm, whereas  $\hat{l}_t > 0$  is an autonomous expenditure component.<sup>3</sup>

In the remainder of this paper, I follow Farmer (2008) by assuming that the autonomous component of eq. (9) is an additional predetermined variable of the model economy that conveys the value of the investment expenditure carried out by the representative firm measured in wage units. Such a variable allows extrinsic uncertainty to affect realized allocations and its dynamics is assumed to be given by the following stochastic AR(1) process:

$$\hat{l}_t = \kappa + \rho \hat{l}_{t-1} + \varepsilon_t \tag{10}$$

where  $\kappa > 0$  is a positive drift,  $\rho \in (0,1)$  measures the persistence of the exogenous investment sequence, whereas  $\varepsilon_t$  is a stochastic disturbance that is assumed to be normally distributed with a mean equal to zero and a constant variance equal to  $\sigma_{\varepsilon}^2$ .

<sup>&</sup>lt;sup>3</sup> Dutt (1991-1992) provides a microfoundation of eq. (9) by assuming that the model economy is populated by workers that consume a fixed share of their labour income and capitalists undergoing lump-sum consumption. More recently, Guerrazzi (2011, 2012) does the same by considering hand-to-mouth workers (wage earners) together with entrepreneurs – or profit-earners – that invest their whole income to boost capital accumulation (cf. Woodford, 1986).

The erratic process in eq. (10) looks qualitatively similar to the one in eq. (2). Nevertheless, its meaning is completely different. Specifically, the expression in eq. (10) does not represent the evolution of technology shocks – or any other variation of fundamentals – but the realization of self-fulfilling long-run expectations of the long-run state of the economy underlying the investment decision of the firm and it represents the analytical device that in the present context allows us to skip intertemporal optimization over an infinite horizon and to consider the macroeconomic effects of extrinsic uncertainty. In other words, eq. (10) is assumed to formalize in a simple way a central issue of the *General Theory*, i.e., the idea that the value of the investment expenditure exogenously evolves by mirroring the animal spirits of entrepreneurs with no regards for the value of an additional unit of capital to the flow of their expected profits (cf. Keynes, 1936, Chapter 12, p. 149).

Interestingly, Kurz (2008, pp. 778-779) provides a microfoundation for eq. (10) by deriving an equivalent expression as a limit posterior of a Bayesian learning inference process in a non-stationary environment in which financial-market traders are committed in forecasting the liquidation value of a risky asset in a situation in which its future prices are unknown. Such a Bayesian criterion implies that what is actually learnable in a non-stationary economy – characterized by pervasive uncertainty – should be described by a stable process and this explicitly suggests a value of  $\rho$  inside the unit circle (cf. Guerrazzi, 2011; Guerrazzi and Gelain, 2015).<sup>4</sup>

Given the equilibrium level of employment that equalizes the supply function and the expected demand function conveyed by eq. (6), the realization of  $\hat{I}_t$  determines – through eq. (10) – the actual value of produced output measured in wage units implied by effective market transactions. Consequently, considering the production function in eq. (1), the actual value of the real wage – say  $W_t^*$  – will be given by the ratio between the value of  $Y_t^*$  and the expenditure function evaluated in  $L_t^*$ .<sup>5</sup> Formally speaking, this means that the actual real wage rate is given by the following expression:

$$W_t^* = \frac{K_t(W_t^e)^{-\frac{1-\alpha}{\alpha}}}{(1-\alpha)^{-\frac{1-\alpha}{\alpha}}A_t^{-\frac{1}{\alpha}}\hat{l}_t + cK_t(W_t^e)^{-\frac{1}{\alpha}}}$$
(11)

At the macroeconomic level, the mechanism of real-wage determination implied by eq. (11) has two important implications. On the one hand, the elasticity of the wage with respect to equilibrium employment is given by the difference between the elasticity of produced output with respect to the labour input – which is constant and equal to  $1 - \alpha$ 

<sup>&</sup>lt;sup>4</sup> In his *Treatise on Probability*, Keynes (1921, Chapter 32, p. 391) had a similar intuition. In fact, he did explicitly state: "No one supposes that a good induction can be arrived at merely by counting cases. The business of strengthening the argument chiefly consists in determining whether the alleged association is *stable*, when the accompanying conditions are varied." (italics from the original author).

<sup>&</sup>lt;sup>5</sup> The fact the expenditure function determines simultaneously the price of the good and the nominal wage can be rationalized by assuming the presence of a mechanism of indexation that in each period links the two nominal references (cf. Gray, 1976; Guerrazzi, 2010).

- minus the elasticity of the expenditure function with respect to  $L_t^*$  – which is given instead by  $cL_t^*/((1-\alpha)\hat{I}_t + cL_t^*)$ . Even if the firm is always on its downward-sloping labour demand schedule, such a feature of eq. (11) reveals that the cyclical behaviour of real wages crucially depends on the reactivity of the expenditure function to the employment rate so that the pro-cyclicality of real wages – a pattern that conventional RBC models find difficult to explain without technical progress – cannot a priori be ruled out (cf. Guerrazzi, 2023).

On the other hand, recalling the choice of units exploited in eq.s (9) and (10), the expression in eq. (11) implies that the capital accumulation rule in eq. (3) can be rewritten as

$$K_{t} = \left(\frac{(W_{t-1}^{e})^{-\frac{1-\alpha}{\alpha}}}{(1-\alpha)^{-\frac{1-\alpha}{\alpha}}A_{t-1}^{-\frac{1}{\alpha}} + c\frac{K_{t-1}(W_{t-1}^{e})^{-\frac{1}{\alpha}}}{\hat{I}_{t-1}}} + 1 - \delta\right)K_{t-1}$$
(12)

Everything else being equal, the non-linear difference equation in (12) reveals that productivity improvements (losses) as well as exogenous increases (reductions) in the level of the expenditure function measured in wage units tend to boost (restrain) capital accumulation.<sup>6</sup> By contrast, the effect on firm's investment decisions triggered by revisions in relative price expectations depends on the cyclicality of real wage rates explained above. Specifically, whenever real wages tend to be pro-cyclical (anticyclical) by moving in the same (opposite) direction of equilibrium employment, i.e., when the elasticity of the expenditure function with respect to  $L_t^*$  is lower (higher) than  $1 - \alpha$ , an upward revision of the expected real wage will be associated – ceteris paribus – to a deceleration (an acceleration) of capital accumulation. Interestingly, the path of real investment implied by pro-cyclical real wages is perfectly consistent with the positive relation between short-run price expectations – that determine employment – and long-run expectations – that, in turn, determines investment – assumed by Dutt (1991-1992) in his shifting equilibrium model (cf. Kregel, 1976).

As far as the erratic component of the expenditure function is concerned, there is no certainty that the actual relative prices implied by eq. (11) coincide with their predetermined expectation reported in the denominator of eq. (5). Specifically, as illustrated by the diagram in Figure 2, depending on the actual realization of  $\hat{I}_t$  and value taken by  $L_t^*$ , there are three possible outcomes. On the one side, the value achieved by the expenditure function measured in wage units – as indicated by  $D_t(L_t^*)$  in Figure 2 – can well coincides with the corresponding value of the effective demand  $\overline{D}_t^e$ . In such a situation, it will hold that the real-wage expectation of the firm is equal to the actual value of the real wage rate. As argued by Casarosa (1981, 1984), the corresponding allocation of produced output and employment achieved when  $W_t^e$  is

<sup>&</sup>lt;sup>6</sup> The same positive relationship between productivity shocks and real investment is found also in a dynamic q-model of investment (cf. Lucas, 1967).

equal to  $W_t^*$  will represent a short-run equilibrium point for the model economy that does not require any expectation revision of relative prices.<sup>7</sup>



Figure 2: Market-period equilibrium and the expenditure function

On the other side, the value achieved by the expenditure function measured in wage units – alternatively denoted by  $D_t'(L_t^*)$  and  $D_t''(L_t^*)$  in Figure 2 – can be lower or higher than the corresponding value of the effective demand indicated by  $\overline{D}_t^e$ . In the former case, the expected real wage will be higher than its actual value whereas in the latter it will hold exactly the opposite. In both cases, however, since its short-run expectations are disappointed, the representative firm will somehow change its relative price expectations and the corresponding allocation of output and employment won't be a short-run equilibrium.

Whenever the relative-price expectations of the representative firm are not fulfilled, i.e., outside of any short-run equilibrium, the firm itself will tend to revise them on the basis of realized outcomes. Consequently, following Guerrazzi (2023), I assume that the adjustment of real-wage expectations is described by the following adaptive – or error learning – process:

$$W_t^e = W_{t-1}^e + \lambda \left( \frac{K_{t-1}(W_{t-1}^e)^{-\frac{1-\alpha}{\alpha}}}{(1-\alpha)^{-\frac{1-\alpha}{\alpha}} A_{t-1}^{-\frac{1}{\alpha}} \hat{I}_{t-1} + cK_{t-1}(W_{t-1}^e)^{-\frac{1}{\alpha}}} - W_{t-1}^e \right)$$
(13)

<sup>&</sup>lt;sup>7</sup> It is worth noticing that in a short-run equilibrium, the value of equilibrium employment is given by the intersection of the supply function in eq. (4) and the expenditure function in eq. (9) so that it can be written as a function of the exogenous investment expenditure only. Specifically, in a short-run equilibrium  $L_t^* = (1 - \alpha)\hat{l}_t/(1 - c)$ .

where  $\lambda \in (0,1)$  is a parameter that conveys how the representative firm revises its relative price expectations depending on the forecasting error experienced in the previous period.

The adjustment process described by eq. (13) implies that the firm tends to adjust the value of its real wage expectation over time in response to deviations of actual gross profits from their expected value. Specifically, the firm tends to reduce (increase) the value of  $W_t^e$  whenever the realized value of its profits conveyed by the expenditure function is above (below) its expected value.<sup>8</sup> In other words, the process of short-run expectations implied by eq. (13) implies that the behaviour of representative firm is aimed at targeting a point on its marginal labour-productivity schedule in which the actual real wage rate is equal to its expected value so that its gross profit expectations are perfectly verified (cf. Guerrazzi, 2023).

The dynamic expression in eq. (13) allows us to close the model in a well-determined manner; indeed, given the value of each parameter and the realizations of TFP and expenditure – or confidence – shocks, eq.s (2), (10), (12) and (13) provide the evolution of all the predetermined variables of the model economy. Thereafter, eq.s (6), (7), (8) and (11) pin down the values of the remaining non-predetermined variables. Furthermore, it is worth noticing that since all the involved dynamic processes have roots belonging to the unit circle, in the long run all the variables will display a firm tendency to converge towards a deterministic steady-state allocation whose detailed determination is illustrated in Appendix.

#### **III. Numerical Properties**

The numerical properties of the theoretical framework developed above are explored through some numerical simulations by focusing on the dynamic behaviour of output, (un)employment and real wages.<sup>9</sup> In this direction, I calibrate the model on a quarterly basis by taking as reference the US economy and targeting the RBC estimations provided by Ravn and Simonelli (2007). Before assigning point values to each model's parameter, however, it is necessary to say something about the behaviour of workers in the labour market. For this purpose, I normalize the long-run value of labour supply to 1 and I assume that labour participation is somehow procyclical (cf. Tüzemen, 2017). Specifically, I make the hypothesis that in each period labour supply is given by the following expression:

$$L_t^S = 1 \left( 1 + \gamma \frac{L_t^* - L_{\rm SS}}{L_{\rm SS}} \right) \tag{14}$$

where  $\gamma \in (0,1)$  is a measure of the procyclicality degree of labour participation, whereas  $L_{SS}$  is the steady-state level of employment.

<sup>&</sup>lt;sup>8</sup> In nominal terms, the reduction (increase) of the real wage rate can happen through an increase (a reduction) of the price of the produced good, a reduction (an increase) of the nominal wage or a combination of the two.

<sup>&</sup>lt;sup>9</sup> MAT LAB codes are available from the author upon reasonable request.

The expression in eq. (14) completes the description of the economic environment and it straightforwardly implies that labour supply is above (below) its constant longrun value when employment is above (below) its steady-state level. Consequently, when real wages are pro-cyclical, such an expression is equivalent to an upward-sloped labour supply schedule.

In order to be consistent with a long-run unemployment rate of 5.5%, a figure often mentioned as the structural level of US unemployment, the model economy is calibrated by targeting a value of  $L_{SS}$  equal to 0.945 (cf. Turner et al. 2001). Specifically, the point values of each parameter are set as follows. First, the value of the output elasticity with respect to capital ( $\alpha$ ) as well as the capital depreciation rate ( $\delta$ ) as set at the values suggested by Kydland and Prescott (1982). Second, the persistence of TFP ( $\xi$ ) is set at the value estimated by Chang (2000), its drift ( $\mu$ ) is set according to the fixed value of  $L_{\rm SS}$ , whereas its volatility  $(\sigma_n)$  is set by targeting the observed standard deviation of output that amounts to 1.56% (cf. Ravn and Simonelli 2007). Third, the value of reactivity of the expenditure function (c) is the taken from the estimations of the marginal propensity to consume retrieved by Souleles (2002). Fourth, the persistence of the exogenous investment sequence ( $\rho$ ) is taken from Farmer (2008), its drift ( $\kappa$ ) is set by targeting a long-run value of the expenditure function consistent with  $L_{SS}$ , whereas its volatility ( $\sigma_{\varepsilon}$ ) is set by targeting short-run employment rates in the same range of the available labour supply.<sup>10</sup> Fifth, without any loss of generality, the value of  $\lambda$  is taken from the work by Adam and Padula (2011) on subjective inflation expectations. In addition, the degree of procyclicality of labour force participation is set according to the estimates provided by Tüzemen and Van Zandweghe (2018). All the parameter's values and their respective description are summarized in Table 1.

Parameter	Description	Value			
α	Output elasticity with respect to capital	0.3600			
μ	u Drift of productivity shocks				
ξ	Persistence of productivity shocks	0.9500			
$\sigma_\eta$	Volatility of productivity shocks	0.0053			
δ	Capital depreciation rate	0.0250			
С	Reactivity of the expenditure function	0.6000			
κ	Drift of exogenous investment	0.2969			
ρ	Persistence of exogenous investment	0.5000			
$\sigma_{\varepsilon}$	Volatility of exogenous investment	0.0068			
λ	Wage expectations' reactivity				
γ	γ Procyclicality of labour supply				
Table 1: Calibration					

<sup>&</sup>lt;sup>10</sup> It is worth noticing that – together with a value of  $L_{SS}$  equal to 0.945 – the selected values for  $\alpha$ , c,  $\rho$  and  $\kappa$  imply that the long-run value of the wage elasticity with respect to employment is equal to 0.5464. Moreover, the model economy is simulated by assuming that TFP and expenditure shocks are orthogonal, i.e., by assuming that  $E_t[\eta_t \varepsilon_t] = 0$ , for all t.

Given the parameter values in Table 1, the cyclical properties of output, (un)employment, and real wages are evaluated as follows. First, under the assumption that each predetermined variable starts 1% below its stochastic steady-state value retrieved in Appendix, I generated 1,171 observations for each variable of interest. Second, I threw away the first 1,000 observations for each simulated series in order to remain with 171 data points that correspond to the length of US quarterly data analysed by Ravn and Simonelli (2007). Third, I detrended the log of the model-generated data by using the Hodrick-Prescott (HP) filter with a smoothing parameter set at 1,600 (cf. Hodrick and Prescott, 1997).<sup>11</sup> Fourth, I computed the deviations of all the artificial series from the HP trend and then I took the respective standard errors and their correlation. Sixth, I repeated the previous steps for 10,000 times. Finally, I took the mean of all the statistics obtained in the fourth step. The results of the numerical procedure described above are collected in Table 2 (observed values in parenthesis).

Variable		$Y^*$	$L^*$	$W^*$	$U^*$
Standard deviation		1.56	1.71	0.83	11.02
(%)		(1.56)	(1.46)	(0.86)	(10.76)
	<i>Y</i> *	1	0.93	0.68	-0.92
Completion			(0.81)	(0.18)	(-0.87)
Correlation	$L^*$	-	1	0.45	-0.99
maurx	$W^*$	-	-	1	-0.44
	$U^*$	-	-	-	1

Table 2: Simulation results (average values)

The figures in Table 2 reveal that – on average – the theoretical model developed above tends to somehow overstate the volatility of employment as well as the correlation between output, employment, and real wages. Nevertheless, along the other explored dimensions, despite its analytical parsimony, it fairly replicates the cyclical behaviour of the involved variables taken at their macroeconomic level. Specifically, the model appears to be able to display the observed degree of real-wage stickiness as the well as the strong volatility of unemployment and its negative correlation with produced output (cf. Shimer, 2005). In addition, under the suggested calibration, the model does not fall prey of the countercyclical pattern of real wages advocated by prototypical RBC models (cf. Lucas, 1981).

Sample plots of the simulated series together with their respective HP trend are illustrated in the four panels of Figure 3, whereas the impulse response function to a 1%

<sup>&</sup>lt;sup>11</sup> Different values of the initial conditions and different values of the smoothing parameter do not alter the results in a significant manner (cf. Shimer, 2005).

shock in TFP and in the investment expenditure measured in wage units are tracked, respectively, in the panels of Figures 4 and 5.



Figure 3: Simulated series

The diagrams in Figures 4 and 5 allows us to clarify the dynamics and the transmission mechanisms generated by the model economy described above and to assess how it differs from conventional RBC models. On the one hand, the four panels of Figure 4 show that a 1% shock to TFP has – on impact – an amplified impact on output, employment, and unemployment and about the same effect on the real wage rate. Thereafter, the observed increase in  $W^*$  triggers an upward revision of real wage expectations which is responsible for the decline in output and employment as well as for the increase in unemployment observed in the period just after the occurrence of the productivity shock. In turn, the fall in output is then responsible for the consequent decline in real wage rate that leads to a downward revision of real wage expectation that pushes output and employment upward – and unemployment downward – in the direction of their respective steady-state values. The hump-shaped impulse response functions in Figure 4 are completely at odds with respect to the ones retrieved in the workhorse RBC model where TFP shocks imply monotonic responses for all the involved variables (cf. King et al. 1988).

On the other hand, the panels of Figure 5 show that – at the beginning – a 1% shock to the investment expenditure measured in wage units has no impact on output, employment, and unemployment but it triggers only a dumped reduction in the real wage rate. Thereafter, the initial reduction in  $W^*$  determines a downward revision of

real wage expectations which is responsible for the lagged increase observed in output and employment and the corresponding lagged decline in the unemployment rate. In turn, the increase in output leads to a persistent increase in the real wage rate and in its expectations that triggers the reduction of output and employment and the increase of the unemployment rate in the direction of their respective steady-state values. Obviously, being related to shocks associated to our measure of extrinsic uncertainty, these adjustments are not contemplated by conventional RBC models.



Figure 4: Impulse response functions to TFP shocks (average values)





Figure 5: Impulse response functions to expenditure shocks (average values)

#### **IV. Concluding Remarks**

In this paper, exploiting the nexus between the market for goods and the labour market offered by the Keynesian principle of effective demand, I tried to integrate the marketperiod adjustments of relative price expectations with the short-run and the long-run dynamics generated by the occurrence of demand and supply shocks typical of RBC models with capital accumulation (cf. Guerrazzi, 2023). Specifically, I built a theoretical framework in which demand shocks were assumed to be driven by self-fulfilling entrepreneurial expectations about the long-run state of the economy that – in turn – were assumed to affect actual relative prices and the investment expenditure measured in wage units (cf. Farmer, 2008; Guerrazzi, 2011, 2012; Guerrazzi and Gelain, 2015). At the same time, in a more conventional way, supply shocks were assumed to affect the production possibilities of firms through their impact on TFP (cf. Kydland and Prescott, 1982; Long and Plosser, 1983; King et al. 1988).

The cyclical behaviour of the theoretical framework developed in this paper suggests that the Keynesian principle of effective demand can be thought as the channel through which both demand and supply shocks exhibit their effects on real macroeconomic magnitudes. In other words, shocks to the expenditure function as well as TFP shocks manifest their effect on output and (un)employment through their impact on the expected demand function, the former through a revision of relative prices and a change in the employed capital stock, and the latter through a change in the joint productivity of the available production factors. Furthermore, assuming a procyclical labour supply and adopting a quite conventional calibration, such a theoretical framework fairly replicated the dynamics of output, (un)employment and real wages observed in the US before the burst of the Great Recession and the COVID-19 pandemic.

There are many important factors that the present analysis left outside. For instance, financial markets and their convulsive evolution certainly have a crucial role in the investment decisions of firms (cf. Guerrazzi, 2015). This is the reason why I didn't explore the dynamics of productive investment and the one of the other components of aggregate demand. This gap could be probably closed by augmenting the model with an

interest rate rule and making some hypothesises about the dynamics of nominal magnitudes. In the same way, government activities and international trade are likely to deeply affect macroeconomic performances (cf. Parui, 2021; Giles and Williams, 1999). The analytical integration of these aspects within the theoretical model introduced in this paper is left to further developments.

#### **Appendix: Steady-state determination**

A steady-state equilibrium is defined as a situation in which the expected value of all the endogenous variables of the model economy described in the main text take a constant value that depends only on the parameters of the model itself. Using the subscript SS to denote the constant expected value of each variable under scrutiny, the elements of such a long-run allocation around which actual values will tend to fluctuate over time can be derived as follows.

First, eq. (13) straightforwardly implies that in a steady-state equilibrium, the expected real wage rate coincides its actual level. Hence,

$$W_{\rm SS}^e = W_{\rm SS}^* \tag{A1}$$

Second, eq.s (2), (10) and (11) allows us to write the equilibrium real wage rate as

$$W_{\rm SS}^* = \left(\frac{(1-c)(1-\alpha)^{\frac{1-\alpha}{\alpha}}A_{\rm SS}^{\frac{1}{\alpha}}K_{\rm SS}}{\hat{l}_{\rm SS}}\right)^{\alpha} \tag{A2}$$

where  $A_{SS} = \exp(\ln \mu/(1-\xi))$  and  $\hat{I}_{SS} = \kappa/(1-\rho)$ .

Third, plugging eq. (A2) into eq. (3) allows us to find the equilibrium stock of capital. Specifically,

$$K_{\rm SS} = (1 - \alpha) \left(\frac{(1 - c)^{\alpha} A_{\rm SS}}{\delta}\right)^{\frac{1}{1 - \alpha}} \hat{I}_{\rm SS} \tag{A3}$$

Fourth, plugging eq. (A3) into eq. (A2), the equilibrium real wage can be written as

$$W_{\rm SS}^* = (1-\alpha) A_{\rm SS}^{\frac{1}{1-\alpha}} \left(\frac{1-c}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{A4}$$

Fifth, eq.s (6), (A3) and (A4) allows us to convey equilibrium employment a

$$L_{\rm SS}^* = \frac{(1-\alpha)I_{\rm SS}}{(1-c)A_{\rm SS}^{\frac{1}{\alpha}}}$$
(A5)

Finally, considering the expressions in eq. (A3)–(A5), eq.s (7) and (8) pins down, respectively, the equilibrium unemployment rate  $(U_{SS}^*)$  and the equilibrium output  $(Y_{SS}^*)$ . Given the parameters values in Table 1, the steady-state values of all the endogenous variables as well as their description are collected in Table A1.

Variable	Description	Value
$Y_{\rm SS}^*$	Output	21.4454
A <sub>SS</sub>	TFP	2.7183
K <sub>SS</sub>	Capital	343.1266
$L^*_{SS}$	Employment	0.9450
$U_{\rm SS}^*$	Unemployment	0.0550
Î <sub>SS</sub>	Investment expenditure (wage units)	0.5906
W <sub>SS</sub>	Real wage	14.5239

 Table A1: Steady-state values

#### References

- Adam, K., and Padula, M. (2011). "Inflation Dynamics and Subjective Expectations in the United States", *Economic Inquiry*, 49-1: 13-25.
- Casarosa, C. (1981). "The Microfundation of Keynes's Aggregate Supply and Expected Demand Analysis", *Economic Journal*, 91-361: 188-94.
- Casarosa, C. (1984). "Aggregate Supply and Expected Demand Analysis in Keynes' *General Theory*: An Essay on the Micro-Foundations", in Baranzini, M. (ed.). *Advances in Economic Theory*, Oxford: Basil Blackwell.
- Cass D., and Shell, K. (1983), "Do Sunspot Matter?", *Journal of Political Economy*, 91-2: 193-227.
- Chang, Y. (2000). "Comovement, Excess Volatility, and Home Production", *Journal of Monetary Economics*, 46-2: 385-96.
- Davidson, P. (1978). *Money and the Real World*, 2<sup>nd</sup> Edition, London and Basingstoke: Macmillan Press.
- Domar, E. D. (1946). "Capital Expansion, Rate of Growth, and Employment", *Econometrica*, 14-2: 137-47.
- Dutt, A. K. (1991-1992). "Expectations and Equilibrium: Implications for Keynes, the Neo-Ricardian Keynesians, and the Post Keynesians", *Journal of Post Keynesian Economics*, 14-2: 205-24.
- Farmer, R. E. A. (2008). "Old Keynesian economics", in Farmer, R. E. A. (ed.). Macroeconomics in the Small and the Large: Essays on Microfoundations, Macroeconomic Applications and Economic History in Honor of Axel Leijonhufvud, Northampton, MA: Edward Elgar Publishing.
- Farmer, R. E. A. (2010). *Expectations, Employment and Prices*, Oxford: Oxford University Press.
- Fazzari, S. M., Ferri, P., and Variato, A. M. (2020). "Demand-Led Growth and Accommodating Supply", *Cambridge Journal of Economics*, 44-3: 583–605.

- Giles, J. A., and Williams, C. L. (1999), "Export-led Growth: A Survey of the Empirical Literature and Some Noncausality Results", *Econometrics Working Papers*, 9901.
- Gray, J. A. (1976), "Wage Indexation: A Macroeconomic Approach", *Journal of Monetary Economics*, 2-2: 221-35.
- Guerrazzi, M. (2010). "Nominal Wage Indexation, Quasi Equilibria and Real Wage Dynamics", *Bulletin of Economic Research*, 62-3: 279-94.
- Guerrazzi, M. (2011). "Search and Stochastic Dynamics in the Old Keynesian Economics: A Rationale for the Shimer's Puzzle", *Metroeconomica*, 62-4: 561-86.
- Guerrazzi, M. (2012). "Expectations, Employment and Prices: A Suggested Interpretation of the New «Farmerian» Economics", *Economia Politica*, 28-3: 369-95.
- Guerrazzi, M. (2015). "Animal Spirits, Investment and Unemployment: An Old Keynesian View of the Great Recession", *EconomiA*, 16-3: 343-58.
- Guerrazzi, M. (2023). "The Keynesian Nexus Between the Market for Goods and the Labour Market", *International Review of Economics*, 70-2: 195-216.
- Guerrazzi, M., and Gelain, P. (2015). "A Demand-Driven Search Model with Self-Fulfilling Expectations: The New 'Farmerian' Framework Under Scrutiny", *International Review of Applied Economics*, 29-1: 81-104.
- Harrod, R. F. (1939). "An Essay in Dynamic Theory", *Economic Journal*, 49-193: 14-33.
- Hodrick, R. J., and Prescott, E. C. (1997). "Postwar U.S. Business Cycles: An Empirical Investigation", *Journal of Money, Credit and Banking*, 29-1: 1-16.
- Keynes, J. M. (1921). A Treatise on Probability, London: MacMillan and Company.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*, London: MacMillan and Company.
- King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). "Production, Growth and Business Cycles: I. The Basic Neoclassical Model", *Journal of Monetary Economics*, 21-2-3: 195-232.
- Kurz, H. D. (1990). "Effective Demand, Employment and Capital Utilisation in the Short Run", *Cambridge Journal of Economics*, 14-2: 205-17.
- Kurz, M. (2008). "Beauty Contests Under Private Information and Diverse Beliefs: How Different?", *Journal of Mathematical Economics*, 44-7-8: 762–84.
- Kregel, J. A. (1976). "Economic Methodology in the Face of Uncertainty: The Modelling Methods of Keynes and the Post-Keynesians", *Economic Journal*, 86-342: 209-25.
- Kydland, F. E., and Prescott, E. C. (1982). "Time to Build and Aggregate Fluctuations", *Econometrica*, 50-6: 1345-70.
- Long, J. B., and Plosser, C. I. (1983), "Real Business Cycles", Journal of Political Economy, 91-1: 39-69.
- Lucas, R. E. (1967). "Adjustment Costs and the Theory of Supply", *Journal of Political Economy*, 75-4: 321-34.

- Lucas, R. E. (1981). *Studies in Business-Cycle Theory, Cambridge*, Cambridge MA: MIT Press.
- Nerlove, M., and Arrow, K. J. (1962). "Optimal Advertising Policy under Dynamic Conditions", *Economica*, 29-114: 129-142.
- Parui, P. (2021). "Government Expenditure and Economic Growth: A Post-Keynesian Analysis", *International Review of Applied Economics*, 35-3-4: 597-625.
- Ravn, M. O., and Simonelli, S. (2007). "Labor Market Dynamics and the Business Cycle: Structural Evidence for the United States", *Scandinavian Journal of Economics*, 109-4: 743-77.
- Robbins, L. (1968). *The Theory of Economic Development in the History of Economic Thought*, London: Palgrave Macmillan.
- Shiozawa, Y. (2021). "The Principle of Effective Demand: A New Formulation", *Review* of Keynesian Studies, 3: 67-95.
- Shimer, R. (2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", *American Economic Review*, 95-1: 25-49.
- Silverberg, G. (1987). "Technical Progress, Capital Accumulation, and Effective Demand: A Self-Organization Model", in Batten, D., Casti, J. L., and Johansson, B. (eds.). Economic Evolution and Structural Adjustment. Lecture Notes in Economics and Mathematical Systems, Berlin: Springer, Heidelberg.
- Smith, M. (2012). "Demand-led Growth Theory: A Historical Approach", *Review of Political Economy*, 24-4: 543-73.
- Solow, R. M. (1956). "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics*, 70-1: 65-94.
- Souleles, N. S. (2002). "Consumer Response to the Reagan Tax Cuts", *Journal of Public Economics*, 85-1: 99-120.
- Turner, D., Boone, L., Giorno, C., Meacci, M., Rae, D. and Richardson, P. (2001), "Estimating the Structural Rate of Unemployment for the OECD Countries", OECD Economic Studies, 33-2: 171-216.
- Tüzemen, D. (2017). "Labor Market Dynamics with Endogenous Labor Force Participation and On-the-Job Search", *Journal of Economic Dynamics and Control*, 75-C: 28-51.
- Tüzemen, D., and Van Zandweghe, W. (2018). "The Cyclical Behavior of Labor Force Participation", *Research Working Papers of the Federal Reserve of Kansas City*, 18.
- Vandenborre, H. (1958). "An Integration of Employment Economics within the Keynesian Theory of Money Flows", Oxford Economic Papers, 10-2: 205-19.
- Woodford, M. (1986). "Stationary Sunspot Equilibria in a Finance Constrained Economy", *Journal of Economic Theory*, 40-1: 128-37.

## Marco Guerrazzi

Department of Economics – DIEC University of Genoa Genoa, Italy