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ORIGINAL RESEARCH

A new born-approximation approach to compute the effects of motion on the solution of electromagnetic problems involving moving materials with stationary boundaries

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Abstract

The non-relativistic motion of media induces very weak effects on electromagnetic fields. For this reason, it is extremely difficult to compute such effects with traditional techniques. To overcome this difficulty, a novel approach is introduced in this work. It is based on the Born approximation of the solution of electromagnetic problems involving media in motion. The formulation of these problems makes use of equivalent field sources and is specifically tailored to find the effects of motion on the electromagnetic field. The new methodology exploits traditional simulators able to deal with media at rest. The motion of the materials has to be managed by specific programs performing simple algebraic calculations. The simplest of the methods proposed does not present any additional complexity with respect to more traditional approaches to the same problems. All methods deal with time-harmonic electromagnetic fields and, for this reason, they can manage materials in motion with stationary boundaries. A complete set of simulations for cylinders moving in the axial direction show that the new methods outperform traditional numerical approaches and, in some significant cases, traditional approaches based on semi-analytical techniques. The good features of the new methodology are shown to hold true for a range of velocity values spanning 11 decades; such a range covers most of the applications of practical interest.

1 | INTRODUCTION

The interaction of electromagnetic waves with media in motion has always been an important research topic. It has significant theoretical implications [1–6] and a plethora of applications [7], including the tachometry of celestial bodies [8] or the flow measurement of liquids, gases or solids in industrial processes [9], biological systems or plasma physics [10]. For the indicated reasons, the research community has always tried to find reliable techniques to solve these problems. The first of these efforts have produced several analytical methods [8, 11–16]. An extension has recently been proposed [17]. However, all these approaches can manage very specific problems and their usefulness is often limited to provide results for checking the reliability of numerical techniques. In order to deal with problems of practical interest, in the last few years most of the work has been devoted to the development of computational methods and now, for problems involving media in motion with stationary boundaries, well-posedness and finite element approximability results are available [18, 19]. Commercial simulators with appropriate integrations can be used to find the approximate solutions to these problems [20].

This situation can be considered satisfactory from several perspectives. For example, for a given problem involving media in motion with stationary boundaries one can refer to its solution, to the solution of the same problem with all media at rest and to the difference between these fields [21]. The difference field, in particular, is due to the effects of motion of the media involved. By using the above well-posedness results one can know in advance under which conditions both solutions and, then, the difference field are well defined. However, the approximation of the difference field, obtained by the calculation of the difference between the approximations of the solutions of the two problems indicated above, could be unreliable. This could happen in the presence of objects or materials moving with non-relativistic velocities because, under the indicated conditions, the effects of motion on the electromagnetic field are so weak [11, 17, 22] that the difference of the two numerical solutions could be smaller than the approximations accepted on the single solutions.

In this paper we propose a new methodology for the reliable calculation of the very weak effects of the non-relativistic motion of media on the electromagnetic field.

The new set of methods is obtained by using Born approximations [23, 24] of different orders for electromagnetic scattering problems formulated in terms of the difference field of interest. Since any medium in motion in a reference frame is perceived as a bianisotropic material in that frame [25, 26], one could expect that the new methods have to be able to manage bianisotropic media. This is clearly the case when the effects of motion on the electromagnetic field are computed by calculating the difference of numerical solutions. On the contrary, this is not necessary for the new methods: a single code able to manage media at rest is enough. Motion is just taken into account in the definition of equivalent sources and it does not require any simulation but just simple calculations involving known quantities. For this reason, the new methods can be implemented by using a single traditional simulator and do not require any generalization of the available codes.

The simplest of the proposed methods requires the numerical solution of just two problems, like the traditional approach which calculates the difference field by solving the problem of interest with the media in motion and then the same problem with the media at rest. Higher order approximations, on the contrary, require additional simulations.

The new methods can be used for cylinders moving in the axial direction or for axisymmetric objects in rotation around their axis of symmetry. For the former class of problems, a deeper analysis is possible, in the presence of TE or TM polarized plane wave illuminations. It allows us to deepen our understanding of some features of the new approach. This is the reason why a specific part of the theory and all numerical results refer to cylinders moving in the axial direction. Numerical results for problems involving rotating objects will be presented in a future work.

Although the new methodology can be based on different time-harmonic electromagnetic simulators, all numerical results presented in this manuscript are obtained using traditional finite element simulators. The results show, on the one hand, the apparent problems of the traditional approach which approximates the difference field by computing the difference between two approximations and, on the other hand, the superiority of the new class of numerical methods. They are by far more reliable than the traditional approach and, moreover, this happens for a range of velocity values spanning several decades. In particular, they show that the new methods are able to provide reliable results for all speed values of interest in applications.

The above considerations could also be important for testing or even designing inverse scattering techniques of interest in industrial processes [9, 27, 28] or in astrophysics [8], which aim at recovering information on the velocity fields of moving objects [20, 21, 29–31]. As a matter of fact, in practice it could be difficult to understand which component of the difference field is able to provide significant information about the velocity field and in which region or direction this is true. This happens for example when complex, rotating, axisymmetric scatterers are considered [20] or in the presence of cylinders moving in the axial direction in complex environments, like cavities or waveguides [30, 31]. In these cases the new class of methods can be of help in choosing the number, the positions and the type of sensors to be used and, then, it can give a significant contribution to the design of inverse scattering techniques. Moreover, once the above problems are overcome, the testing phase of such techniques usually requires reliable synthetic data to which noise with a given signal-to-noise ratio is added [20, 21, 29]. For this phase, too, the new approach could be very useful. Our research activity on the proposed methodology actually started when we noticed that in many cases the synthetic data we provided to inverse procedures were not able to give significant information about the velocity field and, moreover, when such information was present, the data were affected by errors which were several orders of magnitude larger than the effects of motion, for velocity values of interest in many practical applications.

As a final comment on the importance of the new methods, we observe that they are expected to be useful to compute in a reliable way the small effects induced on the electromagnetic field by the small bianisotropy of some natural media or artificial metamaterials [32, 33]. In this case, too, for the same reason as above, there will be no need to develop new simulators to obtain such results. These considerations will not be further developed to limit the length of the manuscript. They will be addresses in a future work, as well.

The paper is organized as follows. In Section 2, the class of problems of interest is defined. An exact alternative formulation for calculating the relevant difference field for this study is proposed in Section 3, together with the formulations giving its Born approximations. Section 4 is devoted to the finite element approximations of the solutions to all problems introduced in previous sections. In order to get a deeper understanding of the properties of the new approach, a specific analysis is carried out in Section 5, for cylinder moving in the axial direction under TM illumination. Finally, in Section 6 several numerical results clearly show the reliability of the new methods and their superiority with respect to more traditional numerical techniques.

2 | DEFINITION OF THE PROBLEM OF INTEREST AND FIRST CONSIDERATIONS ON THE EFFECTS OF MOTION

The time-harmonic electromagnetic boundary value problems of interest in this paper are summarized here. All mathematical details, however, can be found in [18] for cylinders moving in the axial direction and in [19] for rotating axisymmetric objects.

As it was pointed out in the Introduction, we are interested in electromagnetic problems involving media in motion. In order to retain the possibility to deal with time-harmonic fields, we will consider problems in which the boundaries of the moving objects are stationary [25]. Since all moving media are perceived as bianisotropic [25, 26], the formulation of the problem of interest is the following:

$$\begin{cases} \operatorname{curl}\mathbf{H} - j\omega\mathbf{D} = \mathbf{J}_e & \text{in } \Omega, \\ \operatorname{curl}\mathbf{E} + j\omega\mathbf{B} = -\mathbf{J}_m & \text{in } \Omega, \\ \mathbf{D} = \varepsilon\,\mathbf{E} + \xi\,\mathbf{H} & \text{in } \Omega, \\ \mathbf{B} = \zeta\,\mathbf{E} + \mu\,\mathbf{H} & \text{in } \Omega, \\ \mathbf{H}\times\mathbf{n} - Y\,(\mathbf{n}\times\mathbf{E}\times\mathbf{n}) = \mathbf{f}_R & \text{on } \Gamma, \end{cases} \tag{1}$$

where Ω is the domain in which the problem is formulated and Γ is its boundary. Most of the other symbols are standard. ε, ξ, ζ and μ denote the constitutive parameters of the bianisotropic media we have to deal with in one of their four possible notations [34] (pp. 4-9), [32] (Section 5). The previous formulation is based on Maxwell curl equations, constitutive relations and boundary conditions, as it is usually the case [18, 19]. It is written by using the notation usually adopted for three-dimensional problems and, due to its generality, it can refer to open (radiation or scattering) or cavity problems [19]. However, it can be easily modified to manage cylindrical objects moving in the axial direction by considering two-dimensional domains and three-dimensional fields, as it is shown in [18].

It is very well known how to establish the well-posedness of this kind of problems under very weak and non-restrictive hypotheses. The details can be found in [19] and [18].

The same problem is considered in the presence of all media at rest, as discussed in [21]. In this case the problem formulation is the following:

$$\begin{cases} \operatorname{curl}\mathbf{H}_0 - j\omega\mathbf{D}_0 = \mathbf{J}_e & \text{in } \Omega, \\ \operatorname{curl}\mathbf{E}_0 + j\omega\mathbf{B}_0 = -\mathbf{J}_m & \text{in } \Omega, \\ \mathbf{D}_0 = \varepsilon^0\,\mathbf{E}_0 & \text{in } \Omega, \\ \mathbf{B}_0 = \mu^0\,\mathbf{H}_0 & \text{in } \Omega, \\ \mathbf{H}_0\times\mathbf{n} - Y\,(\mathbf{n}\times\mathbf{E}_0\times\mathbf{n}) = \mathbf{f}_R & \text{on } \Gamma, \end{cases} \tag{2}$$

where ε^0 and μ^0 denote the constitutive parameters of the media involved in (1) when they are at rest (and then we assume that they are at most anisotropic in their rest frame).

Under these simpler conditions the well-posedness of problem (2) can be proven [35].

As a consequence of the well-posedness of problems (1) and (2), the difference fields $\mathbf{E}-\mathbf{E}_0$ and $\mathbf{H}-\mathbf{H}_0$ exist, are unique and depend continuously on the source terms $\mathbf{J}_e, \mathbf{J}_m$ and $\mathbf{f}_R$, for almost all applications. They are a direct consequence of motion since they become zero if media at rest are considered for (1). For this reason, they are of particular interest for this study. In particular, the goal of this manuscript is to define techniques for the reliable evaluation of them, when the media involved in problem (1) are in motion with velocity values of interest in most engineering applications, which are very small fractions of the speed of light in vacuum. On the other hand, when motion takes place at very large velocities and the indicated well-posedness results still apply [18, 19], there is no need to look for such techniques. As a matter of fact, under the indicated conditions, the solution of problem (1) is significantly affected by motion and the difference field can be calculated in a very reliable way even by using approximations of the solutions of problems (1) and (2). This simple approach, which calculates the difference of solutions, was adopted in our former manuscripts dealing with the reconstruction of velocity profiles [20, 21, 29].

3 | ALTERNATIVE FORMULATIONS FOR THE DIRECT CALCULATION OF THE DIFFERENCE FIELDS

The simple approach described above, which was adopted in several of our former manuscripts, suffers from severe limitations even for velocity values considered large in engineering applications. This is essentially due to the fact that the calculations of difference fields become unreliable, when the effects of motion are small, even in the presence of very good approximations of the single solutions of problems (1) and (2).

The indicated difficulties were noticed in our former publications related to the reconstruction of velocity profiles [20, 21, 29]. They will be fully analyzed in Section 6, where comparisons of the results obtained by different techniques will be presented.

Using a mix of well known results [23, 36, 37], it is possible to deduce alternative approximate formulations having the difference fields as unknowns. These formulations are based on Born approximations [23] (Sec. 10.2.B, pp. 464–465), [36] (Sec. 10.4, pp. 388–397) of an alternative formulation of the electromagnetic problems of interest, which is obtained by the so called volume equivalent theorem [37] (Sec. 7.7, pp. 327–328).

In particular, as discussed in [21], from problems (1) and (2), we can deduce:

$$\begin{cases} \operatorname{curl}(\mathbf{H}-\mathbf{H}_0) - j\omega\big(\varepsilon\,\mathbf{E} + \xi\,\mathbf{H} - \varepsilon^0\,\mathbf{E}_0\big) = 0 & \text{in } \Omega, \\ \operatorname{curl}(\mathbf{E}-\mathbf{E}_0) + j\omega\big(\zeta\,\mathbf{E} + \mu\,\mathbf{H} - \mu^0\,\mathbf{H}_0\big) = 0 & \text{in } \Omega, \\ (\mathbf{H}-\mathbf{H}_0)\times\mathbf{n} - Y\,\big(\mathbf{n}\times(\mathbf{E}-\mathbf{E}_0)\times\mathbf{n}\big) = 0 & \text{on } \Gamma. \end{cases} \tag{3}$$

It is very common, in order to deal with scattering problems [37] (p. 328), to add $j\omega\varepsilon^0\,\mathbf{E} - j\omega\varepsilon^0\,\mathbf{E}$ to the first equation of (3) and $j\omega\mu^0\,\mathbf{H} - j\omega\mu^0\,\mathbf{H}$ to the second one. In this way we get:

$$\begin{cases} \operatorname{curl}(\mathbf{H}-\mathbf{H}_0) + \\ \quad -j\omega\big(\varepsilon\,\mathbf{E} + \xi\,\mathbf{H} - \varepsilon^0\,\mathbf{E}_0 + \varepsilon^0\,\mathbf{E} - \varepsilon^0\,\mathbf{E}\big) = 0 & \text{in } \Omega, \\ \operatorname{curl}(\mathbf{E}-\mathbf{E}_0) + \\ \quad +j\omega\big(\zeta\,\mathbf{E} + \mu\,\mathbf{H} - \mu^0\,\mathbf{H}_0 + \mu^0\,\mathbf{H} - \mu^0\,\mathbf{H}\big) = 0 & \text{in } \Omega, \\ (\mathbf{H}-\mathbf{H}_0)\times\mathbf{n} - Y\,\big(\mathbf{n}\times(\mathbf{E}-\mathbf{E}_0)\times\mathbf{n}\big) = 0 & \text{on } \Gamma. \end{cases} \tag{4}$$

Then we deduce

$$\begin{cases} \text{curl}\,(\mathbf{H}-\mathbf{H}_0) - j\omega\varepsilon^0(\mathbf{E}-\mathbf{E}_0) = \mathbf{J}_{e,eq} & \text{in } \Omega, \\ \text{curl}\,(\mathbf{E}-\mathbf{E}_0) + j\omega\mu^0(\mathbf{H}-\mathbf{H}_0) = -\mathbf{J}_{m,eq} & \text{in } \Omega, \\ (\mathbf{H}-\mathbf{H}_0)\times\mathbf{n} - Y\left(\mathbf{n}\times(\mathbf{E}-\mathbf{E}_0)\times\mathbf{n}\right) = 0 & \text{on } \Gamma, \end{cases} \tag{5}$$

where

$$\mathbf{J}_{e,eq} = j\omega\left(\varepsilon-\varepsilon^0\right)\mathbf{E} + j\omega\xi\,\mathbf{H} \tag{6}$$

and

$$\mathbf{J}_{m,eq} = j\omega\left(\mu-\mu^0\right)\mathbf{H} + j\omega\zeta\,\mathbf{E} \tag{7}$$

are, in this more general context involving media in motion (and, then, bianisotropic materials), the well known volume equivalent electric and magnetic current densities [37] (p. 328). The two equivalent sources $\mathbf{J}_{e,eq}$ and $\mathbf{J}_{m,eq}$ retain most of the features of the original approach [37] (p. 328). In particular, they are different from zero in the region occupied by the media in motion [21]. As a matter of fact $\varepsilon-\varepsilon^0$, ξ, $\mu-\mu^0$ and ζ are different from zero in the domain occupied by those media. Moreover, from problem (5) we deduce that they radiate in the presence of all media at rest [21].

From this perspective problem (5) is like problem (2). Then, by using again classical results [35], it is well posed and, in particular, its solution (that is the difference fields $\mathbf{E}-\mathbf{E}_0$ and $\mathbf{H}-\mathbf{H}_0$ of interest) depends continuously on the equivalent sources.

These sources are unknown unless the solution of problem (1) is known [37] (p. 328). However, as it has already been pointed out, for velocity values of interest in engineering applications the difference fields $\mathbf{E}-\mathbf{E}_0$ and $\mathbf{H}-\mathbf{H}_0$ are very small [18] (for cylinders moving in the axial direction), [19] (for rotating axisymmetric objects).

Then, by exploiting the indicated continuous dependence of the solution on the sources, we can get a very good approximation of the solution of problem (5), the so-called first-order Born approximation [23] (Sec. 10.2.B, pp. 464-465), [36] (Sec. 10.4, pp. 388-397), by replacing the equivalent sources $\mathbf{J}_{e,eq}$ and $\mathbf{J}_{m,eq}$ by, respectively:

$$\mathbf{J}_{e,eq,a1} = j\omega\left(\varepsilon-\varepsilon^0\right)\mathbf{E}_0 + j\omega\xi\,\mathbf{H}_0, \tag{8}$$

and

$$\mathbf{J}_{m,eq,a1} = j\omega\left(\mu-\mu^0\right)\mathbf{H}_0 + j\omega\zeta\,\mathbf{E}_0. \tag{9}$$

The approximate solution obtained in this way will be denoted by $(\mathbf{E}-\mathbf{E}_0)_{ba1}$ and $(\mathbf{H}-\mathbf{H}_0)_{ba1}$.

Once these approximating fields are available, we can get a second-order Born approximation, $(\mathbf{E}-\mathbf{E}_0)_{ba2}$ and $(\mathbf{H}-\mathbf{H}_0)_{ba2}$, by solving problem (5) with the following approximate equivalent sources:

$$\mathbf{J}_{e,eq,a2} = j\omega\left(\varepsilon-\varepsilon^0\right)\left((\mathbf{E}-\mathbf{E}_0)_{ba1}+\mathbf{E}_0\right) + j\omega\xi\left((\mathbf{H}-\mathbf{H}_0)_{ba1}+\mathbf{H}_0\right), \tag{10}$$

and

$$\mathbf{J}_{m,eq,a2} = j\omega\left(\mu-\mu^0\right)\left((\mathbf{H}-\mathbf{H}_0)_{ba1}+\mathbf{H}_0\right) + j\omega\zeta\left((\mathbf{E}-\mathbf{E}_0)_{ba1}+\mathbf{E}_0\right). \tag{11}$$

Higher-order Born approximations can be obtained in an analogous way. As it will be pointed out in Section 6, there is no practical need to consider orders higher than the second. However, in order to deduce this result, in Section 6, we will also consider the third order approximation.

The proposed approaches do not suffer from the difficulties described above which affect the reliability of the method based on the computation of the difference of the solutions of problems (1) and (2). In particular, the accuracy of the approximations of the difference fields achieved in these ways improves as the largest value of the velocity field becomes smaller.

It is interesting to observe that the new approach based on the first-order Born approximation requires the solution of two problems, like the original and unsatisfactory approach that calculates the difference between the solutions of problems (1) and (2). As a matter of fact, problem (2) has to be solved to find $\mathbf{E}_0$ and $\mathbf{H}_0$ and these fields allow us to determine $\mathbf{J}_{e,eq,a1}$ and $\mathbf{J}_{m,eq,a1}$ which, in turn, are necessary to approximate the difference fields of interest by solving problem (5) with the indicated approximate current densities.

In general, the n-th-order Born approximation requires the solution of $n+1$ problems.

Problems (1), (2) and (5) can be formulated in alternative equivalent [38] ways. We recall the variational formulation for Problem (1) because it is the most general of the three.

Let U be the space where one looks for the solutions. We have [19]:

$$U = H_{L^2,\Gamma}(\text{curl},\Omega) = \{\mathbf{v}\in H(\text{curl},\Omega) \mid \mathbf{v}\times\mathbf{n}\in L_t^2(\Gamma)\}, \tag{12}$$

where

$$H(\text{curl},\Omega) = \{\mathbf{v}\in (L^2(\Omega))^3 \mid \text{curl}\,\mathbf{v}\in (L(\Omega))^3\}, \tag{13}$$

and

$$L_t^2(\Gamma) = \{\mathbf{v}\in (L^2(\Gamma))^3 \mid \mathbf{v}\cdot\mathbf{n} = 0 \text{ almost everywhere on } \Gamma\}. \tag{14}$$

$(L^2(\Omega))^3$ is the usual Hilbert space of complex-valued square integrable vector fields on Ω with scalar product given by $(\mathbf{u},\mathbf{v})_{0,\Omega} = \int_\Omega \mathbf{v}^*\mathbf{u}\,dV$ (* denotes the conjugate transpose). $(L^2(\Gamma))^3$ is defined in an analogous way on Γ.

The variational formulation of Problem (1) is [19]:

Given $\omega > 0, \mathbf{J}_e \in (L^2(\Omega))^3, \mathbf{J}_m \in (L^2(\Omega))^3$ and $\mathbf{f}_R \in L_t^2(\Gamma)$, find $\mathbf{E}\in U$ such that

$$a(\mathbf{E},\mathbf{v}) = l(\mathbf{v}) \quad \forall \mathbf{v}\in U, \tag{15}$$

where

$$a(\mathbf{u},\mathbf{v}) = \left(\mu^{-1}\operatorname{curl}\mathbf{u},\operatorname{curl}\mathbf{v}\right)_{0,\Omega} - \omega^2\left((\varepsilon - \xi\mu^{-1}\zeta)\,\mathbf{u},\mathbf{v}\right)_{0,\Omega} + j\omega\left(\mu^{-1}\zeta\,\mathbf{u},\operatorname{curl}\mathbf{v}\right)_{0,\Omega} - j\omega\left(\xi\mu^{-1}\operatorname{curl}\mathbf{u},\mathbf{v}\right)_{0,\Omega} + j\omega\left(Y\,(\mathbf{n}\times\mathbf{u}\times\mathbf{n}),\mathbf{n}\times\mathbf{v}\times\mathbf{n}\right)_{0,\Gamma}, \tag{16}$$

and

$$l(\mathbf{v}) = -j\omega\left(\mathbf{J}_e,\mathbf{v}\right)_{0,\Omega} - \left(\mu^{-1}\mathbf{J}_m,\operatorname{curl}\mathbf{v}\right)_{0,\Omega} + j\omega\left(\xi\mu^{-1}\mathbf{J}_m,\mathbf{v}\right)_{0,\Omega} - j\omega(\mathbf{f}_R,\mathbf{n}\times\mathbf{v}\times\mathbf{n})_{0,\Gamma}. \tag{17}$$

The corresponding variational formulations of Problems (2) and (5), which are the same apart from the obvious differences in the source terms, are simpler because the addends involving ζ or ξ are not present. Finally, the variational formulations for problems involving cylindrical objects moving in axial direction can be found in [18]. The simpler formulations of Problems (2) and (5) in the presence of cylinders at rest can be deduced as indicated above because some of the terms of the variational formulation shown in [18] are trivial.

4 | IMPLEMENTATION OF THE METHODOLOGY USING FINITE ELEMENT METHODS

The approach proposed in Section 3 places no restrictions on the method used to calculate the solutions of the different problems.

In practice, however, most of the times the solutions of the problems presented in Sections 2 and 3 will be computed by numerical methods.

Although we can use any numerical method, in the following we will specifically refer to the finite element method for which results of convergence [39] (p. 112) were established in [19] and [18], for problems involving media in motion, or, for example, in [35] for problems in which all media are at rest.

By introducing a triangulations $\mathcal{T}_h$ of Ω and a specific finite element on it one can then refer to a finite dimensional subspace U_h of U to evaluate the so-called Galerkin approximation [35]. As usual, h denotes the maximum diameter of all elements of the triangulation [40] (p. 131).

We will denote by $\mathbf{E}_h$ and $\mathbf{H}_h$ the approximate finite element solution of problem (1) [18, 38]. $\mathbf{E}_{0,h}$ and $\mathbf{H}_{0,h}$ will refer to the finite element approximation of the solution of problem (2). In this way, the difference field is approximated as $\mathbf{E}_h - \mathbf{E}_{0,h}$ and $\mathbf{H}_h - \mathbf{H}_{0,h}$.

On the other hand, from $\mathbf{E}_{0,h}$ and $\mathbf{H}_{0,h}$ approximate equivalent sources for the first-order Born approximation are obtained:

$$\mathbf{J}_{e,eq,a1,h} = j\omega\left(\varepsilon - \varepsilon^0\right)\mathbf{E}_{0,h} + j\omega\xi\,\mathbf{H}_{0,h}, \tag{18}$$

and

$$\mathbf{J}_{m,eq,a1,h} = j\omega\left(\mu - \mu^0\right)\mathbf{H}_{0,h} + j\omega\zeta\,\mathbf{E}_{0,h}. \tag{19}$$

By a finite element simulator we can then solve problem (5) with the equivalent sources $\mathbf{J}_{e,eq}$ and $\mathbf{J}_{m,eq}$ replaced by, respectively, $\mathbf{J}_{e,eq,a1,h}$ and $\mathbf{J}_{m,eq,a1,h}$.

In this way, by a two-step procedure we get the finite element solution of the first-order Born approximation $(\mathbf{E}-\mathbf{E}_0)_{ba1,h}$, $(\mathbf{H}-\mathbf{H}_0)_{ba1,h}$.

In order to get the finite element solution of the second-order Born approximation, $(\mathbf{E}-\mathbf{E}_0)_{ba2,h}$ and $(\mathbf{H}-\mathbf{H}_0)_{ba2,h}$, one can calculate the following equivalent sources:

$$\mathbf{J}_{e,eq,a2,h} = j\omega\left(\varepsilon - \varepsilon^0\right)\left((\mathbf{E}-\mathbf{E}_0)_{ba1,h} + \mathbf{E}_{0,h}\right) + j\omega\xi\,\left((\mathbf{H}-\mathbf{H}_0)_{ba1,h} + \mathbf{H}_{0,h}\right), \tag{20}$$

and

$$\mathbf{J}_{m,eq,a2,h} = j\omega\left(\mu - \mu^0\right)\left((\mathbf{H}-\mathbf{H}_0)_{ba1,h} + \mathbf{H}_{0,h}\right) + j\omega\zeta\,\left((\mathbf{E}-\mathbf{E}_0)_{ba1,h} + \mathbf{E}_{0,h}\right), \tag{21}$$

and solve once more problem (5), with the indicated equivalent sources, by a finite element simulator.

In Section 6, it will be shown that the new approaches, even for the first order of the Born approximation, guarantee reliable results for the difference fields for a huge range of velocity values and that, in particular, they can be used to estimate the effects of motion on the electromagnetic field for all problems of practical interest involving moving objects with stationary boundaries.

For the indicated reason, we do not worry about the convergence of the Born approximations of order n, that is of $(\mathbf{E}-\mathbf{E}_0)_{ban}$ to $\mathbf{E}-\mathbf{E}_0$ and $(\mathbf{H}-\mathbf{H}_0)_{ban}$ to $\mathbf{H}-\mathbf{H}_0$ (in the appropriate norms), as $n \to \infty$. By the same token, we are not worried by the lack of convergence of $(\mathbf{E}-\mathbf{E}_0)_{ban,h}$ to $\mathbf{E}-\mathbf{E}_0$ and $(\mathbf{H}-\mathbf{H}_0)_{ban,h}$ to $\mathbf{H}-\mathbf{H}_0$ as h goes to 0, when one considers a sequence of finer and finer meshes, for any fixed order of the Born approximation.

5 | ADDITIONAL CONSIDERATIONS ON THE EFFECTS OF MOTION FOR CYLINDERS MOVING IN THE AXIAL DIRECTION

In order to validate the proposed approach it is convenient to consider problems with analytical solution. We can choose from a limited set of alternatives, which include cylinders moving in the axial direction [12, 17] or rotating spheres [8] in free space. The former class, in the presence of incident fields having TE or TM polarizations, gives crucial additional advantages with respect to the latter because, due to a clear separation of the effects of motion on the different components of the scattered field, it allows for a deeper understanding of the way the new approach works.

For this reason, in the following we will refer to electromagnetic scattering problems involving cylinders moving in the axial direction, made up of isotropic materials (in their rest frames) and illuminated by time-harmonic uniform plane waves which propagate in the plane orthogonal to the cylinder axis. Without loss of generality, the cylinders axes will always be the z-axis and the impinging waves will always propagate along directions lying in the (x, y) plane. Moreover, we will consider incident fields having a TM polarization (for a TE polarization the conclusions could be deduced in an analogous way). Finally, analytical techniques can solve problems with multilayer cylinders when each layer has a constant axial velocity value [17]. However, since, on the one hand, the focus of this study is on the effects of motion when the axial speed is not relativistic and, on the other hand, no difficulty arises if the cylinders are multilayer [17, 18], in the following we will consider problems involving homogeneous cylinders moving in the axial direction with a uniform axial velocity $v_z \in \mathbb{R}$, $v_z \neq 0$. We will refer to this quantity, as usual, by considering $\beta = \frac{v_z}{c_0}$, where c_0 is the speed of light in vacuum.

Remark 1. The consideration of homogeneous cylinders in uniform motion in the axial direction allows us to reduce the mathematical formalism. However, it is important to point out that the next analysis holds true under much weaker hypotheses and applies, for example, to problems involving inhomogeneous media in motion with non-constant velocity fields [18].

Problems of the considered class have some interesting properties. We recall a couple of them which are crucial for our next developments:

- the tensor fields $(\varepsilon - \varepsilon^0), \xi, (\mu - \mu^0)$ and ζ have all xz, yz, zx, yz, zz entries equal to zero (see [21], equations (19) and (20)), so that they process just the transverse x and y components of fields and determine the x and y components of the equivalent current densities (see any of equations (6)-(21)),
- the tensor fields $(\varepsilon - \varepsilon^0)$ and $(\mu - \mu^0)$ present a common factor β^2 while ξ and ζ depend on β, for small values of β (see [21], equations (19), (20) and the comment just above equations (21)),
- in the presence of cylinders at rest made up of isotropic materials a transverse electric (respectively, magnetic) current density determines a solution having a trivial TM (respectively, TE) part of the solution and a non-trivial TE part (respectively, TM) [37] (see also [21], below equation (28)).

The last of the above considerations implies that in the presence of a TM illumination the solution $\mathbf{E}_0, \mathbf{H}_0$ of problem 2 has only the TM part, that is:

$$E_{0,x} = E_{0,y} = H_{0,z} = 0, \tag{22}$$

while the other components, $E_{0,z}$, $H_{0,x}$ and $H_{0,y}$ are, in general, different from zero. The same conclusions apply to $\mathbf{E}_{0,b}$, $\mathbf{H}_{0,b}$ [41].

In the following we will use a more compact notation for a vector $\mathbf{A}$:

$$\mathbf{A} = A_x \mathbf{x} + A_y \mathbf{y} + A_z \mathbf{z} = \mathbf{A}_t + A_z \mathbf{z}. \tag{23}$$

It is now clear that Equation (22) implies $\mathbf{E}_{0,t} = 0$. Moreover, by the first item of the previous list $(\varepsilon - \varepsilon^0) E_{0,z} \mathbf{z} = 0$ and $\zeta E_{0,z} \mathbf{z} = 0$. Then:

$$\begin{aligned} \mathbf{J}_{e,eq,a1} &= j\omega(\varepsilon - \varepsilon^0)\mathbf{E}_0 + j\omega\xi\,\mathbf{H}_0 = \\ &= j\omega(\varepsilon - \varepsilon^0)\mathbf{E}_{0,t} + j\omega\xi\,\mathbf{H}_{0,t} = j\omega\xi\,\mathbf{H}_{0,t}, \end{aligned} \tag{24}$$

and

$$\begin{aligned} \mathbf{J}_{m,eq,a1} &= j\omega(\mu - \mu^0)\mathbf{H}_0 + j\omega\zeta\,\mathbf{E}_0 = \\ &= j\omega(\mu - \mu^0)\mathbf{H}_{0,t} + j\omega\zeta\,\mathbf{E}_{0,t} = j\omega(\mu - \mu^0)\mathbf{H}_{0,t}. \end{aligned} \tag{25}$$

The same conclusion applies to $\mathbf{J}_{e,eq,a1,b}$ and $\mathbf{J}_{m,eq,a1,b}$. We will not repeat this statement later on since our next conclusions will also apply to the corresponding finite element quantities (current densities or fields).

Since $\mathbf{E}_0$ and $\mathbf{H}_0$ are independent of β by definition, the second item of the previous list guarantees that the $(L^2(\boldsymbol{\Omega}))^2$ norm of $\mathbf{J}_{e,eq,a1}$ is controlled by β, at least for small values of β. As a matter of fact, from Equation (24) and taking account of the properties of ξ we deduce:

$$\|\mathbf{J}_{e,eq,a1}\|_{(L^2(\Omega))^2} = \|\omega\xi\,\mathbf{H}_{0,t}\|_{(L^2(\Omega))^2} \leq C\beta, \tag{26}$$

where C is a real constant independent of β. Analogously, $\mathbf{J}_{m,eq,a1}$ is controlled by β^2, for small values of β.

Then, by the linearity of problem (5), the continuous dependence of its solution on the sources and the property recalled in the third item of the previous list we deduce:

- the norms of $(\mathbf{E} - \mathbf{E}_0)_{ba1,z}$ and $(\mathbf{H} - \mathbf{H}_0)_{ba1,t}$ are controlled by β^2,
- the norms of $(\mathbf{H} - \mathbf{H}_0)_{ba1,z}$ and $(\mathbf{E} - \mathbf{E}_0)_{ba1,t}$ are controlled by β.

Now from Equations (10) and (11) we get

$$\begin{aligned} \mathbf{J}_{e,eq,a2} = {} & j\omega(\varepsilon - \varepsilon^0)\big((\mathbf{E} - \mathbf{E}_0)_{ba1} + \mathbf{E}_0\big) + \\ & + j\omega\xi\,\big((\mathbf{H} - \mathbf{H}_0)_{ba1} + \mathbf{H}_0\big) = \end{aligned}$$

$$= j\omega\left(\varepsilon - \varepsilon^0\right)\mathbf{E}_0 + j\omega\xi\,\mathbf{H}_0 + $$
$$+ j\omega\left(\varepsilon - \varepsilon^0\right)(\mathbf{E} - \mathbf{E}_0)_{ba1} + j\omega\xi\,(\mathbf{H} - \mathbf{H}_0)_{ba1} =$$
$$= \mathbf{J}_{e,eq,a1} + \tag{27}$$
$$+ j\omega\left(\varepsilon - \varepsilon^0\right)(\mathbf{E} - \mathbf{E}_0)_{ba1} + j\omega\xi\,(\mathbf{H} - \mathbf{H}_0)_{ba1}$$
$$= \mathbf{J}_{e,eq,a1} +$$
$$+ j\omega\left(\varepsilon - \varepsilon^0\right)(\mathbf{E} - \mathbf{E}_0)_{ba1,t} + j\omega\xi\,(\mathbf{H} - \mathbf{H}_0)_{ba1,t}.$$

In an analogous way we deduce:

$$\mathbf{J}_{m,eq,a2} = \mathbf{J}_{m,eq,a1} + \tag{28}$$
$$+ j\omega\left(\mu - \mu^0\right)(\mathbf{H} - \mathbf{H}_0)_{ba1,t} + j\omega\zeta\,(\mathbf{E} - \mathbf{E}_0)_{ba1,t}.$$

One then obtains that the L^2 norm of $\mathbf{J}_{e,eq,a2} - \mathbf{J}_{e,eq,a1}$ is controlled by β^3 whereas the same norm of $\mathbf{J}_{m,eq,a2} - \mathbf{J}_{m,eq,a1}$ presents a term which depends on β^4 and another one which is controlled by β^2 (the overall dependence of $\mathbf{J}_{m,eq,a2} - \mathbf{J}_{m,eq,a1}$ is then on β^2).

Again, by the properties of problem (5) recalled above, we can say that

- the norms of $(\mathbf{E} - \mathbf{E}_0)_{ba2,z} - (\mathbf{E} - \mathbf{E}_0)_{ba1,z}$ and $(\mathbf{H} - \mathbf{H}_0)_{ba2,t} - (\mathbf{H} - \mathbf{H}_0)_{ba1,t}$ are controlled by β^2,
- the norms of $(\mathbf{H} - \mathbf{H}_0)_{ba2,z} - (\mathbf{H} - \mathbf{H}_0)_{ba1,z}$ and $(\mathbf{E} - \mathbf{E}_0)_{ba2,t} - (\mathbf{E} - \mathbf{E}_0)_{ba1,t}$ are controlled by β^3.

From these considerations we deduce that the Born approximation of the second order provides negligible corrections to $(\mathbf{H} - \mathbf{H}_0)_{ba1,z}$ and $(\mathbf{E} - \mathbf{E}_0)_{ba1,t}$ obtained by the Born approximation of the first order, when β is small. On the contrary, it provides significant corrections, proportional to β^2, on $(\mathbf{E} - \mathbf{E}_0)_{ba1,z}$ and $(\mathbf{H} - \mathbf{H}_0)_{ba1,t}$, which have norms of the same order of magnitude of the rectifications. In other words, the Born approximation of the first order is not reliable for all components of the difference field, no matter how small β is. It is reliable just for the TE part of such a field. On the contrary, the Born approximation of the second order is fully reliable for both, for small values of β.

Without going into the details it is easy to verify with some additional steps that the Born approximation of the third order provides in any case negligible corrections to the results obtained by the Born approximation of the second order, when β is small: as a matter of fact such corrections are proportional to β^4 on the TM part and to β^3 on the TE part.

These results prove that in order to fully understand the effects of motion it is convenient to consider at least a second-order Born approximation. Moreover, they suggest that there is no practical need to consider a third order Born approximation, for small values of β.

Remark 2. One could expect that just the first order effects of motions are important in practice. However, this is not always the case. For example, for applications related to the reconstruction of velocity profiles one could use sensors measuring single electromagnetic field components. As a consequence, it could happen, as we have shown for cylinders moving in the axial direction, to be forced to deal with components affected by motion by second order effects. This is the reason why first and second order effects are carefully analyzed in this work. By the same token, higher order effects are not studied in details because the authors of this manuscript are not aware of applications of practical interest in which components are affected by motion just by n-th order effects, with $n > 2$.

6 | NUMERICAL RESULTS

A useful byproduct of our former analysis is that the comparison of the numerical results can be carried out by presenting figures related just to the axial components $(\mathbf{E} - \mathbf{E}_0)_z$ and $(\mathbf{H} - \mathbf{H}_0)_z$ or to their approximations. As a matter of fact, the transverse parts of the difference fields share their properties with the axial ones. In particular, for our problems $\mathbf{H}_{0,z} = 0$ and $\mathbf{H}_{0,b,z} = 0$ so that $(\mathbf{H} - \mathbf{H}_0)_z = H_z$ and $\mathbf{H}_{b,z} - \mathbf{H}_{0,b,z} = \mathbf{H}_{b,z}$.

Let us now consider some specific examples. According to our previous considerations, in the following we will present results obtained when cylinders are homogeneous, have a circular cross section and are illuminated by a TM-polarized uniform plane wave propagating in vacuum with $\mathbf{E}_{inc} = \mathbf{z}e^{j2\pi f\sqrt{\mu_0\varepsilon_0}y}$, where $f = 1$ GHz (the factor $e^{j\omega t}$ is assumed and suppressed, as usual).

Cylinders will be made up of isotropic media characterized in their rest frame by $\mu = \mu_0$, $\sigma = 0$ [18], $1 < \varepsilon_r \leq 2$. The wavelength in such media will be in the range $[\frac{\lambda_0}{\sqrt{2}}, \lambda_0)$, with $\lambda_0 = 0.299792458$ m. β will belong to $(0, \frac{1}{\sqrt{\varepsilon_r}})$ (the indications would be the same if β assumed negative values with magnitude in the indicated range). The radius of cylinders is in any case $R_c = \frac{\lambda_0}{\sqrt{2}}$.

For all our examples the domain will be a disk, having a radius $R_d = 14R_c$, which is almost equal to $10\lambda_0$. All finite element solutions will be computed by using a first-order Lagrangian approximation for the axial component of the unknown field and a first-order edge element approximation for its transverse part [18, 41]. We will use almost uniform meshes with p concentric circles. An example obtained for $p = 20$ is shown in Figure 1. We will always choose the number p in such a way that the meshes will have about 14 elements per wavelength. This number is slightly larger than the usual value, which is equal to 10 for first-order finite element approximations. Our choice is useful to limit the errors due to finite element discretizations and ease the analysis of the approximations introduced by the Born approximations of order 1 and 2.

In the following we will present comparisons among the results obtained by the difference of finite element solutions of problems (1) and (2) $(\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$ or $\mathbf{H}_{b,z} - \mathbf{H}_{0,b,z})$, by the difference of semi-analytical solutions [12] of the same

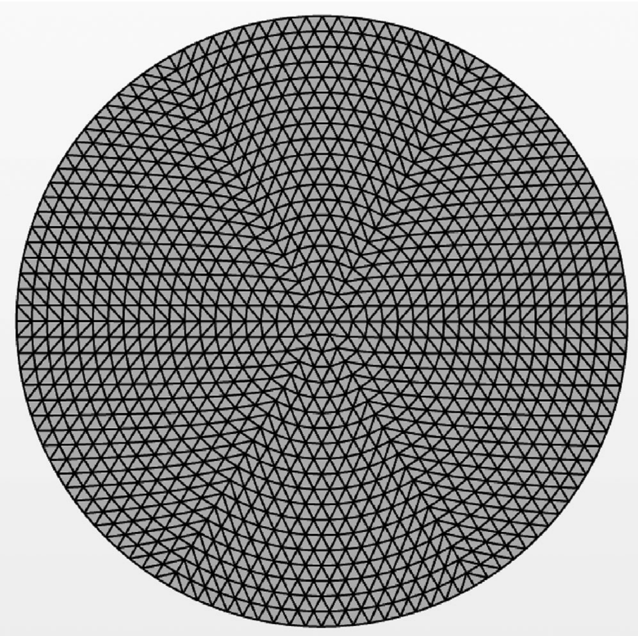

FIGURE 1 An example of the quasi-uniform meshes used in our simulations. It was obtained for $p = 20$.

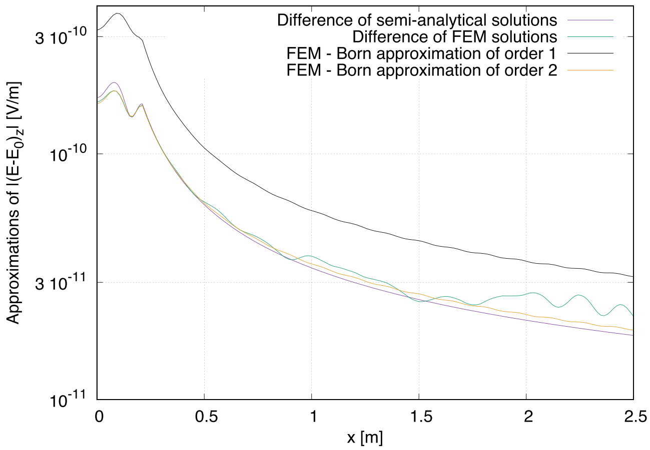

FIGURE 2 Approximations of $|(\mathbf{E}-\mathbf{E}_0)_z|$ along the x axis for $\beta = 10^{-5}$, $\delta = 10^{-12}$, $p = 196$ and $\varepsilon_r = 2$. The result obtained by computing the difference of semi-analytical solutions are well approximated by $(\mathbf{E}-\mathbf{E}_0)_{ba2,b,z}$ and $\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$, even though the latter has spurious oscillations where the effects of motions are smaller. Instead, $(\mathbf{E}-\mathbf{E}_0)_{ba1,b,z}$ presents errors of the same order of magnitude of the quantity it has to approximate, as theoretically expected.

problems or directly by finite element solutions of Born approximations of order one ($(\mathbf{E}-\mathbf{E}_0)_{ba1,b,z}$ or $((\mathbf{H}-\mathbf{H}_0)_{ba1,b,z})$ and two ($(\mathbf{E}-\mathbf{E}_0)_{ba2,b,z}$ or $(\mathbf{H}-\mathbf{H}_0)_{ba2,b,z}$).

Finite element solutions are usually computed by iterative solvers [42] (p. 307). Since we are interested in the calculation of small quantities the stopping criterion and the involved parameters are particularly important. We will adopt "criterion 2" of [43] (p. 60), with a residual $\delta = 10^{-n}$, $n \in \{8, 12\}$. These values slightly extend the range usually adopted for computations with double precision arithmetic (e.g. [43] (pp. 58–60) suggests the range [9,12]) to get clearer indications of the effects of this parameter. Some additional details about the stopping criterion can be found in [41].

On the contrary, the unknown coefficients of all semi-analytical expansions of the solutions are calculated by direct methods and then they will not be affected by δ.

In the first set of simulations we consider a cylinder with $\varepsilon_r = 2$ in its rest frame, for different values of $\beta = 10^{-m}$, $m \in \{1, 2, 3, \ldots, 11, 12\}$. n is set to 12 and p to 196.

The numerical results for the axial component of the magnetic field are almost the same for $m \in \{1, 2, 3, \ldots, 10, 11\}$. Moreover, they are very good approximations of the corresponding semi-analytical solutions. Instead, for $m = 12$, that is for $m \geq n$, $\mathbf{H}_{b,z} - \mathbf{H}_{0,b,z}$ is corrupted by numerical noise, whereas the other three solutions retain the same features as for $m < 12$. The wrong results of $\mathbf{H}_{b,z} - \mathbf{H}_{0,b,z}$, which in this case are not of interest for its practical implications since the axial velocities are smaller than 0.3 mm per second, are due to the fact that $|H_z|$ is smaller than the main TM component of the solution by a factor equal to β. Since it is equal to 10^{-12}, it is of the same order of magnitude of $\delta = 10^{-12}$ and there is no guarantee that the iterative solver will compute it in a reliable way.

This behaviour was already pointed out in [41] (see Figure 5 of that paper). For this reason we do not present any figure related to these results.

On the contrary, the results on the effects of motion on the axial component of the electric field are different and new. For example, the fields $\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$ are completely overlapped to the corresponding solutions obtained by the Born approximation of order 2 for $m \in \{1, 2, 3, 4\}$. Both are very good approximations of the differences of semi-analytical solutions. For $m = 5$ $\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$ presents errors which are not negligible, while the second-order Born approximation and the difference of semi-analytical solutions retain the same features they had for $m \leq 4$. Figure 2 presents the magnitudes of the results obtained in this case along the x axis. All plots have an even symmetry and just the data for $x \geq 0$ are reported.

$\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$ becomes totally unreliable for $m \geq 6$ and the other comparisons become more challenging. This is because the difference of semi-analytical solutions is slightly corrupted by numerical noise when $m = 6$, is significantly affected by noise when $m = 7$, becomes totally unreliable when $m = 8$ (but it is still different from zero) and is equal to zero when $m \geq 9$. The results for the case $m = 8$ are shown in Figure 3.

The behaviours of the differences of semi-analytical solutions change with β because the effects of motion on the axial component of the electric field get smaller and smaller, by a factor β^2, with respect to the main TM component of the field and no algorithm based on double precision arithmetic can reliably calculate such effects when $\beta \leq 10^{-8}$.

The Born approximations, instead, have the expected behaviours for all values of m considered. In particular, $(\mathbf{E}-\mathbf{E}_0)_{ba2,b,z}$, as it has already been pointed out, approximate very well the differences of semi-analytical solutions when they are reliable (for $m \leq 6$; see Figure 2) and gets smaller and smaller with β, by a factor equal to β^2, for the other values of m. For

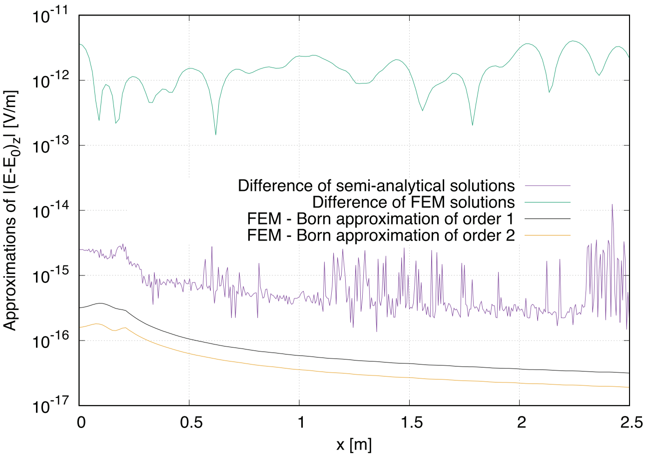

FIGURE 3 Approximations of $|(\mathbf{E}-\mathbf{E}_0)_z|$ along the x axis for $\beta = 10^{-8}$, $\delta = 10^{-12}$, $p = 196$ and $\epsilon_r = 2$. $\mathbf{E}_{h,z} - \mathbf{E}_{0,h,z}$ is clearly completely unreliable in this case. The difference of semi-analytical solutions is unreliable, too. In particular, it is evidently corrupted by numerical noise and, moreover, it is not 10^6 times smaller than the corresponding plot shown in Figure 2, as it should be. Instead, $(\mathbf{E}-\mathbf{E}_0)_{ba2,h,z}$ and $(\mathbf{E}-\mathbf{E}_0)_{ba1,h,z}$ look the same as the corresponding plots reported in Figure 2, with the correct scaling. $(\mathbf{E}-\mathbf{E}_0)_{ba1,h,z}$ has the usual expected problem.

example, the magnitude of $(\mathbf{E}-\mathbf{E}_0)_{ba2,h,z}$ in Figure 3 has the same behaviour as that of Figure 2, apart from the factor 10^{-6}.

Finally, first-order Born approximation share most of the features of the second-order one but presents errors on $(\mathbf{E}-\mathbf{E}_0)_{ba1,h,z}$ of the same order of magnitude of the field itself, as it is theoretically expected for cylinders moving in the axial direction under a TM illumination.

The first set of simulations points out that the second-order Born approximation is not only better than the classical numerical approach which calculates $\mathbf{E}_{h,z} - \mathbf{E}_{0,h,z}$ but it is also more reliable than the approach based on the calculation of the difference of semi-analytical solutions, for $\beta \leq 10^{-8}$. With an additional simulation (with $\beta = 3.336\ 10^{-8}$) we have verified that this is true for velocity values up to 10 [m s $^{-1}$], a large velocity value for many applications. Moreover, the Born approximations are reliable for a huge range of velocity values, spanning 11 decades. The new approach was expected to be able to deal very well with small values of β. It is a surprise, instead, its reliability for β of the same order of magnitude of its physical upper bound ($\simeq 0.707$) or of the upper bound for which a well-posedness result for the scattering problem at hand is available ($\beta \simeq 0.26$; see [18]).

For most applications users of finite element simulators do not have to worry about the values of the parameters of the algebraic solvers. Unfortunately, the lack of control on parameters like δ could have very bad consequences on the reliability of the calculation of interest in this paper. In order to show them, we repeat our analysis by setting $n = 8$ ($\delta = 10^{-8}$). Such a change does not affect semi-analytical solutions. However, all finite element solutions are affected in a very significant way. From the considerations reported above one can deduce that $\mathbf{H}_{h,z} - \mathbf{H}_{0,h,z}$ is reliable just when $m < n$, that is for $m \in \{1, 2, 3, \dots, 6, 7\}$, whereas $(\mathbf{H}-\mathbf{H}_0)_{ba2,h,z}$ and $(\mathbf{H}-\mathbf{H}_0)_{ba1,h,z}$

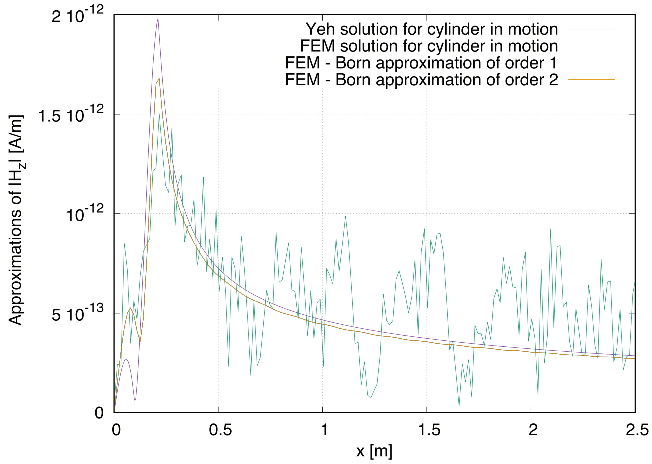

FIGURE 4 Approximations of $|(\mathbf{H}-\mathbf{H}_0)_z| = |H_z|$ along the x axis for $\beta = 10^{-9}$, $\delta = 10^{-8}$, $p = 196$ and $\epsilon_r = 2$. $\mathbf{H}_{h,z} - \mathbf{H}_{0,h,z} = \mathbf{H}_{h,z}$ is clearly unreliable in this case. Instead, $(\mathbf{H}-\mathbf{H}_0)_{ba2,h,z}$ and $(\mathbf{H}-\mathbf{H}_0)_{ba1,h,z}$ approximate very well the semi-analytical solution.

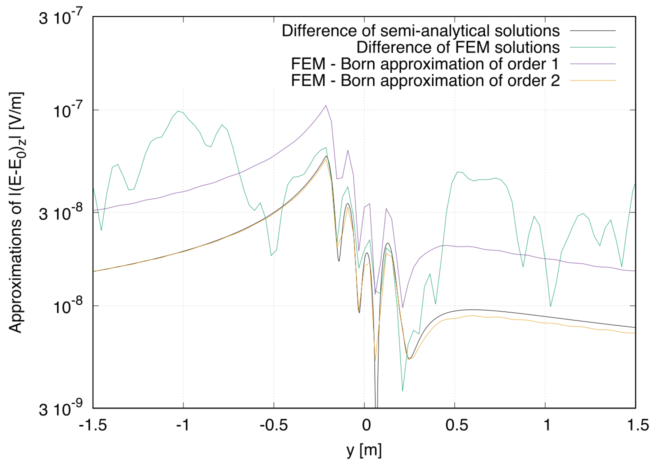

FIGURE 5 Approximations of $|(\mathbf{E}-\mathbf{E}_0)_z|$ along the y axis for $\beta = 10^{-4}$, $\delta = 10^{-8}$, $p = 196$ and $\epsilon_r = 2$. The result obtained by computing the difference of semi-analytical solutions is well approximated by $(\mathbf{E}-\mathbf{E}_0)_{ba2,h,z}$. $(\mathbf{E}-\mathbf{E}_0)_{ba1,h,z}$ is affected by errors of the same magnitude of the field it has to approximate, as usual. Finally, $\mathbf{E}_{h,z} - \mathbf{E}_{0,h,z}$ is completely wrong.

have satisfactory behaviours for all values of m. Figure 4 presents for example the effects of motion on the axial component of the magnetic field along the x axis, when $m = 9$. It shows that Born approximations of order 1 and 2 give very good results, with negligible differences between them, whereas $\mathbf{H}_{h,z} - \mathbf{H}_{0,h,z}$ is wrong. It is interesting to observe that, for cylinders moving in the axial direction illuminated by a TM polarized wave, such results are not so wrong as those related to the electric field, when the conditions for reliability are not verified. This is because for the magnetic field we do not have to calculate any difference since $\mathbf{H}_{0,h,z} = 0$.

A reduction of n has also a significant impact on our capability to estimate the effects of motion on the axial component of the electric field. For example, $\mathbf{E}_{h,z} - \mathbf{E}_{0,h,z}$ is totally unreliable for all values of $m \geq \frac{n}{2} = 4$. Figure 5 reports the effects

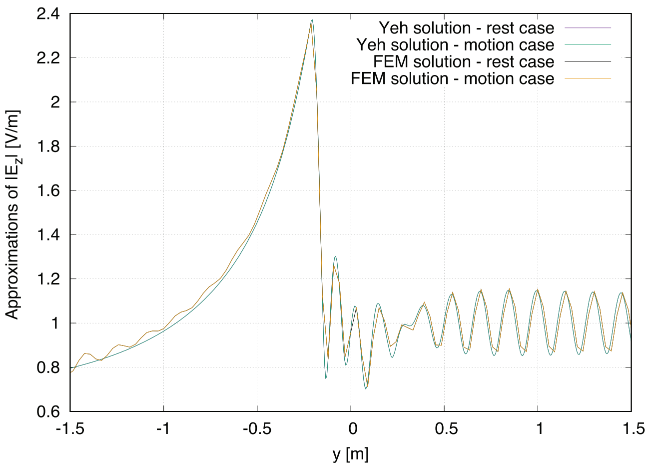

FIGURE 6 Behaviour of $|\mathbf{E}_{b,z}|$ and $|\mathbf{E}_{0,b,z}|$ and of the corresponding semi-analytical solutions along the y axis for $\beta = 10^{-4}$, $\delta = 10^{-8}$, $p = 196$ and $\varepsilon_r = 2$. The two numerical results are completely overlapped. The same holds true for the two semi-analytical solutions.

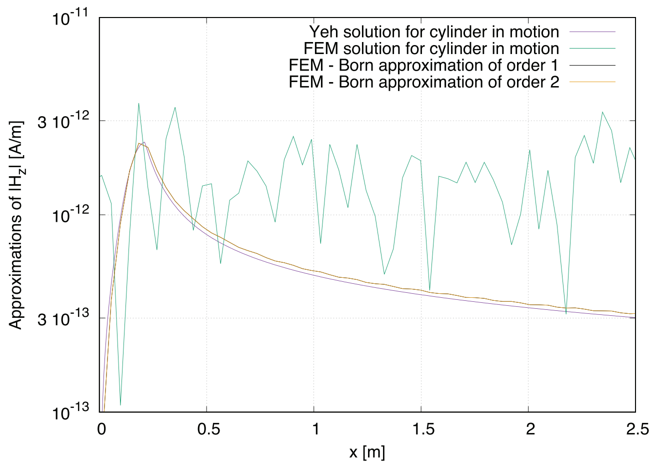

FIGURE 7 Approximations of $|(\mathbf{H} - \mathbf{H}_0)_z| = |H_z|$ along the x axis for $\beta = 10^{-7}$, $\delta = 10^{-8}$, $p = 140$ and $\varepsilon_r = 1.01$. $\mathbf{H}_{b,z} - \mathbf{H}_{0,b,z} = \mathbf{H}_{b,z}$ is clearly unreliable in this case. Instead, $(\mathbf{H} - \mathbf{H}_0)_{ba2,b,z}$ and $(\mathbf{H} - \mathbf{H}_0)_{ba1,b,z}$ approximate very well the semi-analytical solution.

of motion on the axial component of the electric field along the y axis when $m = 4$. It is interesting to observe that the errors affecting $\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$ in Figure 5 are much larger than those shown in Figure 2 for the same component, even though the velocity is ten times larger and should guarantee better results. This is simply due to the different values of n adopted in the two simulations.

As for $(\mathbf{E} - \mathbf{E}_0)_{ba2,b,z}$ and $(\mathbf{E} - \mathbf{E}_0)_{ba1,b,z}$, they are correct for $m \leq n + 1$, that is for $m \in \{1, 2, 3, \ldots, 8, 9\}$ (even though the order one presents the usual error of the same order of β^2). Both of them are contaminated by numerical noise for $m \in \{10, 11, 12\}$ because the considered component is too small, with respect to the transverse ones, to be computed in a reliable way by iterative solvers with the selected value of n. We can then deduce that even with $n = 8$ the second-order Born approximation provides the same quality of results which can be obtained by the difference of semi-analytical solutions, for all values of β. However, any additional reduction of n has dramatic effects because the second-order Born approximation cannot provide reliable results for $(\mathbf{E} - \mathbf{E}_0)_{ba2,b,z}$ when $\beta \leq 10^{-(n+2)}$, that is when $|v_z|$ is up to $3\ 10^{8-(n+2)}$ [m s $^{-1}$]. It is clear that such a range includes impressive axial velocity values for small values of n ($n \geq 1$).

The results of this new set of simulations indicate that it is not a good idea to reduce the computational time required by finite element simulators by using small values of n, for the type of applications of interest in this manuscript.

For the specific case $m = 4$ it could be useful to analyze the behaviour of the finite element component $\mathbf{E}_{b,z}$ and to compare it with $\mathbf{E}_{0,b,z}$ and with the corresponding semi-analytical components. In Figure 6, it can be observed that the two finite element plots are completely overlapped and that the same happens for the two plots of the semi-analytical solutions.

Moreover, the order of magnitude of the components is 10^8 times larger than that of their differences, which are reported in Figure 5. The plots in Figures 5 and 6 allow us to understand, on the one hand, the reason for the unreliability of the classical numerical approach which calculates $\mathbf{E}_{b,z} - \mathbf{E}_{0,b,z}$ and the need for a new way to deal with the problem. On the other hand, they explain why the Born approximations are able to provide exceptionally good results.

The cylinders considered so far are all characterized by $\varepsilon_r - 1 = 1$. Since the dielectric parameters of the media in motion have important consequences on the motion effects, in the third and last set of simulations we study the scattering by cylinders having $\varepsilon_r = 1.01$, so that $\varepsilon_r - 1 = 0.01$. Such a value could be of interest when the medium in motion is, for example, a gas [28]. The mesh for all these cases is obtained by setting $p = 140$, in order to have again about 14 elements per wavelength.

Figure 7 shows the behaviour of $|(\mathbf{H} - \mathbf{H}_0)_z| = |H_z|$ along the x axis for $\beta = 10^{-7}$ and $\delta = 10^{-8}$. The small density of the scatterer is a big problem for the traditional approach based on the direct calculation of $\mathbf{H}_{b,z}$, in spite of the large velocity value considered (about 30 m/s) and of a significant first order effect of motion on such a component. This unreliability is like the one shown in Figure 4 for the same component and is due to the fact that the quantity of interest is of the same order of magnitude (in both figures the largest magnitude is about $2\ 10^{-12}$ [A/m]). The effects of a much larger velocity value are balanced by a much small value of $\varepsilon_r - 1$ of the scatterer. On the contrary, $(\mathbf{H} - \mathbf{H}_0)_{ba2,b,z}$ and $(\mathbf{H} - \mathbf{H}_0)_{ba1,b,z}$ approximate very well the semi-analytical solution.

The results for $|(\mathbf{E} - \mathbf{E}_0)_z|$ along the y axis for $\beta = 10^{-6}$ and $\delta = 10^{-12}$ are reported in Figure 8. It shows that the usual finite element approach provides unreliable results. Also the difference between semi-analytical solutions is slightly corrupted by numerical noise, especially where it assumes small values. On the contrary, the two Born approximations provide the best results. It is important to observe that, in this case, the first-order Born approximation is not affected by errors as it was the case for scatterers with larger values of ε_r.

The reduced effects of motion for small values of $\varepsilon_r - 1$ can be easily understood by a more traditional Born

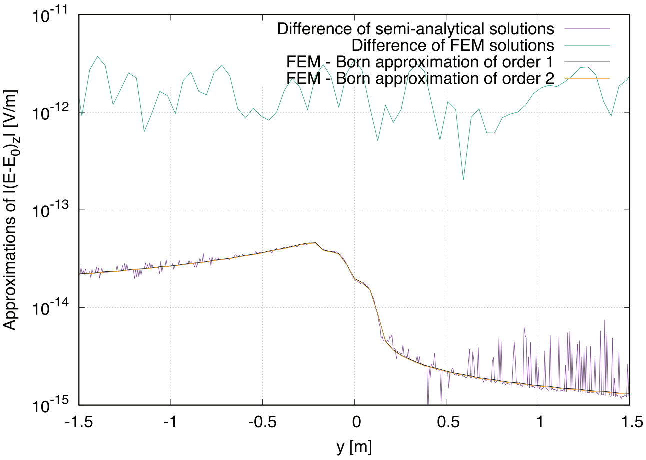

FIGURE 8 Approximations of $|(\mathbf{E}-\mathbf{E}_0)_z|$ along the y axis for $\beta = 10^{-6}$, $\delta = 10^{-12}$, $p = 140$ and $\varepsilon_r = 1.01$. The result obtained by computing the difference of semi-analytical solutions is corrupted by noise where the magnitudes are small. The two Born approximations, $(\mathbf{E}-\mathbf{E}_0)_{ba2,h,z}$ and $(\mathbf{E}-\mathbf{E}_0)_{ba1,h,z}$ provide the same results, in this case, and are the most reliable solutions. Finally, $\mathbf{E}_{h,z} - \mathbf{E}_{0,h,z}$ is completely wrong.

approximation. As a matter of fact, the field $\mathbf{H}_{0,t}$, which appears in equations (24) and (25), can be thought as the superposition of the incident and the scattered field [37] (p. 328)

$$\mathbf{H}_{0,t} = \mathbf{H}_{inc,t} + \mathbf{H}_{0,sc,t}, \tag{29}$$

where the scattered field is generated by the equivalent source [37] (p. 328)

$$\mathbf{J}_{e,eq,standard} = j\omega\varepsilon_0(\varepsilon_r - 1)\mathbf{E}_0. \tag{30}$$

For small values of $\varepsilon_r - 1$ and limited extensions of the scatterers a very good approximation of $\mathbf{E}_0$ is given by $\mathbf{E}_{inc}$, which is independent of ε_r by definition. Then $\mathbf{H}_{0,t}$ in Equation (29) can be very well approximated by $\mathbf{H}_{0,t,a}$, which is the sum of a field independent of $\varepsilon_r - 1$ and of another field whose norm is controlled by $\varepsilon_r - 1$, at least for small values of the same quantity.

Taking into account that, for small values of β the tensor fields $(\varepsilon - \varepsilon^0)$ and $(\mu - \mu^0)$ depend on $\beta^2(\varepsilon_r - 1)$ while ξ and ζ present a factor $\beta(\varepsilon_r - 1)$ [41] (see the second paragraph below equation (24) of that paper), one can then repeat the procedure presented in Section 5 by replacing $\mathbf{H}_{0,t}$ with $\mathbf{H}_{0,t,a}$ in equations (24) and (25).

At the end of the procedure, for small values of β and $\varepsilon_r - 1$, one obtains that the norms of $(\mathbf{E}-\mathbf{E}_0)_{ba1,z}$ and $(\mathbf{H}-\mathbf{H}_0)_{ba1,t}$ are controlled by $\beta^2(\varepsilon_r - 1)$, while the norms of $(\mathbf{H}-\mathbf{H}_0)_{ba1,z}$ and $(\mathbf{E}-\mathbf{E}_0)_{ba1,t}$ are smaller than or equal to a constant independent of β and ε_r multiplied by $\beta(\varepsilon_r - 1)$. It is easy to verify that such a quantity assume the same value in the cases considered in Figures 4 and 7. This clarifies the meaning of our previous comment on the results of Figure 7.

Moreover, the same procedure guarantees that the norm of $(\mathbf{E}-\mathbf{E}_0)_{ba2,z} - (\mathbf{E}-\mathbf{E}_0)_{ba1,z}$ is controlled by $\beta^2(\varepsilon_r - 1)^2$. It has an additional $(\varepsilon_r - 1)$ factor with respect to the norm of $(\mathbf{E}-\mathbf{E}_0)_{ba1,z}$. This explains why the difference between the second order Born approximation and the first order one cannot be detected in Figure 8.

The same conclusion does not apply to our former analysis, which considered $\varepsilon_r = 2$.

The final set of simulations show that the upper bound for m for the reliability of the different approximations is affected by ε_r but this is in any case in agreement with our previous deductions. For example, the difference of finite element solutions is not reliable for the calculation of $(\mathbf{E}-\mathbf{E}_0)_z$ when $m \geq \frac{n}{2}$ if $\varepsilon_r = 2$ and when $m \geq \frac{n}{2} - 2$ if $\varepsilon_r = 1.01$. The same reduction by 2 is present for all upper bounds of m which guarantee the reliability of the different approaches, when the ε_r of the scatterer at rest is reduced from 2 to 1.01.

At the same time, the final results confirm that the approximation of H_z obtained by the Born approximations are extremely good, for any of the values considered for m, n and ε_r.

7 | CONCLUSIONS

In this paper a new methodology is presented that aims at the reliable calculation of the very weak effects of the non-relativistic motion of media on the electromagnetic field.

The new methods do not require the development of numerical simulators because they need to solve problems involving just media at rest. The bianisotropic constitutive parameters of media in motion are managed in the definition of equivalent sources and require simple algebraic calculations involving known quantities. Moreover, the simplest of the new methods requires the solution of two problems, like the traditional approach.

Even though the new methods can be used to deal with cylinders moving in the axial direction or with axisymmetric objects in rotation around their axes of symmetry, in this first work the numerical results are focused just on the first of the two classes of problems.

All results are computed by using finite element simulators. They show that the traditional approach provides unreliable results in many situations of practical interest. Moreover, they clearly show the reliability of the new methods. This was largely expected for non-relativistic velocity values. However, the extension of the range of speed values for which the new methods are shown to provide satisfactory results is beyond authors' expectations. As a matter of fact, the reliability is guaranteed for large fractions of the speed of light in vacuum, too. Finally, it is shown that for velocity values up to several meters per second the results provided by the new methods could be better than those deduced by the calculation of the difference of semi-analytical solutions.

The effects of the approximations introduced by iterative solvers are considered together with a study of the effects of the density of the scatterer on the performances of the new methods.

AUTHOR CONTRIBUTIONS

Mirco Raffetto: Conceptualization; data curation; formal analysis; investigation; methodology; project administration; software; supervision; validation; writing—original draft; writing—review and editing. **Mario Rene Clemente Vargas**: Conceptualization; data curation; formal analysis; investigation; methodology; software; supervision; validation; writing—original draft; writing—review and editing. **Kirill Zeyde**: Conceptualization; data curation; formal analysis; investigation; methodology; software; supervision; validation; writing—original draft; writing—review and editing.

CONFLICT OF INTEREST STATEMENT

The author declares no conflicts of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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REFERENCES

1. Einstein, A.: Zur Elektrodynamik bewegter Körper. Ann. Phys. 17(322), 891–921 (1905)
2. Tai, C.T.: Present views on electrodynamics of moving media. Radio Sci. 2(2), 245–248 (1967)
3. Costen, R.C., Adamson, D.: Three-dimensional derivation of the electrodynamic jump conditions and momentum-energy laws at a moving boundary. Proc. IEEE 53(9), 1181–1196 (1965)
4. Weiglhofer, W.S.: Analytic methods and free-space dyadic green's functions. Radio Sci. 28(05), 847–857 (1993)
5. Ferencz, C.: Electromagnetic wave propagation in inhomogeneous, moving media: A general solution of the problem. Radio Sci. 46(05), 1–14 (2011)
6. Deck-Leger, Z.-L., Zheng, X., Caloz, C.: Electromagnetic wave scattering from a moving medium with stationary interface across the interluminal regime. Photonics 8(6), 202 (2021)
7. Van Bladel, J.G.: Electromagnetic Fields. 2nd ed. IEEE Press, Piscataway, NJ (2007)
8. De Zutter, D.: by a rotating dielectric sphere. IEEE Trans. Antennas Propag. 28(5), 643–651 (1980)
9. Yan, Y.: Mass flow measurement of bulk solids in pneumatic pipelines. Meas. Sci. Technol. 7(12), 1687 (1996)
10. Messiaen, A.M., Vandenplas, P.E.: High-frequency effect due to the axial drift velocity of a plasma column. Phys. Rev. 149, 131–140 (1966)
11. Yeh, C.: Reflection and transmission of electromagnetic waves by a moving dielectric medium. J. Appl. Phys. 36(11), 3513–3517 (1965)
12. Yeh, C.: Scattering obliquely incident microwaves by a moving plasma column. J. Appl. Phys. 40(13), 5066–5075 (1969)
13. De Zutter, D.: Scattering by a rotating circular cylinder with finite conductivity. IEEE Trans. Antennas Propag. 31(1), 166–169 (1983)
14. Shiozawa, T., Hazama, K.: General solution to the problem of reflection and transmission by a moving dielectric medium. Radio Sci. 3(6), 569–575 (1968)
15. Censor, D.: Scattering of a plane wave at a plane interface separating two moving media. Radio Sci. 4(11), 1079–1088 (1969)
16. Huang, Y.-X.: Reflection and transmission of electromagnetic waves by a dielectric medium moving in an arbitrary direction. J. Appl. Phys. 76(5), 2575–2581 (1994)
17. Pastorino, M., Raffetto, M.: Scattering of electromagnetic waves from a multilayer elliptic cylinder moving in the axial direction. IEEE Trans. Antennas Propag. 61(9), 4741–4753 (2013)
18. Brignone, M., Raffetto, M.: Well posedness and finite element approximability of two-dimensional time-harmonic electromagnetic problems involving non-conducting moving objects with stationary boundaries. ESAIM: Math. Model. Numer. Anal. 49(4), 1157–1192 (2015)
19. Kalarickel Ramakrishnan, P., Raffetto, M.: Well posedness and finite element approximability of three-dimensional time-harmonic electromagnetic problems involving rotating axisymmetric objects. Symmetry 12(2), 1–28 (2020)
20. Kalarickel Ramakrishnan, P., Clemente Vargas, M.R., Raffetto, M.: Electromagnetic inverse scattering of rotating axisymmetric objects. IEEE Access 9, 168185–168192 (2021)
21. Pastorino, M., Raffetto, M., Randazzo, A.: Electromagnetic inverse scattering of axially moving cylindrical targets. IEEE Trans. Geosci. Remote Sens. 53(3), 1452–1462 (2015)
22. Van Bladel, J.: Electromagnetic fields in the presence of rotating bodies. Proc. IEEE 64(3), 301–318 (1976)
23. Jackson, J.D.: Classical Electrodynamics. 3rd ed. Wiley, New York (1999)
24. Morse, P.M., Feschback, H.: Methods of Theoretical Physics. McGraw-Hill, New York (1953)
25. Cheng, D.K., Kong, J.-A.: Covariant descriptions of bianisotropic media. Proc. IEEE 56(3), 248–251 (1968)
26. Sommerfeld, A.: Electrodynamics. In: Lectures on Theoretical Physics. Academic Press, New York (1952)
27. Huang, Y.-X.: Identification of the property of a moving medium by the reflected electromagnetic wave. Radio Sci. 38(3), 1044 (2003)
28. Censor, D.: Scattering of electromagnetic waves by a cylinder moving along its axis. IEEE Trans. Microwave Theory Tech. 17(3), 154–158 (1969)
29. Brignone, M., Gragnani, G.L., Pastorino, M., Raffetto, M., Randazzo, A.: Noise limitations on the recovery of average values of velocity profiles in pipelines by simple imaging systems. IEEE Geosci. Remote Sens. Lett. 13(9), 1340–1344 (2016)
30. Zeyde, K., Sharov, V.: Fluid flow sensors design based on electromagnetic drag effect. In: 2019 International Conference on Control, Artificial Intelligence, Robotics Optimization (ICCAIRO), pp. 48–53. IEEE, Piscataway (2019)
31. Zeyde, K.: Resonant cavity second order perturbation by moving medium with stationary domain. In: 2021 IEEE 22nd International Conference of Young Professionals in Electron Devices and Materials (EDM), pp. 132–136. IEEE, Piscataway (2021)
32. Fernandes, P., Ottonello, M., Raffetto, M.: Regularity of time-harmonic electromagnetic fields in the interior of bianisotropic materials and metamaterials. IMA J. Appl. Math. 79(1), 54–93 (2014)
33. Marqués, R., Medina, F., Rafii-El-Idrissi, R.: Role of bianisotropy in negative permeability and left-handed metamaterials. Phys. Rev. B 65, 144440 (2002)
34. Kong, J.A.: Theory of Electromagnetic Waves. Wiley, New York (1975)
35. Monk, P.: Finite Element Methods for Maxwell's Equations. Oxford Science Publications, Oxford (2003)
36. Griffiths, D.J., Schroeter, D.F.: Introduction to Quantum Mechanics. Cambridge University Press, Cambridge (2018)
37. Balanis, C.A.: Advanced Engineering Electromagnetics. Wiley, New York (1989)
38. Fernandes, P., Raffetto, M.: Well-posedness and finite element approximability of time-harmonic electromagnetic boundary value problems involving bianisotropic materials and metamaterials. Math. Models Methods Appl. Sci. 19(12), 2299–2335 (2009)
39. Ciarlet, P.G.: Basic Error Estimates for Elliptic Problems. Elsevier Science Publishers B. V. (North-Holland), Amsterdam (1991)
40. Ciarlet, P.G., Lions, J.L. (eds.): Handbook of Numerical Analysis. In: Finite Element Methods, Part 1, vol. II. North-Holland, Amsterdam (1991)

41. Kalarickel Ramakrishnan, P., Raffetto, M.: Accuracy of finite element approximations for two-dimensional time-harmonic electromagnetic boundary value problems involving non-conducting moving objects with stationary boundaries. ACES J. 33(6), 585–596 (2018)
42. Volakis, J.L., Chatterjee, A., Kempel, L.C.: Finite Element Method for Electromagnetics: Antennas, Microwave Circuits, and Scattering Applications. IEEE Press, Piscataway, NJ (1998)
43. Barrett, R., Berry, M., Chan, T.F., Demmel, J., Donato, J., Dongarra, J., Eijkhout, V., Pozo, R., Romine, C., Van der Vorst, H.: Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods. 2nd ed. SIAM, Philadelphia, PA (1994)

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