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RESEARCH ARTICLE

The Keynesian nexus between the market for goods and the labour market

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Abstract
In this paper, I build on the Keynesian analysis of the market for goods to draw some implications on the dynamic behaviour of some typical labour market indicators. Specifically, focusing on real magnitudes and distinguishing between the aggregate expected demand function and the aggregate expenditure function, I discuss the implied "daily" adjustments of expected and actual real wages that allow to achieve a short-run equilibrium. In addition, in order to show that the suggested picture of market for goods does not require a distinct setting to describe the transactions of labour services, I offer a rationale for equilibrium unemployment due to deficient demand grounded on the searching-and-matching theory.

Keywords Keynesian economics · Expected demand · Expenditure function · Aggregate supply

JEL Classification E12 · E24 · J31 · J64

1 Introduction

Starting from the introductory textbook on economics by Samuelson (1948, Chapter 12), the macroeconomic representation of the market for goods and services in the short run relies on a two-dimensional diagram which is known in the literature as the Keynesian cross (cf. Mankiw 1988). In sharp contrast with the Marshallian cross that relates prices and quantities in the conventional picture of the market for a given commodity, the Keynesian cross completely abstracts from prices and explains the determination of national income through the interaction of two quantity schedules. Specifically, considering a closed economy without any government expenditure, the former conveys the aggregate expenditures of households and firms—namely, consumption and investment—for any level of

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income. Such a schedule sometimes has been called aggregate expenditure—or aggregate demand—function (cf. Blanchard 2020). Given that some expenditures are assumed to be unrelated to income—for instance, for reasons of subsistence and/or for the urge of action of entrepreneurs—and taking into account that economic agents usually reply to increases in their income by saving a share of their additional resources, the vertical intercept of the aggregate expenditure function is positive, but its slope is lower than one. The latter schedule of the Keynesian cross simply represents all the pairs in which the national product is exactly equal to the aggregate expenditure and therefore its slope is identically equal to one. Such a 45°-line is often dubbed as the aggregate supply function even if it does not collect any reference to output technology and production costs (cf. Casarosa 1998).

The intersection between the aggregate expenditure and the aggregate supply function pins down the real value of national income in the short run; indeed, in that equilibrium point, firms are producing exactly the amount of goods that consumers and investors as a whole intend to purchase. Consequently, none of these actors have any incentive to change its behaviour even if the achieved allocation is different from full employment and some workers remain involuntarily unemployed. Furthermore, the Keynesian cross is also used to provide a graphical rationale for the Keynesian multiplier, i.e., the dynamic process according to which an increase in the autonomous components of aggregate expenditure may be able to generate—after some periods of time—an increase of equilibrium output which is strictly higher than the initial stimulus (cf. Perotti 2005).

Despite its clarity and its didactic worth, the theory underlying the functioning of the market for goods encapsulated into the textbook Keynesian cross is quite distant from the formulation of the central ideas crystallized by Keynes (1936) in his *General Theory*. Specifically, when he introduced the principle of effective demand (Chapter 3) and when he fixed the units of measure of his theoretical analysis (Chapter 4), Keynes (1936) implicitly traced out a strong link between the market for goods and the labour market without neglecting the possibility that the achievement of a short-run equilibrium—qualitatively similar to the one described above—may involve some adjustments in wages and prices (cf. Hartwig 2006). Consequently, the conventional picture of the Keynesian cross has to be somehow enlarged in order to accommodate the possibility of simultaneous price and quantity adjustments.

In this paper, drawing on an array of works by Casarosa (1981, 1984), I build on the analysis of the market for goods developed by Keynes (1936) in the *General Theory* to draw some of implications on the dynamic behaviour of some typical labour market indicators by extending the traditional analysis underlying the Keynesian cross. Specifically, focusing on real magnitudes and distinguishing between the aggregate expected demand function and the aggregate expenditure function, I discuss the implied "daily"—or market period—adjustments of real wages that may lead to the simultaneous balance between the aggregate expenditure, the aggregate expected demand and the aggregate supply functions that qualify a short-run equilibrium. In addition, drawing on some recent papers and a book by Farmer (2008, 2010, 2013), I offer a rationale for short-run equilibrium unemployment due to deficient demand grounded on the searching-and-matching theory (cf. Pissarides 2000).

From a theoretical perspective, the analysis developed in this paper aims at achieving two different goals. On the one hand, making some assumptions about the "daily" revision of entrepreneurs' wage expectations, I explore the stability of the Keynesian short-run equilibrium, an issue that has never been explicitly explored before (cf. Dutt 1987, 1991–1992; Rivot 2020). On the other hand, avoiding to refer to the traditional representation of the labour market grounded on labour demand and supply schedules, I give some insights about the institutional setting underlying labour transactions in a model economy in which equilibrium output and employment are driven by aggregate demand and some workers may remain without an occupation despite their willingness to work at the prevailing level of the real wage (cf. Guerrazzi 2011, 2012; Guerrazzi and Gelain 2015).

The paper is arranged as follows. Section 2 explores the microfoundation of firms' behaviour offered by Keynes (1936) in the *General Theory*. Section 3 derives the "daily" and the short-run equilibrium of the economic system as a whole. Section 4 addresses the stability of the short-run equilibrium by addressing the implied adjustments of real wages and employment. Section 5 reinterprets the short-run equilibrium of the economic system as the resting point of a searching-and-matching process with persistent unemployment. Finally, Sect. 6 concludes.

2 The microfoundation of firms' behaviour in the *General Theory*

Drawing on Casarosa (1981, 1984), here I develop an analytical framework that summarizes the microfoundation of firms' behaviour suggested by Keynes (1936). Specifically, I consider a model economy in which there are r identical firms and where the time horizon of entrepreneurial choices is so limited that it does not allow for variations in the installed productive capacity. Nevertheless, producers are assumed to be in the position to revise their decisions on how much workers to employ in their plants. Therefore, arranged plans for output and employment can be compared with realized outcomes (cf. Gnos 2004). Within this economy, each firm has access to the following production technology:

$$y_i = f\left(n_i\right) \quad i = 1, \dots, r \tag{1}$$

where y_i is the output of the representative firm, $f(\cdot)$ is its production function, whereas n_i is the level of employment at the i-th firm.

The production function of the representative firm is assumed to be well-behaved so that for each producer it holds true that

$$f(0) = 0$$

$$f'\left(n_i\right) > 0 \quad \forall n_i > 0 \quad f''\left(n_i\right)\langle 0 \quad \forall n_i \rangle 0 \quad i = 1, \dots, r \tag{2}$$

$$\lim_{n_i \to 0} f'\left(n_i\right) = \infty$$

$$\lim_{n_i \to \infty} f'\left(n_i\right) = 0$$

The properties of the production function summarized in (2) reveals that labour is an essential production factor and that its marginal productivity is positive but decreasing. In addition, $f(\cdot)$ is assumed to satisfy the Inada conditions.

From a behavioural perspective, Keynes (1936) acknowledged the theory of the competitive firm as developed by Marshall (1920). Aiming at aggregating the choices of individual entrepreneurs in a laissez-faire environment, however, he introduced the concepts of supply and expected demand functions defined in terms of proceeds—or revenues—for the individual firm. At first, the supply function for the i-th firm is given by the actual proceeds generated by selling the output produced by employing a given number of workers. Formally, speaking the supply function of the representative firm can be written as

$$z_i = p_i^S f\left(n_i\right) \quad i = 1, \dots, r \tag{3}$$

where z_i are the nominal revenues deriving from selling $f\left(n_i\right)$ units of goods whereas p_i^S is the supply price of a unit of output.

In a competitive economy, recalling the labour is the only variable factor over the time horizon under scrutiny, p_i^S is necessarily equal to the marginal cost of employing an additional employee. Given that employment and output at the firm level are linked by the technological constraint summarized by Eq. (1), the marginal cost of employing an additional employee is equal to the nominal wage rate paid to the individual worker divided by the additional output generated by the employment of that worker. Consequently, the analytical expression for the supply price of a unit of output will be given by

$$p_i^S = \frac{w}{f'\left(n_i\right)} \quad i = 1, \dots, r \tag{4}$$

where w is nominal wage rate taken as given by each producer.

Plugging the expression in Eq. (4) into Eq. (3), the supply function of the representative firm becomes

$$z_i = w\frac{f\left(n_i\right)}{f'\left(n_i\right)} \quad i = 1, \dots, r \tag{5}$$

Given the assumptions on technology itemized in (2), for any given level of w, the supply function in Eq. (5) is increasing in n_i. Specifically, if $f\left(n_i\right)$ has a constant elasticity with respected to the labour input and the nominal wage is exogenously given, then z_i is simply a linear function of n_i.

According to the principle of effective demand introduced in the Chapter 3 of the *General Theory*, the individual firm would tend to employ the number of workers which is consistent with its exogenous expectations for the price of the produced good and the wage to pay to the employed workers (cf. Keynes 1936). Consequently, the expected demand function for the i-th firm can be written as

$$d_i^e = p_i^e f(n_i) \quad i = 1, \ldots, r \tag{6}$$

where d_i are the expected revenues from employing n_i workers whereas $p_i^e > 0$ is the expected price of good produced by the representative firm.

Given the exogenous value of p_i^e, the derivation of the firm's expected revenues directly from the values of produced output means that each entrepreneur atomistically believes that she/he can sell everything that she/he is able to produce at that price (cf. Torr 1984). Consequently, net of the scalar p_i^e, the expected demand function in Eq. (6) will share the same geometrical properties of the production function catalogued in (2).

As correctly argued by Casarosa (1981, 1984), the maximization of the expected profits of the representative firm implies the equilibrium between the supply and the expected demand function as defined in Eqs. (3) and (6). In fact, the expected profits of the i-th firm are given by

$$p_i^e f(n_i) - wn_i - SC \quad i = 1, \ldots, r \tag{7}$$

where $SC > 0$ is the fixed user cost of employed capital.

Considering the expression in Eq. (7), the first-order condition (FOC) for profit maximization implies that

$$p_i^e f'(n_i) - w = 0 \quad i = 1, \ldots, r \tag{8}$$

Given the values of p_i^e and w, Eq. (8) provides the number of workers that the firm will find profitable to employ when it expects the revenues conveyed by Eq. (6).

After a trivial manipulation, Eq. (8) can be written as

$$d_i^e = z_i \quad i = 1, \ldots, r \tag{9}$$

As anticipated above, the expression in Eq. (9) explicitly reveals that the maximization of the firm's profits implies the equality between the supply and the expected demand function as defined by Eqs. (5) and (6) for all the firms operating in the economy. As effectively argued by Farmer (2010), such an equilibrium condition suggests that the supply price conveyed by Eq. (4) is perfectly consistent with the idea of entrepreneurs that compete one another for the production factors—in this case the labour input only—by means of price adjustments. Considering a given value of p_i^e and a given value of w, the equilibrium condition in Eq. (9) is illustrated in Fig. 1.

The level of employment denoted by $\overline{n}_i$ in Fig. 1 is the one that fulfils Eq. (9) and it can be defined as the equilibrium employment for the "day"—or the Marshallian market period—such that the expected revenues of the representative firm are exactly equal to the revenues deriving from selling the corresponding amount of produced output.[1] In other words, according to the production technology summarized

[1] Keynes (1936, Chapter 5) defined the "day" as "the shortest interval after which the firm is free to revise its decision as to how much employment to offer. It is, so to speak, the minimum effective unit of economic time."

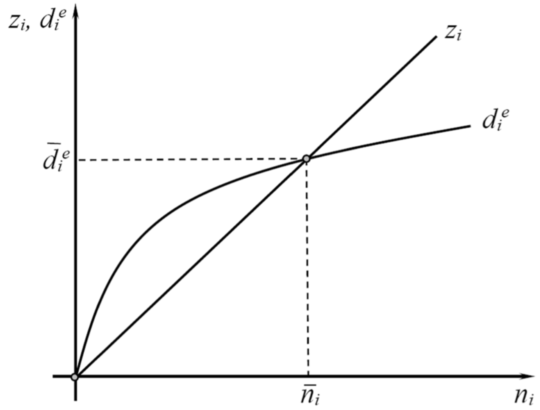


Fig. 1 The "daily" equilibrium for the single firm

by Eq. (1), $\bar{n}_i$ is the level of employment that realizes the equality between the expected output price of the individual producer and its supply price of each unit of output.[2] In parallel, $\bar{d}_i^e$ is the "daily" equilibrium of the expected demand—or the effective demand—of the i-th firm.

3 The "daily" and the short-run equilibrium of the economic system

Given the entrepreneurs' price expectations, the aggregate "daily" equilibrium of employment and the aggregate expected demand could be derived by summing up the "daily" equilibrium level of employment for the single firm and the corresponding level of the expected—or effective—demand. Keynes (1936), however, derived aggregate magnitudes by extending to the whole economic system the microeconomic analysis developed in the previous section (cf. Casarosa 1981, 1984). In other words, Keynes (1936) derived the "daily" equilibrium of aggregate employment and the aggregate effective demand relying on the definition of an aggregate supply function and an aggregate expected demand function both measured in wage units. Such a choice of the units of measure for the aggregate supply and the aggregate expected demand functions is motivated by the Keynes' (1936) willingness to work with macroeconomic schedules that mirror the volume of nominal transitions in the market for goods but depend only on aggregate employment. Indeed, the definition of an aggregate price index allows us to sum the values of all the heterogeneous commodities that form the estimate of national output. Although the labour input is just heterogeneous as produced commodities, however, dividing the value of such a set of variegated commodities by the nominal wage allows us to measure the

[2] It is worth noticing that the shutdown equilibrium $n_i = 0$ implies the equality between the supply and the expected demand functions. Such an allocation, however, is not consistent with profit maximization.

aggregate supply and the aggregate expected demand functions in terms of "effective" units of labour.[3]

Let us now define the aggregate supply function. If we assume that there no production externalities, then the aggregate output of the model economy can be easily obtained by summing up the individual output of the single firms. Formally speaking, considering the expression in Eq. (1) and recalling that r is the number of firms in the model economy, the aggregate output is given by

$$Y(N) = rf\left(\frac{N}{r}\right) \tag{10}$$

where $N \equiv rn_i$ is the aggregate level of employment.

Given the properties of the individual production function collected in (2) and the fixed value of r, the expression in Eq. (10) is a two-time differentiable function that depends only on N. Consequently, the aggregate supply function can be written as

$$Z = w\frac{Y(N)}{Y'(N)} \tag{11}$$

where Z are the nominal revenues collected by all the firms in the economy.

Dividing the two sides of Eq. (11) by w, we find the aggregate supply function in wage units that will depends on aggregate employment only. Specifically,

$$Z_w = \frac{Y(N)}{Y'(N)} \tag{12}$$

where $Z_w \equiv Z/w$ are the nominal revenues collected by all the firms in the economy in terms of the money wage.

According to the properties of the individual production function itemized in (2), the aggregate supply function in Eq. (12) will be unambiguously an increasing function of aggregate employment.

Let us now consider the demand side of the model economy. Assuming that entrepreneurs have identical price expectations, i.e., making the assumption that $p_i^e = p_e > 0 \; \forall \; i = 1, \ldots r$, the aggregated expected demand function can be written as

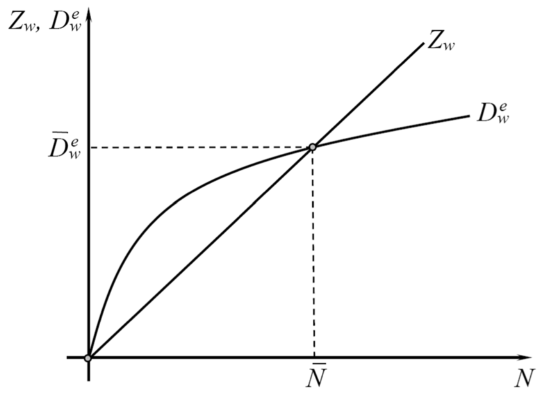
$$D^e = p^e Y(N) \tag{13}$$

where D^e are the expected revenues of all the firms operating in the economy.

Dividing the two sides of Eq. (13) by w, we find the aggregate expected demand function in wage units that will depends on the expected price-wage ratio—or the inverse of the expected real wage—and on aggregate employment. Specifically,

[3] In Keynes's (1936, Chapter 4) words: "the quantity of employment can be sufficiently defined for our purpose by taking an hour's employment of ordinary labour as our unit and weighting an hour's employment of special labour in proportion to its remuneration; i.e. an hour of special labour remunerated at double ordinary rates will count as two units".

Fig. 2 The "daily" equilibrium for the whole economy



$$D_w^e = \frac{Y(N)}{W^e} \tag{14}$$

where $D_w^e \equiv D^e/w$ are the aggregate expected revenues measured in wage units whereas $W^e \equiv w/p^e$ is the expected real wage rate or, equivalently, the inverse of the expected price-wage ratio.

Given the exogenous value of W^e, the expression in Eq. (14) straightforwardly reveals that the aggregate expected demand function mirrors the properties of the aggregate output defined in Eq. (10) so that it will be an increasing function of aggregate employment. Obviously, this means to assume—as we did at the micro level—that relative prices are unrelated to aggregate employment (cf. Torr 1984).

At the aggregate level, a "daily" equilibrium for the model economy is given by the following equality:

$$D_w^e = Z_w \tag{15}$$

As illustrated in Fig. 2, given the level of the expected real wage, the equilibrium condition in Eq. (15) provides the level of aggregate employment—denoted by $\overline{N}$—such that the aggregate expected demand is equal to the aggregate supply function. For the same arguments underlying the diagram in Fig. 1, $\overline{D}_w^e$ is the "daily" equilibrium of the aggregate expected demand—or the aggregate effective demand—for the whole firms operating in the economy and it is consistent with the maximization of aggregate profits. Obviously, the retrieved value of $\overline{N}$ is indexed by the level of W^e so that the aggregate employment for the "day" is not unique and it depends on the real wage expectations of the entrepreneurs. Specifically, the higher the value of W^e, the lower the value of $\overline{N}$ and vice versa.

When the r productive firms operating in the economy employ the $\overline{N}$ workers implied the "daily" equilibrium illustrated in Fig. 2 their choices generate—in terms of produced output—a certain purchasing power for consumers and investors and such a purchasing power will generate a given level of aggregate expenditure. Each individual firm, consumer and investor take their respective decisions and form their expectations in isolation from the other actors, so there is no guarantee that the purchasing power generated by the employment of $\overline{N}$ workers is actually equal to the

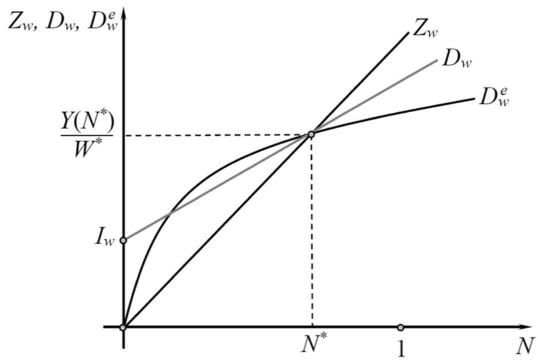


Fig. 3 The short-run equilibrium

aggregate expenditure. Consequently, in order to determine the short-run equilibrium prevailing at the macroeconomic level and to analyse the interaction between the decisions of producers and buyers, we have to introduce an aggregate expenditure function that conveys the behaviour of consumers and investors as a whole according to the value of realized proceeds. Along the lines of Casarosa (1981, 1984), I assume that the aggregate expenditure function is given by a linear function of the aggregate supply function such as

$$D_w = cZ_w + I_w \tag{16}$$

where D_w is the value of the aggregate expenditure of consumers and investors measured in wage units, $c \in (0, 1)$ is a measure the of the reactivity of aggregate expenditures with respect to the aggregate proceeds pocketed by firms, whereas $I_w > 0$ is the autonomous component of aggregate demand.

As recalled in the introduction, the hypotheses on c and I_w introduced above follow from the observation that usually consumers and investors tend to reply to increases in their economic means by saving a share of their additional resources and the fact that a share of the aggregate expenditure is unrelated to Z_w. Regarding the second point, the constancy of I_w can be explained by the constancy of some consumption expenditures and/or the animal spirits of entrepreneurs that are willing to adjust their productive capacity no matter the path of their actual proceeds (cf. Keynes 1936, Chapter 12).[4]

For a given value of W^e, a short-run equilibrium is given by a situation in which the purchasing power generated by the employment of the implied number of workers of the "day" is exactly equal to the aggregate expenditure of consumers and investors so that the prevailing value of real wage is equal to its expected counterpart. Consequently, given the expressions in Eqs. (12), (14) and (16), the condition for a short-run equilibrium necessarily involves the intersection of three distinct relationships. Specifically, from a formal point of view, we have a short-run equilibrium whenever

[4] To be precise, in the present context the constancy of I_w implies to assume that the short run the autonomous component of aggregate demand is proportional to the real wage rate.

$$D_w = D_w^e = Z_w \tag{17}$$

As Illustrated in Fig. 3, when the condition in Eq. (17) holds true, aggregate employment is equal to N^* and the aggregate expenditures that consumers and investors intend to make as a whole are exactly equal to the expected revenues of producers. In terms of wages, this means that the real wage expected by entrepreneurs lead them to employ an amount of workers that—in turn—generates a purchasing power such that consumers and investors find profitable to purchase the corresponding amount of output. In other words, the prevailing level of the real wage —denoted by W^*—implies that the profit-maximizing level of employment for each firm is consistent with the equilibrium between the aggregate supply function and the aggregate expenditure function expressed in wage units conveyed by Eqs. (12) and (14). Obviously, in this situation the real wage expectations of producers coincide with actual outcomes and so no agent has any incentive to change its behaviour.

The diagram in Fig. 3 deserves some additional remarks. First, given the assumed shapes of D_w and Z_w, there is only one meaningful short-run equilibrium allocation despite the multiplicity of the expectational-driven "daily" equilibria described in the previous section. Second, there is no reason to expect that N^* coincides with the full employment allocation. By contrast, according to Keynes (1936), it may well happen that at the real wage W^* some unemployed workers would be willing to work so that—normalizing to 1 the measure of the aggregate labour force—it will usually be that $N^* < 1$. In addition, even if they coincide when aggregate employment reaches its short-run equilibrium value, the aggregate demand function and the aggregate expenditure function remain distinct objects. In this regard, some authors argued that the expected demand function should be considered as the entrepreneurs' expectations of the expenditure function (cf. Millar 1972; Patinkin 1976; Wells 1978; Davidson 1978). As revealed by the analysis of the present and the previous sections, however, this reading is quite misleading; indeed, as pointed out by Casarosa (1981, 1984), such an outcome could be achieved only in a non-competitive environment in which producers are not price takers. Stated differently, the expected demand function of the entrepreneurs can be actually considered as the expectation of the aggregate expenditure function only by assuming that each producer is trying to guess the impact of her/his output and employment decisions on the demand function of the commodity she/he produces and hence on its supply price. In turn, this implies that each firm has to make a fair guess on how the output and employment decisions of the other firms are related to its own decisions and on how consumers and investors react to the output and employment decisions of the firms as a whole. Obviously, these assumptions would make sense only if the production of commodities were concentrated in the plants owned by only one or few producers, but they cannot be accepted in an atomistic competitive market for goods and services.

4 The stability of the short-run equilibrium

Considering that entrepreneurs form their real wage expectations in an atomistic and uncoordinated manner, there is no certainty that the "daily" equilibrium will coincide with the unique short-run equilibrium illustrated in Fig. 3. In other words, the expected wage rate does not necessary coincide with the actual real wage rate so that the equilibrium level of employment observed in a given "day" does not necessarily coincide with its short-run equilibrium level. Specifically, when W^e is above (below) W^* so that employment is below (above) its short-run equilibrium level, the aggregate expenditure function is above (below) the aggregate expected demand function so that firms realize a level of profits above (below) the expected level either because the actual "daily" price is higher (lower) than expected. In this case, the entrepreneurs' expectation will be proved wrong and therefore they will tend to revise them and to change their employment and output decisions by targeting a different level of profits. According to Keynes (1936), such a "daily" revision process should lead the economic system as a whole to gravitate closely around its short-run equilibrium (cf. Dutt 1987, 1991–1992; Rivot 2020).

In order to explore the stability of a short-run equilibrium from an analytical perspective, it is necessary to make some assumptions about the shape of the individual production function and the way in which producers adjust "day-by-day" their real wage expectations when they are inconsistent with actual outcomes. On the one hand, aiming at excluding the presence of production externalities, I will consider an individual production function such that aggregate output depends on aggregate employment only so that—at the aggregate level—there are no scaling effects. In this direction, for the sake of simplicity, I will assume that the individual production function is given by the following constant-elasticity function:

$$f\left(n_i\right) = \frac{n_i^\alpha}{r^{1-\alpha}} \quad i = 1, \dots, r \tag{18}$$

where $\alpha \in (0, 1)$ is the elasticity of output with respect to employment.

Given the level of n_i, the production function in Eq. (18) conveys the idea that the larger the number of firms in the model economy, the lower the output supply of the single producer. Considering Eq. (18), such an expression implies that aggregate output simply reduces to

$$Y(N) = N^\alpha \tag{19}$$

Equation (19) straightforwardly implies that the aggregate supply function expressed in wage units is a linear function of aggregate employment whose slope is equal to α^{-1} (cf. Davidson 1962).

On the other hand, I will assume that entrepreneurs adjust their expected value of the real wage rate according to the following adaptive process:

$$W_t^e = W_{t-1}^e + \lambda\left(\overline{W}_{t-1} - W_{t-1}^e\right) \tag{20}$$

where t denotes the "day", $\overline{W}_t$ is the actual "daily" real wage rate in t, whereas $\lambda \in (0, 1)$ is a parameter that conveys how firms revise "day-by-day" their expectations for the real wage on account of the forecasting error experienced in the previous period.

The adjustment process of the expected real wage described by Eq. (20) implies that entrepreneurs tend to adjust the value of W^e for the "day" in response to deviations of actual "daily" profits from their expected value. Specifically, entrepreneurs tend to reduce (increase) W^e whenever their actual "daily" profits are above (below) their expected value by targeting a point on their marginal labour productivity schedules in which profit expectations are perfectly verified.

Considering the expression in Eq. (19), Eqs. (12) and (16) imply that the aggregate employment level and the real wage prevailing in the short run are respectively given by

$$N^* = \frac{\alpha I_w}{1 - c} \tag{21}$$

$$W^* = \alpha^\alpha \left(\frac{1 - c}{I_w} \right)^{1-\alpha} \tag{22}$$

The expression in Eq. (21) reveals that—in the short-run—variations in the autonomous component of aggregate expenditure leads to parallel variations in aggregate employment. That was the original intuition underlying the Keynesian multiplier (cf. Kahn 1931).[5] Furthermore, in adherence to the first postulate of the Classical economy, Eq. (22) implies that a short-run increase (decline) in employment can only occur to the accompaniment of a reduction (increase) in the equilibrium real wage (cf. Keynes 1936).

Following a similar procedure, plugging the expression in Eq. (19) into Eqs. (12), (14) and (16) allows to show that the aggregate employment level and the actual real wage for the "day" are respectively given by

$$\overline{N} = \left(\frac{\alpha}{W^e} \right)^{\frac{1}{1-\alpha}} \tag{23}$$

$$\overline{W} = \frac{\left(\frac{\alpha}{W^e} \right)^{\frac{1}{1-\alpha}}}{\frac{c}{\alpha} \left(\frac{\alpha}{W^e} \right)^{\frac{1}{1-\alpha}} + I_w} \tag{24}$$

The expressions in Eqs. (20) and (24) imply that the "daily" adjustment of the real wage expectations is the described by the following non-linear dynamic process:

[5] Equation (21) also implies that the elasticity of the aggregate expenditure function evaluated in N^* is simply equal to c.

Table 1 Baseline calibration

Parameter	Description	Value
α	Output elasticity with respect to labour	0.64
c	Aggregate expenditure reactivity	0.75
I_w	Autonomous expenditure	0.3711
λ	Wage expectations' reactivity	0.26

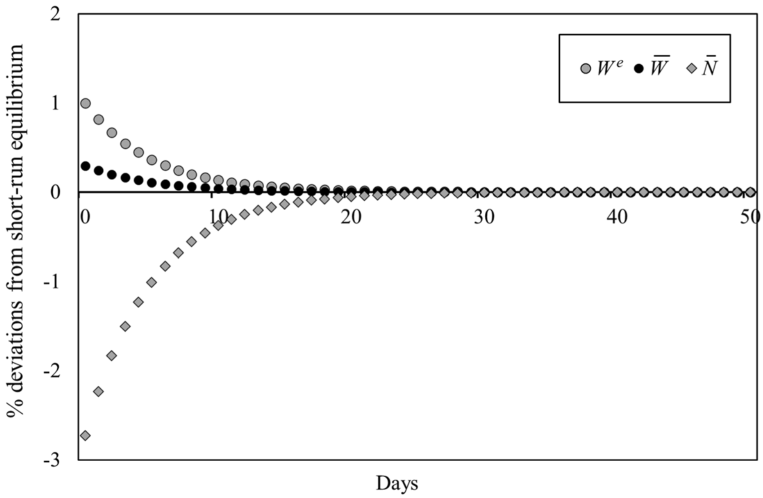


Fig. 4 Wage and employment adjustments towards a short-run equilibrium (Baseline calibration)

$$W_t^e = W_{t-1}^e + \lambda\left(\frac{\left(\frac{\alpha}{W_{t-1}^e}\right)^{\frac{1}{1-\alpha}}}{\frac{c}{\alpha}\left(\frac{\alpha}{W_{t-1}^e}\right)^{\frac{1}{1-\alpha}} + I_w} - W_{t-1}^e\right) \tag{25}$$

Straightforward algebra reveals that the steady-state value of the process in Eq. (25) coincides with the short-run equilibrium value of the real wage conveyed by Eq. (22). Consequently, if the real wage expectations converge towards their short-run reference, then even aggregate employment converges to its short-run equilibrium level conveyed instead by Eq. (21). In order to explore the convergence of the dynamic process in Eq. (25), I rely on some computational experiments grounded on the baseline calibration shown in Table 1.

The model calibration collected in Table 1 takes as a reference the US economy. Specifically, the elasticity of output with respect to the labour input is set at the value chosen by Kydland and Prescott (1982), whereas the value of c is taken by averaging the different estimations of the marginal propensity to consume retrieved by Souleles (2002), who finds point values between 0.6 and 0.9. Thereafter, given the figures for α and c and recalling that 1 is assumed to be the size of the available

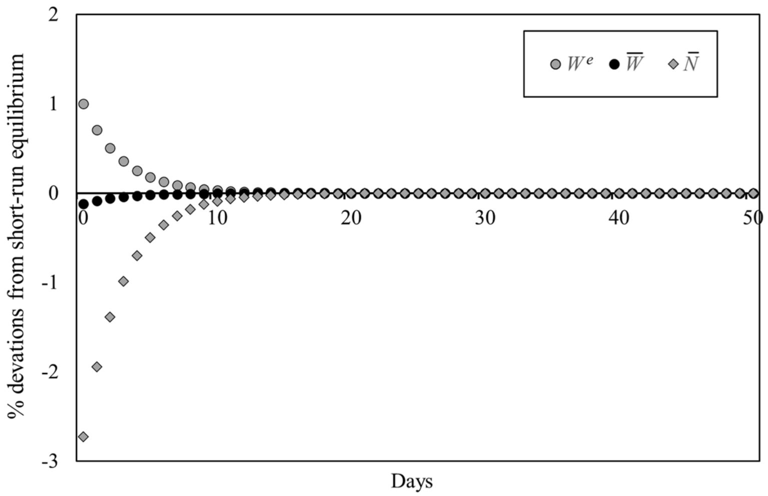


Fig. 5 Wage and employment adjustments towards a short-run equilibrium. ($\alpha = 0.64$, $c = 0.6$, $I_w = 0.5938$, $\lambda = 0.26$)

labour force, the value of the autonomous expenditure I_w is set to be consistent with a short-run unemployment rate of 5% (cf. Guerrazzi 2022). In addition, without any loss of generality, the value of λ is taken from the work by Coibion et al. (2018) on inflation expectations.

Exploiting the parameters' value in Table 1 and assuming that the initial "daily" real-wage expectation is 1% above the short-run equilibrium reference, the adjustments of expected real wages, actual real wage and employment towards a short-run equilibrium implied by Eqs. (23), (24) and (25) are illustrated in Fig. 4.[6]

The diagram in Fig. 4 shows that when the real wage expected for the "day" overshoots its short-run equilibrium value by 1%, the "daily" level of employment (real wage) undershoots (overshoots) its equilibrium reference by 2.72% (0.29%). Thereafter, all the mentioned variables monotonically convergence towards their short-run equilibrium values. Obviously, this dynamic pattern implies that actual real wages and employment tend to move in opposite directions during their adjustment process.

The robustness and the reliability of the trajectories plotted in Fig. 4 can be tested by changing the parameter values used to simulate the model. In this direction, it is worth noticing that baseline calibration collected in Table 1 implies a point value of the multiplier of 2.56, a figure that usually is observed over the medium run (cf. Perotti 2005). A value closer to short-run estimations of the multiplier can be obtained by setting c equal to 0.6 which is lower bound of the estimations of the marginal propensity to consume retrieved by Souleles (2002). Targeting the same level of employment, i.e., setting $I_w = 0.5938$ and exploiting such a value of c, the multiplier

[6] MATLAB codes are available from the author upon reasonable request.

takes the value of 1.6 whereas—using the same values for the remaining parameters—the adjustments of expected real wages, actual real wage and employment towards a short-run equilibrium implied by Eqs. (23), (24) and (25) are illustrated in Fig. 5.

The adjustments of the expected real wages and employment illustrated in Fig. 5 are very similar to the ones reported in Fig. 4. The lower value of the multiplier, however, leads to a completely different path of adjustment for actual wages that now move in the same direction of "daily" employment during their process of convergence; indeed, under this new calibration the real wage for the "day" undershoots its equilibrium value by 0.11% and then it monotonically converges towards its short-run reference.

The output of the trivial computational experiments described above reveal that outside the short-run equilibrium—depending on the value of the multiplier—actual real wages may converge towards their equilibrium value in different ways. In other words, according to the reactivity of the aggregate expenditure function, the point on the marginal labour productivity schedule in which the real wage expectations of entrepreneurs are perfectly verified can be achieved "day-by-day" with real wages that move in the same or in the opposite direction of aggregate employment.[7] By contrast, as we noticed in commenting Eqs. (21) and (22), short-run variations of aggregate employment are always accompanied by real wage movements in the opposite direction. The composition of these dynamic patterns is consistent with the a-cyclical pattern of real wages stressed by many influential contributions (cf. Dunlop 1938; Tarshis 1939; Abraham and Haltiwanger 1995; Huang et al. 2004).

5 The short-run equilibrium as the resting point of a searching-and-matching process

The dynamics of real wages and employment derived in the previous section raise the issue of establishing what kind of institutional setting may be used to describe the transactions of labour services that involve firms and workers. Specifically, while we assumed that entrepreneurs are always on their marginal labour productivity schedules, we didn't make any hypothesis on how workers trade-off consumption against leisure and labour provision.

In a quite recent array of works, Farmer (2008, 2010, 2013) frames an equilibrium qualitatively similar to the one illustrated in Fig. 3 without relying on the traditional—or Marshallian—labour demand and labour supply schedules. By contrast, he views the allocation selected by the aggregate expenditure function on the aggregate supply function as the resting point of a searching-and-matching process in which households have a certain probability to find a job and entrepreneurs have some chances to find suitable candidates for their positions. In the remainder of this section, I will sketch how the short-run equilibrium analysis grounded on aggregate

[7] By continuity, there should be a value of the aggregate expenditure reactivity such that the "daily" real wage jumps immediately to its short-run value without undergoing any adjustment process.

supply and the aggregate expenditure functions can be extended in the direction of the equilibrium unemployment approach popularized by Pissarides (2000).

Farmer (2008, 2010, 2013) acknowledges the concepts of aggregate expenditure and aggregate supply developed in Sect. 3 but he does not consider the issue of "daily" equilibria by working instead on the technological constraints that bind the model economy. Specifically, recognizing that job creation is costly for firms, Farmer (2008, 2010, 2013) assumes that producers have to employ labour not only in the production of goods but also in the workforce recruitment. In other words, he posits that the wasteful recruiting effort that moves jobless workers from home towards production sites is measured in terms of labour instead of produced output. Formally speaking, this assumption on labour allocation will imply that

$$N = X + V \tag{26}$$

where X (V) is fraction of labour allocated in the production of commodities (recruiting activities).

Following Woodford (1986) and Dutt (1987; 1991–1992), in order to ease aggregation, I will assume that in the model economy there are two types of optimizing households that take market prices and matching probabilities as given. In detail, each type refers to an income earners' category which is assumed to be characterized by a specific propensity to consume and a specific task (cf. Guerrazzi 2011, 2012; Guerrazzi and Gelain 2015). On the one hand, there are wage earners—or workers—who are saving-constrained and consume the whole income earned by supplying a fixed amount of labour that—when hired—can be allocated alternatively to production or recruiting activities. On the other hand, there are profit earners—or capitalists—who are more patient and save the whole income earned by employing wage earners and arranging the production process of goods.[8]

Considering a logarithmic utility function that depends only on consumption, the problem of the representative household of wage earners is the following:

$$\max_{H,C} \ln C \tag{27}$$

s.to

$$C \leq WN$$

$$H \leq 1$$

$$U = H - N$$

$$N = hH$$

[8] The proceeds saved by profit earners implicitly define the yield on employed capital and are exploited to finance productive investment.

where C is aggregate consumption, $H \in (0, 1)$ is the measure of wage earners that will search for jobs, U is the unemployment rate whereas $h \in (0, 1)$ is the probability to find a job taken as given by wage earners.[9]

The expressions in (27) reveal that the consumption and the participation choices of the representative household of wage earners are constrained by the fact that consumption (labour provision) cannot exceed the real wage bill (the fixed endowment of labour services). Thereafter, unemployment is given by the member's share who are not employed whereas the share of the employed ones is given by the share of the searching ones who actually find a job.

Because labour does not yield disutility, the solution to the wage earners' problem has the form

$$\begin{aligned} H &= 1 \\ C &= WN \end{aligned} \tag{28}$$

Assuming that the output contribution of wage earners allocated to the production activities has the same elasticity of the individual production function in Eq. (18), the problem of the profit earners' household can be framed as

$$\max_{N} Y - WN \tag{29}$$

s.to

$$I = Y - WN$$

$$Y \leq X^{\alpha}$$

$$N = X + V$$

$$N = vV$$

where I is real investment whereas $v > 1$ is the recruiting effectiveness of employed wage earners taken as given by profit earners.

The expressions in (29) reveal that the employment decisions of the representative household of profit earners are constrained by the fact that real investment has to equal to the share of output not consumed by wage earners whereas produced output cannot exceed the productive contribution of the ones allocated in productive activities. Thereafter, consistently with (26), the total number of employed wage earners is given by the sum between the ones allocated in productive activities and the ones allocated in recruiting activities which are required to hire the desired workforce.

In the present extension of the short-run equilibrium of the market for goods, labour market frictions enter the model economy through an aggregate matching function that combines the searching (recruiting) efforts of wage (profit) earners by remaining unobservable to the two categories of agents. In a time-less short-run equilibrium, such a matching function has to be necessarily equal to the aggregate

9 The adoption of any non-decreasing utility function would not alter the results achieved in this section.

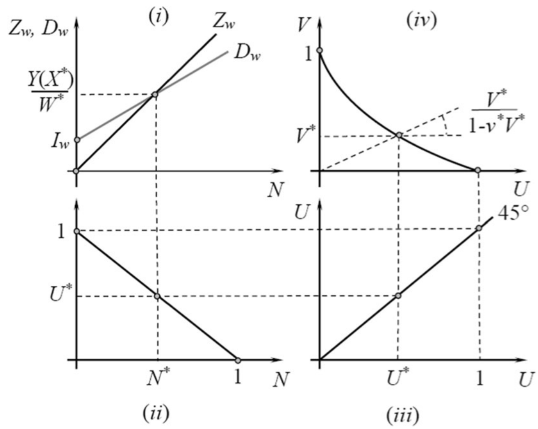


Fig. 6 Short-run equilibrium and matching probabilities

employment generated by the equilibrium on the market for goods (cf. Rogerson et al. 2005). Assuming that the matching function takes a Cobb–Douglas form, this will imply that

$$N = H^{\gamma} V^{1-\gamma} \tag{30}$$

where $\gamma \in (0, 1)$ is the matching elasticity with respect to the searching efforts of wage earners.

Plugging into Eq. (30) the solution of the wage earners' problem itemized in (28) allows us to derive the following version of the Beveridge curve that provides the efficient trade-off between unemployed wage earners and their share employed in recruiting activities:

$$V = (1 - U)^{\frac{1}{1-\gamma}} \tag{31}$$

Furthermore, notice that combining the last three constraints of the profit earners problem, produced output can be written as

$$Y = \left(\left(1 - \frac{1}{v}\right)N\right)^{\alpha} \tag{32}$$

The term $1 - 1/v$ enters the problem in (29) as an aggregate productive externality; indeed, profit earners choose the optimal value of N by taking v as given (cf. Farmer 2008). Once N and U are determined, however, Eq. (31) pins down the optimal fraction of wage earners to allocate in recruiting activities as well as their effectiveness in performing that task.

Given Eq. (32), the FOC for the problem of profit earners is given by

$$\left(\alpha \frac{Y}{N} - W\right)\mu = 0 \tag{33}$$

where μ is the Lagrange multiplier on the real investment constraint in (29).

Recalling the arguments developed in Sect. 3, the schedules that the short-run equilibrium of the market for goods in the searching-and-matching economy can be easily derived. First, the aggregate expenditure function measured in wage units implied by the solution of the wage earners' problem in (28) is given by

$$D_w = N + I_w \tag{34}$$

where $I_w \equiv W^{-1}I$.

Furthermore, the FOC of the problem solved by the household of profit earners in Eq. (33) implies that the aggregate supply function can be written as

$$Z_w = \frac{N}{\alpha} \tag{35}$$

As illustrated in the four panels of Fig. 6, whenever $D_w = Z_w$, the implied level of short-run equilibrium employment together with the optimal searching efforts of wage earners and the equilibrium relationship between (un)employment and recruiters allow to pin down the equilibrium probability to find a job, the equilibrium recruiting effectiveness of labour and the equilibrium real wage rate.

In detail, in panel (*i*) of Fig. 6 there is the short-run equilibrium of the market for goods that pins down the level of employment and the value of national output measured in wage units that according to Eqs. (34) and (35) are given, respectively, by $(\alpha/(1-\alpha))I_w$ and $(1-\alpha)^{-1}I_w$. Moreover, in panel (*ii*) there is the trade-off between employment and unemployment implied by the participation choice of wage earners; indeed, in our model economy the probability to find a job coincides with the fraction of employed wage earners. In addition, in panel (*iv*) there is the Beveridge curve conveyed by Eq. (32) that allows to pin down the short-run equilibrium faction of wage earners allocated in recruiting activities as well as their effectiveness in performing that task. Given these latter variables, the short-run real wage rate can be determined by dividing the real output by the equilibrium value of the aggregate expenditure in wage units.

The intriguing feature of the model economy described above is that the transactions of labour services among wage and profit earners and their price are not mediated neither by a dedicated market nor a bargaining process. The equilibrium of the market for goods determines the real wage, the level of employment and the matching probabilities that lead to the mutual consistencies of the optimal choices of the two categories of agents by avoiding all the difficulties that usually arise in order to explain why workers may be outside their labour supply schedules.

6 Concluding remarks

In this paper, I exploited the microfoundation of the firms' behaviour underlying the analysis of the aggregate supply and the aggregate expected demand functions developed by Casarosa (1981, 1984) to explore the link between the market for goods and the labour market subtly traced out by Keynes (1936) in the *General Theory*.

My theoretical exploration addressed two different issues. On the one hand, drawing on the distinction between aggregate expected demand and aggregate expenditure functions, I studied the "daily" adjustment of real wage expectations by showing that the dynamic correlation between actual wages and employment outside the short-run equilibrium depends on the reactivity of the aggregate expenditure function. Specifically, relying on some computational experiments, I showed that the values of the real wage for the "day" move in the opposite (same) direction of aggregated employment when the reactivity of the aggregate expenditure function and the multiplier of autonomous expenditure are high (low). In addition, I explored the institutional setting that might be used to describe labour transactions by showing that a short-run equilibrium can be considered as the resting point of a searching-and-matching process without relying on labour demand and supply schedules. In other words, augmenting the model economy with a matching function that conveys how the searching efforts of workers and firms generate new employment, I showed that the equilibrium unemployment due to deficient aggregate demand typical of a short-run equilibrium can be seen as a situation in which unemployed workers have a certain probability to find a jobs and—on the other side—firms have some chances to hire eligible employees.

The analysis developed in this paper could be extended in many directions. For instance, it could be interesting to consider different mechanisms of adjustment of wage expectations and their implications for the stability of the short-run equilibrium. In addition, taking into account productivity shocks and productive capital into the model economy would certainly alter in a significant manner the determination and the dynamics of actual output and wages. Furthermore, side by side real magnitudes, even monetary phenomena should be taken into consideration by addressing the determinants of the price of commodities and the nominal wage. All these extensions, however, are left to further developments.

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Competing interests The authors declare no competing interests.

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