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Arithmetic, Working Memory, and Visuospatial Imagery Abilities in Children with Poor Geometric Learning

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Abstract

Many children fail in geometric learning, but factors underlying these failures have not been explored in detail. The present study addresses this issue by comparing fifth and sixth-grade children who had good or poor geometric learning, and were otherwise comparable on verbal intelligence, gender and age. Results showed that children with poor geometric learning have deficits in both arithmetic and geometric problem solving but they are more impaired in the latter. Results also showed that poor geometric learners have weaknesses in working memory, calculation, and visuospatial mental imagery. The results from logistic regressions pointed out that mental imagery skills and arithmetic problem solving ability had the highest discriminatory power in distinguishing between the two groups. Theoretical and practical implications of this research for designing interventions to help poor geometric learners are discussed.

Keywords: geometric learning; visuospatial abilities; mental imagery; arithmetic; working memory; problem solving

Highlights:

- Geometric learning is an important aspect of academic learning, and factors underlying failures in this aspect have not been sufficiently studied
- Failures in tasks related to working memory and arithmetic and calculation are typical in children with poor geometric learning
- Mental imagery and arithmetic problem solving are highly discriminating factors between children with poor or good geometric learning

Arithmetic, Working Memory, and Visuospatial Imagery Abilities in Children with Poor Geometric Learning

The complex set of acquisitions involved in learning geometry, including, for example, knowledge about spatial arrays and their measurement, are linked to students' future academic and professional success (Verstijnen, van Leeuwen, Goldschmidt, Hamel, & Hennessey, 1998). In fact, geometry represents one of the most important forms of mathematical knowledge, relevant in many aspects of everyday life (Cass, Cates, Smith, & Jackson, 2003) and important in fields including science, technology, engineering, and mathematics (Zhang, Ding, Stegall, & Mo, 2012). Nowadays, geometry is included in the majority of mathematical curricula in the world (OECD, 2010).

A relevant body of evidence on students who face specific difficulties in arithmetic, despite having average intelligence and sufficient achievement in other academic areas, has been collected (e.g., Passolunghi & Mammarella, 2010). Conversely, little evidence on students with specific difficulties in geometric learning is available (Mammarella, Giofrè, & Caviola, 2016). Consequently, the cognitive profile of students with difficulties in learning geometry has not been studied in depth. The goal of the present research is to provide insights on factors affecting difficulties in learning geometry.

Recent evidence proposes a distinction between intuitive geometry and geometric learning. Intuitive geometric concepts (e.g., Euclidean geometry) are shared by humans regardless of formal education (Dehaene, Izard, Pica, & Spelke, 2006; Spelke, Lee, & Izard, 2010). In contrast, the geometric learning explored in the present research, operationally defined as the ability to answer typical geometric questions and problems encountered in schools (Giofrè, Mammarella, & Cornoldi, 2014), involves concepts that are predominantly learnt through formal instruction. Geometric learning demands an explicit knowledge of principles and concepts (e.g., diagonals, parallel lines, and right angles) and of rules and their application in representing complex spatial relationships (e.g., imagining the result of the combination of two figures). Such learning also involves applying rules to specific requests (e.g., calculating the area or the perimeter of a figure).

Due to this intrinsic complexity, school curricula in the early grades are focused on basic geometric knowledge (i.e., properties and rules that apply to plane figures such as circles, squares and triangles). Only later on, usually during the fifth and sixth grades, does the curriculum become more complex and structured, and this can create increasing difficulties for some students. It is worth noting, however, that a difficulty in learning geometry may be due not just to complexities involved in geometric learning. This kind of learning difficulty can also stem from a variety of factors that also seem to affect complex geometric learning, including calculation skills, working memory (WM), visuospatial mental imagery, and arithmetic problem solving ability. However, to what extent these aspects are associated with a failure in geometric learning has not been investigated in depth.

A difficulty with calculation seems to be relevant because it impacts students' confidence as they cope with other types of mathematical situations (Aydın & Ubuz, 2010), including processes crucial for geometric learning. In particular, arithmetic is typically involved in many geometric situations requiring the use of measures and calculation (Mammarella et al., 2016). Also, a general problem solving ability is clearly connected to geometric learning, as it is associated with several distinct processes, such as comprehending the problem, building a representation of it, and planning and supervising the solution process (Mammarella et al., 2016; Passolunghi & Pazzaglia, 2004). In particular, arithmetic problem solving, involving not only calculation but also mathematical reasoning, may have a particularly strong impact on geometric learning. In addition, geometric tasks, due to their specific visuospatial features, may require specific abilities, which are not necessarily shared with arithmetic abilities (implied in calculation and arithmetic problem solving), such as spatial skills (e.g., Clements & Battista, 1992) and in particular visuospatial WM (Giofrè, Mammarella, Ronconi, & Cornoldi, 2013) and visuospatial mental imagery (Mammarella et al., 2016). As a result, arithmetic abilities may be necessary but not sufficient for children to master geometry.

In psychological literature, the role of WM has been widely acknowledged in arithmetic learning (e.g., DeStefano & LeFevre, 2004) but examined minimally in relation to geometry. However, WM seems to be involved in geometric learning not only because arithmetic and geometric problem solving share several WM resources (Passolunghi, Cornoldi, Liberto, Passolunghi, & De Liberto, 1999; Passolunghi & Mammarella, 2010; Zheng, Swanson, & Marcoulides, 2011), but also because geometric learning typically requires the temporary maintenance and treatment of both verbal and visuospatial information. This temporary maintenance can be seen, for example, in tasks such as representing geometric forms or memorizing specific geometric formulas (Giofrè et al., 2014). In fact, it has been shown that WM predicts success in school-related tasks that require the maintenance and processing of information, such as reading comprehension (e.g., Carretti, Borella, Cornoldi, & De Beni, 2009; García-Madruga et al., 2013), approximate mental addition (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012; Mammarella, Cornoldi, et al., 2013), multi-digit operations (Heathcote, 1994), magnitude representation (e.g., Pelegrina, Capodieci, Carretti, & Cornoldi, 2014) and mathematical achievement (e.g., Bull, Espy, & Wiebe, 2008; Passolunghi, Mammarella, & Altoè, 2008). Because of this, it seems plausible to hypothesize that WM is similarly involved in learning geometry.

Working memory is a limited-capacity system that enables information to be temporarily stored and manipulated. In the classical dominant tripartite model of WM, the central executive is considered responsible for controlling resources and monitoring the processing of information across domains (Baddeley & Hitch, 1974). In contrast, the storage of information is mediated by two domain-specific slave systems: the phonological loop, which handles the temporary storage of verbal information, and the visuospatial sketchpad, which is specialized in retaining and manipulating visual and spatial information (Baddeley, 1996). A complementary approach distinguishes between many different types of WM processes based not only on the content of the information (visual, spatial and verbal), but also on the degree of cognitive control (Cornoldi & Vecchi, 2003). This distinction has been shown to be particularly relevant in the arithmetic domain, in which different WM components have varying involvement in arithmetic (Mammarella, Pazzaglia, & Cornoldi, 2008). Also, verbal, visuospatial and WM aspects may require different levels of cognitive control, and this distinction seems to be particularly relevant when considering geometric learning. As for visual and spatial tasks, spatial WM (spatial spans) seem to require cognitive control to a lesser extent, while other visual WM tasks seem to require more attentional resources (Cornoldi & Vecchi, 2003).

Geometry deals with spatial information of two and three-dimensional patterns. According to recent reports, visuospatial WM may have a critical role both in arithmetic (Li & Geary, 2013; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2013) and in geometric processes (Giofrè, Mammarella, Ronconi, et al., 2013). Geometry involves processing of figures in space, and it seems plausible that, besides visuospatial WM, other visuospatial abilities affect geometric learning (Hannafin, Truxaw, Vermillion, & Liu, 2008). In particular, it has been argued that geometric learning can be sustained by visuospatial mental imagery (Weckbacher & Okamoto, 2014), which allows people to generate mental representations of geometric figures as they are verbally described and to manipulate, organize and compare elements across imagined figural patterns. In fact, visuospatial mental imagery is not only supported by visuospatial WM processes (Cornoldi & Vecchi, 2003; Logie, 1995), but also involves other skills related to the mental manipulation of forms (Andrade, 2002; Cornoldi, De Beni, & Mammarella, 2008) that may be crucial in geometric learning. Accordingly, a significant correlation between visuospatial mental imagery and geometry has been reported in high-school students, whereas the correlation between mental imagery and algebra was not statistically significant (Weckbacher & Okamoto, 2014).

The present study aimed to investigate which factors underlie the difficulties some children have in geometric learning. To reach this goal, and to identify both factors that cause difficulty in geometric learning and factors that support high geometric achievement, we adopted a good vs. poor ability design (also known as extreme group design). This approach is in fact very common for testing individual differences (Engle, 2010) and has been used extensively and successfully in several studies (e.g., Borella, Ludwig, Fagot, & De Ribaupierre, 2010; Fukuda & Vogel, 2011; Kane et al., 2007; Kane & Engle, 2002; Smeding, Darnon, & Van Yperen, 2015; Unsworth, Schrock, & Engle, 2004). Two groups of children - respectively with good and poor performance in a standardized geometry test but matched for age and verbal intelligence, and with no history of sociocultural challenges, severe arithmetic difficulties, or clinical problems - took part in the study. Children were tested with a large set of tasks related to skills including WM, visuospatial mental imagery, calculation and arithmetic problem solving.

The separate consideration of geometric vs. arithmetic problem solving abilities has not received attention in the literature to date, also because, in other past research there has seldom been a differentiation between children with arithmetic difficulties only, children with geometric difficulties only, and children with difficulties in both areas (Mammarella et al., 2016). Therefore, for the present study, we developed arithmetic and geometric problems that were very similar, in terms of the solving procedures and the computation required, but crucially differed in their content (i.e., arithmetic or geometric). As the skills required for solving both geometric and arithmetic problems are partly overlapping, we hypothesized that children with poor geometric learning would struggle with both geometric and arithmetic problems. However, as the skills required for solving geometric problems also involve specific geometric abilities, we expected children with poor geometric learning to show greater impairment in the geometric problems compared to the arithmetic ones. If confirmed, this result would demonstrate that the difficulties that affect geometric problem solving are not entirely the same as those that affect comparable arithmetic problems.

To examine the role of calculation skills, WM, and visuospatial mental imagery in geometric learning, we compared children with good or poor geometric learning using a large set of tasks including a calculation battery, a WM memory battery, a visuospatial mental imagery test, and a problem solving battery (distinguishing between geometric and arithmetic problems). The arithmetic battery included simple and complex arithmetic calculations, and an approximate calculation task requiring children to decide which choice –between two numbers– better approximated the actual result of a series of calculations. The series of WM tasks assessed both verbal WM in its less controlled (forward digit span) and more controlled (backward digit span) components. We hypothesized that there would be statistically significant differences in almost all the domains, but that differences would be greater in mental imagery and problem solving. In fact, mathematical reasoning, as assessed by problem solving tasks, and the visualization and manipulation of visual representations require some of the processes assessed by WM and arithmetic tasks, while also supporting the complex activities required by geometric learning (see Mammarella et al., 2016 for a discussion). Thus, we predicted that, when arithmetic problems and mental imagery were entered into the equation, the predictive value of the other variables would be diminished.

Method

Participants

Two groups, each including 45 children, were formed on the basis of an initial screening that involved a sample of 309 children (111 in the fifth grade and 198 in sixth grade). Children in the two groups were selected for having good geometric learning (≥ 70° percentile) or poor geometric learning (≤ 30° percentile), assessed by the GEO-P test (Cronbach's α = .71; Mammarella, Todeschini, Englaro, Lucangeli, & Cornoldi, 2012). In this test, children are required to calculate the area of complex figures or to solve complex geometric exercises. Verbal intelligence was evaluated by the Verbal Meaning subtest of the Primary Mental Ability Test, used in two different forms respectively for the younger and the older children (PMA, 2-4 and PMA 11-17; Thurstone & Thurstone, 1963). The Brown-Spearman corrected fidelity index reported for this test in the Italian manual is very high (.96). Children were included in the study if they had not received a clinical diagnosis (e.g., intellectual disability), did not present learning disorders in any other academic area (including arithmetic) according to the school reports, and did not belong to disadvantaged sociocultural or linguistic groups (see for a similar procedure Giofrè, Borella, & Mammarella, 2017; Giofrè et al., 2014; Giofrè, Mammarella, Ronconi, et al., 2013; Mammarella, Giofrè, Ferrara, & Cornoldi, 2013). To confirm that children included in the study did not have a specific arithmetic difficulty, we assessed basic arithmetic abilities through a quick battery, involving arithmetic exercises of different types, used for screening (AC-MT paper-and-pencil battery; Cornoldi, Bellina, & Lucangeli, 2012; Cronbach's $\alpha = .70$).

The two groups were composed of 45 children (20 fourth graders and 25 fifth graders), as follows: the poor geometric learning group (26 females and 19 males; $M_{age} = 134.90$ [8.02] months), and the good geometric learning group (24 females and 21 males; $M_{age} = 136.66$ [8.20] months). These two groups were similar in gender, age, and verbal intelligence, and in basic arithmetic abilities, but differed considerably in terms of geometric learning (Table 1).

Table 1 about here

Materials

Problem solving battery. A problem solving battery, including 12 geometric and 12 arithmetic problems, was explicitly created for the present study. Geometric problems (four with triangles, four with squares, and four with rectangles) were based on knowledge and principles taught to all children in the Italian curriculum. In particular, to solve these problems, children were required to calculate the perimeter of different figures: half of the problems with a direct formula and half with an indirect one. The arithmetic problems were very similar to the geometric ones, meaning that they required the same calculations and procedures, but they did not include any reference to geometric knowledge. In other words, the two kinds of problems (i.e., geometric and arithmetic) had the same mathematical structure, in terms of operations required to solve them, but were crucially differed in their content: geometric problems involved geometric content (e.g., calculating the perimeter), whereas arithmetic problems only involved arithmetic content (e.g., calculating the product of multiplication). An example of a geometric problem was: "Fabio should buy a stick of wood for a frame that has the shape of an isosceles triangle, having the two identical sides measuring 9 cm, and a perimeter of 26 cm. How long is the stick of wood that he needs?"; the corresponding arithmetic one was: "Mr. Peter has to fill 26 bottles with wine and put them in boxes. He has already packaged two boxes with 9 full bottles in each. How many more bottles have to be filled?". In order to avoid practice effects for a particular category of problems, arithmetic and geometric problems were mixed, and problems involving the same algorithm had to be separated by at least four other problems. The overall reliability for the problem solving battery was .87. Correlation between the number of correct responses respectively for geometric and arithmetic problems was adequate (.73), confirming that the two problem types were highly correlated.

Calculation battery. Calculation ability was assessed by three computer based tasks; simple calculation, complex calculation, and approximate calculation. All tasks were presented on a 15-inch laptop and were programmed using the Superlab Software. Stimuli were presented in Arial font, size 16, and each trial started with a fixation marker lasting 1 second and followed by a blank screen. After 1s, the two target operands appeared simultaneously at the center of the screen. The order of the blocks was fixed, but trials within each block were randomized. Cronbach's α of the calculation battery was .76. In the *simple calculation task***,** children were administered with three blocks of twenty-four additions (e.g., 13 + 9), subtractions (e.g., 10 - 8) and multiplications (e.g., 24 × 3), for a total of 92 operations. The time limit was 5 seconds for each operation. A microphone was used to collect the answers. The task lasted about 15 min. In the *complex calculation task*, children were presented with a series of additions (e.g., 39 + 58) and subtractions (e.g., 90 – 34) of increasing difficulty, for a total of 16 operations. The time limit was 10 seconds for each operation. A microphone was again used to collect the answers. The task lasted about 10 min. Finally, in the *approximate calculation task,* children were requested to indicate the most plausible approximation to a given operation between two alternative results. Children were presented with four additions (e.g., 162 + 9: 160 or 190), four subtractions (e.g., 489 – 71: 420 or 460), and four multiplications (e.g., 67 × 5: 340 or 380), for a total of 12 operations. The time limit was 5 seconds for each operation. Two possible answers were simultaneously presented and children had to indicate, pressing one of two different keys on the keyboard, which answer better approximated the correct result. This task also lasted about 10 min.

Working memory battery. Participants were presented with two digit span tasks and five visuospatial WM tasks derived from an Italian standardized WM test battery for children (BVS; Mammarella, Toso, Pazzaglia, & Cornoldi, 2008), which was inspired by the visuospatial WM model proposed by Cornoldi and Vecchi (2003) and includes different aspects of visuospatial WM. The BVS battery manual reports good psychometric properties, a Cronbach's α ranging from .84 to .92, and has been used in other studies (e.g., Caviola, Mammarella, Lucangeli, & Cornoldi, 2014; Giofrè, Mammarella, Ronconi, et al., 2013). All the tasks were administered in a paper and pencil format, except for the Corsi, which was administered in the classical physical board format. All tasks were of increasing difficulty, and children continued as long as they were able to solve at least two items out of three at a given level. The "span" score - which corresponds to the longest list the child is able to perfectly recall at least in 2 out of the 3 presented sequences - was calculated for the digit span and for the spatial span (for the forward and backward version, respectively). In all the other tasks, the score was calculated as the sum of the last three correct responses: for example, if a participant successfully solved two items at the fourth level and one at the fifth, then the score was thirteen. In the *digit span tasks*, digits were presented verbally at a rate of 1 item per second. In the forward digit span, participants were required to recall the digit sequence material in the same order, whereas in the backward digit span they had to recall the series in the backward order. There was no time limit for recalling the digits. The *spatial span tasks* included a forward version of the task, in which participants were required to reproduce the spatial sequence presented by the experimenter in the same order, and a backward version where they had to reproduce the sequence in the backward order (Corsi, 1972). In the v*isual pattern test* (VPT; adapted from Della Sala, Gray, Baddeley, & Wilson, 1997), participants were presented for 3s with random square matrices created by filling half of the squares of a grid. The grids were of increasing size: for example, in the second level, the grids included 6 squares with 3 filled cells, and in the last level, the grids included 22 squares with 11 filled cells. In the presentation phase, participants had to memorize the filled squares. After 3s, the initial stimulus was removed and participants were presented with a blank test matrix in which they had to indicate the previously filled squares. In the *pathway task* (adapted from Mammarella, Toso, et al., 2008), participants were required to mentally visualize a pathway followed by a little man moving on a blank matrix. At the end of a series of directions given by the experimenter (i.e., forward, backward, left, or right), the child had to indicate the man's final position in the matrix. The complexity of the task varied according to the size of the matrix (from 2 × 2 to 6 × 6) and to the length of the pathway described (from 2 to 10). Finally the *jigsaw puzzle task* (adapted from Vecchi & Richardson, 2000) consisted of a series of drawings fragmented into two to ten numbered pieces forming a puzzle. Each whole drawing was presented for 2s, and was then removed. The puzzle pieces were set out in a non-ordered way and a blank matrix with a corresponding number of cells was then displayed in front of the participant. Puzzles had to be solved without moving the pieces, by writing down or pointing to the corresponding number of each piece on a response sheet. The level of complexity was defined by the number of pieces in each puzzle (from 2 to 10).

Visuospatial mental imagery test. The test was developed on the basis of a series of empirical analyses concerning the development of imagery and spatial skills and is included in the "*Geometria test*" battery (Mammarella et al., 2012) with good psychometric properties (Cronbach's $\alpha = .72$). The test involves a series of 16 items that require different mental operations, all of which share a common requirement of holding in memory one or more figures in order to find the correct solution (see Figure 1). More specifically, children are required to compose and decompose spatial patterns (four items each), find an embedded figure (four items), and color the intersection between different figures (four items).

Figure 1 about here

Procedure

Participants were tested in an individual session lasting approximately 1.5 hours in a quiet room outside the classroom. Tasks were administered in the following fixed pseudorandomized order: the problem solving battery, the WM battery, the calculation battery and the visuospatial mental imagery test. The tasks within the batteries were administered in a fixed order, as described in the material section. The research was carried out in accordance with the guidelines of the Bicocca University in Milan (Italy), and the declaration of Helsinki and the Ethical Guidelines of the Italian Association for Psychologists. For all children, parental consent was obtained prior to testing.

Statistical analyses

The R program (R Core Team, 2014) was used with the “rms” package for hierarchical logistic regressions (Harrell, 2016). We considered both the statistical significance, which can be in some cases biased by reduced statistical power (see Tressoldi & Giofrè, 2015 on this point), and the magnitude of the effect expressed in terms of effect size following the Cohen (1988) criteria: .01, .09, and .25 for the partial eta square (η^2_p), and .20, .50, and .80 for the Cohen’s d were considered, small, medium, and large effects, respectively.

As for univariate analysis of variance (ANOVA), skewness and kurtosis were moderated and were under 1.00, which is generally considered acceptable (Tabachnick & Fidell, 2007). Although the ANOVAs are robust against various violations of the assumptions, non-parametric analyses were performed to confirm the results of the parametric ones. As for multivariated analyses, multicollinearity was addressed: tolerance was > .593 and VIF < 1.686, indicating reasonably good values (Neter, Wasserman, & Kutner, 1989), and the correlations between measures were not extremely high (Table 2). Box’s M test was also checked and values were well above .001, which is generally considered as an indication that the null hypotheses of unequal variances can be rejected confidently (Tabachnick & Fidell, 2007). As for logistic regression, this analysis does not make assumptions of linearity, normality, homoscedasticity, and measurement level (Agresti & Kateri, 2011).

Results

Group comparisons.

Problem solving skills. We performed a 2 group [geometric learning: good vs. poor] × 2 problem type [geometric and arithmetic] mixed ANOVA. We found significant main effects of group, $F(1, 88) = 42.93$, $p < .001$, $\eta^2_p = .328$, and problem type, $F(1, 88) = 85.42$, $p = .003$, $\eta^2_p = .324$, with large effect sizes. We also found a significant interaction between group and problem type, $F(1, 88) = 9.25$, $p = .003$, $\eta^2_p = .095$, with a medium effect size. We also performed a post hoc comparison using Bonferroni's correction; all the contrasts were statistically significant. As Figure 2 clearly shows, geometric problems were more difficult than arithmetic problems regardless of the level of geometric learning. Nonetheless, this difference was more pronounced in the poor geometric learning group. The mean numbers of correctly solved problems were 10.18 and 10.91 (geometric and arithmetic respectively) for the good geometric learning group, and 6.67 and 8.69 (geometric and arithmetic respectively) for the poor geometric learning group.

Figure 2 about here

Calculation. We performed a multivariate analysis of variance (MANOVA) comparing calculation skills (simple calculation, complex calculation, and approximate calculation) by group. We found a significant effect of group, $F(3, 86) = 3.84$, $p = .012$, $\eta^2_p = .118$, with a medium effect size. Tests of significance showed that children with good versus poor geometric learning differed statistically on two tasks (i.e., simple calculation and complex calculation), with effect sizes ranging from small to moderate, but did not on approximate calculation task with a small effect size (Table 2).

Working memory. For the WM tasks, we performed a MANOVA comparing WM tests (forward digit span, backward digit span, forward spatial span, backward spatial span, VPT, pathway, and jigsaw puzzle) by group. We found a significant effect of group, $F(7, 82) = 2.37$, $p = .030$, $\eta^2_p = .168$, with a medium effect size. Tests of significance showed that children with poor geometric learning scored significantly lower than those with good geometric learning on almost all the WM tasks (with the exception of the forward digit-span), with effect sizes ranging from small to moderate (Table 3).

Visuospatial mental imagery. An ANOVA comparing the two groups showed that children with good geometric learning scored higher than those with poor geometric learning, with a large effect size (Table 3).

Table 2 and 3 about here

Hierarchical logistic regressions. A series of logistic regression analyses were performed testing five different models to find out which tasks had the highest discriminatory power in distinguishing between children with good versus poor geometric learning. Effect sizes and discriminatory power were estimated using Nagelkerke pseudo R^2 and c-index (or AUC, area under the curve).

Model 1. The logistic regression was performed using WM tasks as predictors (i.e., backward digit span, forward and backward spatial span, VPT, pathway and puzzle). The model was statistically significant and had only a fair predictive value, $\chi^2(6) = 16.29$, $p = .012$, $R^2 = .22$, *c-index* = .739 (Table 4).

Model 2. The logistic regression was performed using calculation tasks as predictors (i.e., simple calculation, and complex calculation). The model was statistically significant, but the predictive value was not very large, $\chi^2(2) = 9.62$, $p = .008$, $R^2 = .14$, *c-index* = .682 (Table 4).

Model 3. The logistic regression was performed using calculation and WM tasks as predictors. The model was statistically significant but had only a fair predictive value, $\chi^2(8) = 20.23$, $p = .010$, $R^2 = .27$, *c-index* = .774 (Table 4).

Model 4. The logistic regression was performed using calculation, WM and the visuospatial mental imagery tasks as predictors. The model was statistically significant and had a good predictive power, $\chi^2(9) = 28.00$, $p = .001$, $R^2 = .36$, *c-index* = .803. Importantly, when visuospatial mental imagery was entered into the model, only this task was statistically significant (Table 4).

Model 5. The logistic regression was performed using calculation and WM tasks, the visuospatial mental imagery test, and the arithmetic problem solving task as predictors. The model was statistically significant and had a good predictive power, $\chi^2(10) = 41.14$, $p < .001$, $R^2 = .49$, *c-index* = .847, and was statistically better compared to all previous models ($p < 001$). In this model, probably due the high correlation between predictors, only visuospatial mental imagery and arithmetic problem solving were statistically significant (Table 4).

Model 6. Considering the high correlation between some of the predictors in model 5, we decided to use a principal component analysis, with a Varimax rotation, to reduce the number of predictors to a smaller set of uncorrelated components for both WM and calculation tasks[1]. The scree-test showed the clear presence of two WM factors and of one calculation factor. These three factors were therefore entered in the logistic regression instead of the original tests. The model was statistically significant and had a similar predictive power compared to model 5, $\chi^2(5) = 36.34$, $p <$

.001, $R^2 = .44$, *c-index* = .831. In this model, only visuospatial mental imagery and arithmetic problem solving were statistically significant (Table 4).

Model 7. In the final step of analysis, we included geometric problem solving, to evaluate whether visuospatial mental imagery and arithmetic problem solving remain significant predictors of group differences in geometric learning, even when geometric problem solving was included in the regression model. The model was statistically significant and had a similar predictive power compared to the previous one, $\chi^2(6) = 46.26$, $p < .001$, $R^2 = .54$, *c-index* = .869. Visuospatial mental imagery and geometric problem solving were statistically significant, whereas arithmetic problem solving ceased to be statistically significant (Table 4).

Table 4 about here

Additional Analyses

All the parametric analysis presented in Table 3 were repeated using a non-parametric approach (Mann-Withney U). The results were very similar and perfectly consistent with the parametric analyses. In particular, the difference between the two groups was not statistically significant for approximate calculation ($U = 823.5$, $p = .122$) or forward digit span ($U = 848.0$, $p = .164$), while all the other effects were statistically significant (*U*s > 775.1, *p*s < .048).

We also performed a series of ANCOVAs and MANCOVAs controlling for the effect of age and gender, and the results were very similar to the original analyses, except for VPT, $F(1, 86) = 3.56$, $p = .062$, $\eta^2_p = .040$, and Puzzle, $F(1, 86) = 3.56$, $p = .056$, $\eta^2_p = .042$, whose results, although not statistically significant, were similar in terms of effect size.

Model 1 and 2 were also repeated using factor scores (see Model 6 for further information). When factor scores were used instead of the original tasks, all predictors were statistically significant (Table 4; Models 1a and 2a), confirming that the relatively high correlation between the tasks was causing some estimation problems.

Discussion

The present study aimed to shed light on geometric learning and on the factors underpinning a failure in this area. We examined whether and to what extent fifth- and sixth-graders with good versus poor geometric learning differed in a series of tasks including arithmetic problem solving, WM, visuospatial mental imagery, and calculation.

Children with poor geometric skills failed in arithmetic problems, but met particular difficulty with geometric problems. In this respect, our results offer more information on the characteristics of children with poor geometric learning. In fact, we designed arithmetic and geometric problems that were very similar, in terms of the solving procedures and the computation required, but not identical, with the latter including additional geometric content, which increased difficulty for children with poor geometric learning (see also Clements & Battista, 1992).

Concerning calculation skills, we found that, in children with poor geometric learning, calculation skills were poorer than in children with good geometric learning, but with very small effect sizes, which is consistent with findings indicating that quantity skills are not directly related to geometry (LeFevre et al., 2010). The result could also be due to the fact that children in the poor geometric learning group were reported not to have disabilities in other academic areas, including arithmetic. However, the result supports the hypothesis that it is possible to individuate children who fail in geometry, and that such children are, at least in part, different from those who only fail in arithmetic (see Mammarella et al., 2016 for a discussion). In further support of this, children with poor geometric learning had minor deficits in calculation skills, but they were struggling in other tasks, confirming that multiple factors are related to deficits in geometric learning.

Children with poor geometric learning were impaired in almost all WM tasks, confirming that verbal, visual and spatial aspects of WM are involved in geometric learning. Only performance on the forward digit span did not differ significantly between the two groups, with a very small effect size. Forward digit span requires verbal short term memory, while backward digit span requires learners to memorize verbally presented pieces of information and to rotate them in order.

Thus, backward digit span may involve some degree of mental rotation or visuospatial skills (Rudel & Denckla, 1974), which might be one reason why the backward digit span had a better discriminatory power compared to the forward one. An alternative explanation is that manipulation of information may be a critical aspect of WM that is involved in mathematical tasks (reviewed by DeStefano & LeFevre, 2004; Raghubar, Barnes, & Hecht, 2010). Forward digit span mainly assesses phonological memory, while assessing manipulation of the information to lesser extent (Alloway, Gathercole, & Pickering, 2006; Cornoldi & Vecchi, 2003; Gathercole, Pickering, Ambridge, & Wearing, 2004; Swanson, 1993). Therefore, it can be argued that, in geometric learning, verbal WM likely plays a critical role only when the task requires more cognitive control and a greater degree of manipulation of information (as it happens, for example, when the task requires the understanding of verbal descriptions of a geometric problems).

It should be noted that specialized geometric vocabulary (e.g., knowledge of shape names; Fisher, Hirsh-Pasek, Newcombe, & Golinkoff, 2013), which was not tested or matched across groups, could play a role in geometric learning. In fact, while children were matched for general verbal intelligence, it still might be the case that knowledge or flexibility with geometric vocabulary could be important and associated with formal geometric knowledge. It can also be argued that children in the poor geometric learning group were not aware of simple rules, for example for calculating the area and perimeter. Future research, should look into this issue for example controlling for children`s knowledge of simple geometric rules.

We also investigated various visuospatial skills in relation to children’s geometric learning. In the present study, visuospatial aspects of the WM system were distinguished according to content (verbal, visual and spatial) and the degree of cognitive control required. Children with poor geometric learning performed poorly in tasks assessing all the visuospatial aspects, a finding that offers further support to the assumption that visuospatial abilities are crucial in geometry (Hannafin et al., 2008). To some extent, this finding indirectly supports the observation that children with poor visuospatial abilities also struggle in geometry (Mammarella, Giofrè, et al., 2013). In fact, results strengthen the hypothesis that both visuospatial WM (Giofrè, Mammarella, Ronconi, et al., 2013; Gittler & Glück, 1998) and visuospatial mental imagery (Cornoldi & Vecchi, 2003) are implicated in geometric learning. It should be noted that visuospatial WM tasks may require more cognitive control compared to verbal simple storage tasks (e.g., Giofrè, Mammarella, & Cornoldi, 2013), and thus may also be more related to geometric learning due to the greater involvement of active manipulation processes (Giofrè et al., 2014). Moreover, the active manipulation of stimuli is crucial in mental imagery tasks (see Cornoldi & Vecchi, 2003). In fact, WM and mental imagery overlap to some extent as they require the maintenance and processing of visual patterns (see Cornoldi & Vecchi, 2003). However, visuospatial mental imagery, compared to WM, can require more attentional resources, and the particular test used in the present study was specifically designed for assessing mental imagery processes involved in geometry. A complementary explanation of the fact that the mental imagery test predicted geometric learning better than the visuospatial WM tests is that WM and visuospatial mental imagery assessments measure geometric learning abilities at different levels. The visuospatial mental imagery task used in the present study can be considered as more geometry specific, tapping resources extensively used in geometric tasks, while WM tasks are in a sense broader and do not involve content typically found in geometric problems.

Results of hierarchical logistic regressions showed that WM and calculation skills had a modest predictive value in discriminating between the two groups (Models 1 and 2; see also Models 1a and 2a). Furthermore, when arithmetic problem solving was not included in the analysis (Model 4), a conspicuous portion of the variance was accounted for by the other predictors (e.g., visuospatial imagery). This finding suggests that solving geometric problems is a complex task which requires not only calculation or arithmetic problem solving but also other skills as well. When visuospatial mental imagery and arithmetic problem solving were included in the model, the predictive value of WM and arithmetic skills was greatly reduced. In fact, our final models showed that only visuospatial mental imagery and arithmetic problem solving were statistically significant when all the variables were entered simultaneously into the equation. It is noteworthy that arithmetic problem solving requires basic calculation skills and WM to some extent (Passolunghi & Pazzaglia, 2004), and that visuospatial mental imagery also requires WM (Cornoldi & Vecchi, 2003). For the same reason, when we included geometric problem solving in our final model (Model 7), the predictive value of the arithmetic problem solving task was greatly reduced and not statistically significant.

The predictive power of arithmetic problem solving seems, at least in part, related to problem solving and simple arithmetic skills also found in geometric problem solving. Basic skills involved in problem solving, such as comprehending the problem, building a representation of it, and planning and supervising the solution processes (see Passolunghi & Pazzaglia, 2004) may be in fact shared by both arithmetic and geometric problem solving. However, our results seem to indicate that failures in arithmetic problem solving do not fully account for failures in geometric problem solving and, in fact, the differences between the two groups were larger in geometric problems compared to arithmetic problems. This is also confirmed by the fact that geometric problem solving proved to be a better predictor of geometric learning compared to arithmetic problem solving.

Our findings have some limitations. In the present study, although the predictors included in the models explain part of the variance in engagement scores, a large portion of the variance still remains unexplained. This finding suggests that future research exploring children's failures in geometric learning will need to address a variety of other potential factors, both cognitive (e.g., other aspects of mental imagery, spatial intelligence, reasoning, overall IQ, and speed of processing) and meta-cognitive/motivational (e.g., interest for mathematics, math anxiety, and self-efficacy) (see Aydın & Ubuz, 2010; Hannafin et al., 2008) factors. Unfortunately, these variables - due to the limitations imposed by the schools participating in the study - could not be assessed in the present study. Furthermore, due to our agreement with the schools, we were only able to test a limited number of students. For this reason, we decided to use an extreme groups approach. This approach is extensively used in the individual difference literature (Engle, 2010), but it may tend to amplify the difference between groups (e.g., Murphy, Mazzocco, Hanich, & Early, 2007). Therefore, future research using a different approach, for example testing a larger group of children, using structural equation models (e.g., Giofrè et al., 2014), may also be needed. Finally, the possibility of using new tasks should be considered, as Cronbach alphas of some of the tasks we used were not particularly high.

Future research should also include children of different ages, to determine whether the present findings could be generalized for younger or older children, and additional groups, in particular children with non-verbal learning disabilities (a condition which is associated with poor visuospatial abilities) who are more likely to obtain low scores in geometry (Mammarella, et al., 2013). In light of this, it would have been interesting to include other measures of visuospatial ability.

In spite of these limitations, the present study has important implications that may help others to better understand why some children struggle with geometric learning. There are many reasons for recommending that teachers pay particular attention to children’s geometric learning. Nowadays, geometry is included in mathematical curricula all over the world, and in international assessments such as the Program for International Student Assessment (PISA; OECD, 2010). It has been suggested that PISA proficiency scores predict educational outcomes (Fischbach, Keller, Preckel, & Brunner, 2013) and that teaching geometry may help to improve spatial intelligence (Gittler & Glück, 1998). Such evidence indicates that geometric learning can be important in many contexts outside of academics and school, and should therefore receive much more attention and research in the near future (Mammarella et al., 2016). Educational implications, which could provide educators with information on the cognitive processes involved in geometric learning, could also be drawn from our findings. For example, knowing that children with poor geometric learning show difficulties in the visual-spatial domain and in WM suggests that teachers should promote activities and strategies designed to compensate for these limits, for example minimizing the load on children’s WM (Alloway, Gathercole, Willis, & Adams, 2004).

Geometric skills supposedly mastered in the fifth and sixth grade are likely built upon more basic geometric concepts acquired earlier in life. Such a possibility has already been explored widely in the domain of arithmetic learning, in which a causal link between an intuitive approximate number sense and learning of formal mathematics has been identified (e.g., Hyde, Khanum, & Spelke, 2014; Park & Brannon, 2013, 2014). Also, there is evidence indicating that formal reasoning and linguistic rules for formal geometric learning are hard for learners to retain, compared to incorrect geometric intuitions (Goldin, Pezzatti, Battro, & Sigman, 2011). This is in accordance with evidence indicating the involvement of a core non-symbolic geometric system in children's geometric learning (Giofrè, Mammarella, Ronconi, et al., 2013) and in the use of geometric abilities, for example in the interpretation of maps (Dillon, Huang, & Spelke, 2013). These latter findings also seem to be significant because it can be argued that the abstract geometric understanding needed to solve geometric problems builds on core mechanics that emerge in infancy and develop throughout life (Dillon & Spelke, 2015). Future studies could investigate whether activities related to basic abilities can affect the emergence of abstract geometric intuitions (Dillon & Spelke, 2015). Finally, it has been found that tasks such as Lego construction are related to mathematical performance (Nath & Szücs, 2014). For these reasons, it would be very interesting to study whether intervention that uses these activities could also improve related abilities such as geometric learning, but more research is needed in this area.

In conclusion, the current study gives some important insights on why many children have difficulty in learning geometry. Geometry is a fundamental skill that can be very important in the complex society in which we live, both in everyday life situations and in the STEM (science, technology, engineering, and mathematics) fields, which nowadays are considered crucial in global society. Therefore, attention should be devoted to identifying factors underlying difficulties in children's geometric learning.

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Footnotes

[1] It is worth noting that even modest correlations can have a relevant impact on path coefficients (see Loehlin & Beaujean, 2016 on this point), because when predictors are entered simultaneously, their single effects are considered over and above the effects of the other predictors.

Table 1

Characteristics of the groups: mean age, verbal intelligence (PMA), score on a standardized test of geometry, standard deviations (SD)

	Geometric learning Groups		Statistical analyses		
	Good	Poor	$F(1,88)$	p	η^2_p
Age (months)	136.66 (8.21)	134.90 (8.02)	1.06	.307	.012
PMA-V [2-4]	28.67 (1.67)	28.09 (2.08)	2.12	.149	.024
PMA-V [11-17]	17.09 (5.70)	16.11 (7.31)	0.50	.481	.006
GEO-P	12.64 (1.69)	3.44 (1.42)	777.95	<.001	.898
AC-MT	27.09 (2.09)	26.37 (2.36)	2.30	.133	.025

Note. PMA-V = Primary mental abilities, verbal; GEO-P = Geometric problem solving; AC-MT = Basic arithmetic abilities

Table 2

Correlations between basic arithmetic battery, WM battery and visuospatial mental imagery tasks

	1	2	3	4	5	6	7	8	9	10	11
1. Simple arithmetic	1										
2. Complex arithmetic	.298**	1									
3. Approximate calculation	.041	.138	1								
4. Forward digit span	.252*	.128	.164	1							
5. Backward digit span	.195	.262*	.153	.430**	1						
6. Forward spatial span	.232*	.175	.031	.197	.392**	1					
7. Backward spatial span	.107	.351**	.168	-.014	.289**	.217*	1				
8. VPT	.264*	.192	.100	.041	.380**	.318**	.325**	1			
9. Pathways	.013	.127	-.007	.087	.205	.166	.265*	.385**	1		
10. Puzzle	.268*	.288**	.096	.003	.278**	.311**	.558**	.446**	.248*	1	
11. Visuospatial mental imagery	.088	.406**	.107	.148	.312**	.326**	.330**	.271**	.264*	.304**	1

Note.
* $p < .05$
** $p < .05$

Table 3

Mean scores and standard deviations (SD) obtained by children with high and low achievement in geometry in the battery of tests, and statistical analyses comparing the two groups.

	Geometric Learning Groups		Statistical analyses			
	Good	Poor	*F(1,88)*	*p*	η^2_p	*Cohen's d*
Arithmetic skills						
Simple arithmetic	69.22 (2.69)	67.31 (3.96)	7.18**	.009	.075	0.56
Complex arithmetic	12.78 (2.30)	11.62 (2.44)	5.35*	.023	.057	0.49
Approximate calculation	8.16 (1.87)	7.56 (1.96)	2.21	.141	.024	0.31
Working Memory						
Forward digit span	5.31 (1.08)	4.98 (0.87)	2.60	.110	.029	0.34
Backward digit span	3.89 (1.19)	3.27 (0.96)	7.43**	.008	.078	0.57
Forward spatial span	5.11 (0.86)	4.58 (0.92)	8.11**	.005	.084	0.60
Backward spatial span	4.91 (1.02)	4.38 (1.17)	5.30**	.024	.057	0.49
VPT	19.64 (4.50)	17.82 (3.91)	4.20*	.043	.046	0.43
Pathway	20.82 (5.34)	18.22 (6.01)	4.71*	.033	.051	0.46
Puzzle	20.07 (4.68)	17.29 (5.12)	7.21**	.009	.076	0.57
Visuospatial mental imagery	12.64 (2.99)	10.11 (2.44)	19.35**	.000	.180	.093

Note. .000 means that the value is zero when approximated to the third decimal.

* $p < .05$

** $p < .01$

Table 4

Standardized beta, standard error and odd rations for each logistic regression model.

Models	B	SE	OR
Model 1			
Backward digit span	.35	.26	1.42
Forward spatial span	.41	.28	1.50
Backward spatial span	.13	.25	1.14
VPT	-.01	.06	0.99
Pathways	.06	.04	1.06
Puzzle	.07	.06	1.07
Model 2			
Simple arithmetic	.14	.07	1.15*
Complex arithmetic	.15	.09	1.16
Model 3			
Backward digit span	.32	.26	1.37
Forward spatial span	.35	.28	1.43
Backward spatial span	.12	.27	1.13
VPT	-.03	.07	0.97
Pathways	.06	.04	1.07
Puzzle	.05	.06	1.05
Simple arithmetic	.12	.08	1.13
Complex arithmetic	.08	.11	1.08
Model 4			
Backward digit span	.28	.28	1.32
Forward spatial span	.20	.30	1.23
Backward spatial span	.08	.28	1.09
VPT	-.04	.07	0.96
Pathways	.05	.05	1.06
Puzzle	.03	.07	1.03
Simple arithmetic	.16	.08	1.17
Complex arithmetic	-.01	.12	0.99
Visuospatial mental imagery	.27	.10	1.31**
Model 5			
Backward digit span	.10	.29	1.11
Forward spatial span	.18	.32	1.20
Backward spatial span	.02	.30	1.02
VPT	-.09	.08	0.91
Pathways	.08	.05	1.08
Puzzle	.04	.07	1.04
Simple arithmetic	.08	.10	1.09
Complex arithmetic	-.17	.14	0.84
Visuospatial mental imagery	.31	.11	1.36*
Arithmetic problem solving	.68	.23	1.98**

Model 6			
Arithmetic factor	-.00	.32	1.00
WM factor 1	.32	.34	1.37
WM factor 2	.27	.30	1.32
Visuospatial mental imagery	.26	.10	1.30**
Arithmetic problem solving	.58	.20	1.73**
Model 7			
Arithmetic factor	-.20	.34	0.82
WM factor 1	.33	.36	1.39
WM factor 2	.21	.31	1.23
Visuospatial mental imagery	.25	.11	1.28*
Arithmetic problem solving	.21	.22	1.23
Geometric problem solving	.44	.16	1.55**
Additional Models			
Model 1a			
WM factor 1	.77	.23	2.15**
WM factor 2	.67	.27	1.94**
Model 2a			
Arithmetic factor	.75	.24	2.12**

Note.

* $p < .05$
** $p < .01$

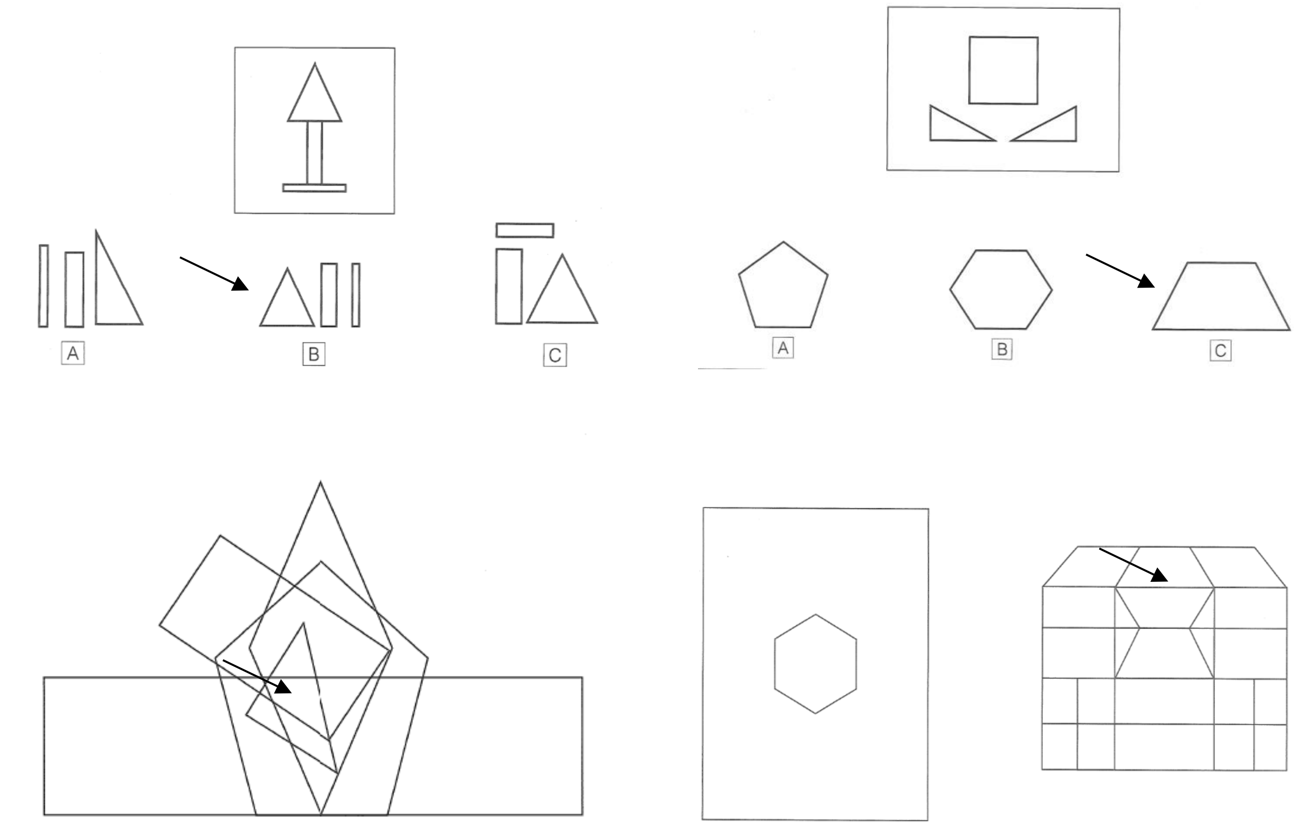

Figure 1. Examples of tasks included in the visuospatial mental imagery test. **TOP LEFT:** requires finding the figures that comprise the composite target figure in the square. **TOP RIGHT:** requires mentally assembling individual pieces to find the target figure. **BOTTIM LEFT:** Asks for the intersection between all the figures. **BOTTOM RIGHT:** asks for the location of the hidden figure embedded in the complex pattern.

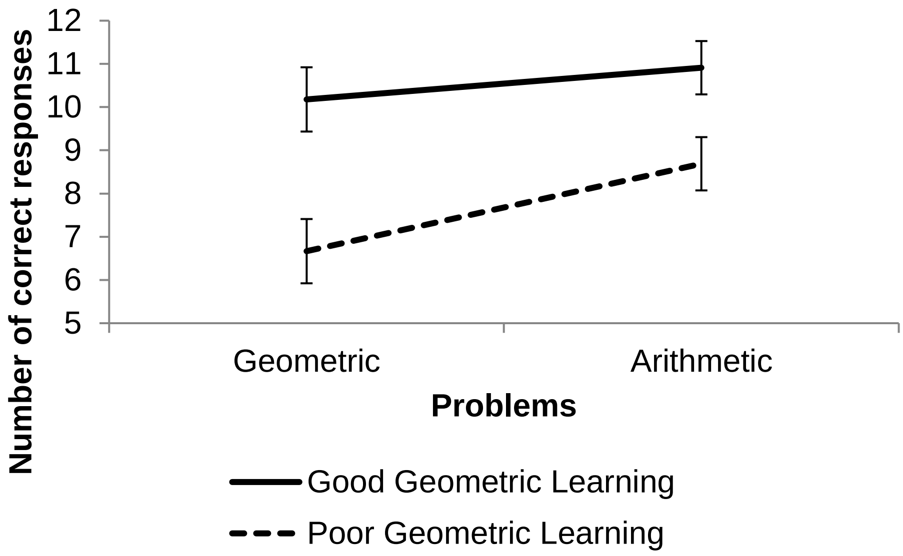

Figure 2. Mean number of correctly solved mathematic problems (geometric and arithmetic) by the two groups (good geometry and poor geometry): with good geometry, M = 10.18 and M = 10.91 for geometric and arithmetic problems respectively; with poor geometry, M = 6.67 and M = 8.69 for geometric and arithmetic problems respectively. The error bars represent 95% confidence intervals.