Optimizing Fuel Consumption in Thrust Allocation for Marine Dynamic Positioning Systems

Miltiadis Kalikatzarakis, Member, IEEE, Andrea Coraddu, Member, IEEE, Luca Oneto, Member, IEEE, and Davide Anguita, Senior Member, IEEE

Abstract—In offshore maritime operations, automated systems capable of maintaining the vessel’s position and heading using its own propellers and thrusters to compensate exogenous disturbances, like wind, waves, and currents, are referred to as marine dynamic positioning (DP) systems. DP systems play a central role in several marine operations, such as drilling, pipe-laying, coring, and ocean observation. These operations are the primary cause of fuel consumption, having a strong impact on the overall footprint of the vessel. For this reason, we will face the problem of optimal thrust allocation of an over-actuated vessel to maintain position and heading with minimal fuel consumption. State-of-the-art approaches simplify this problem by roughly approximating it and obtain a simple, mostly convex, optimization problem that can be solved in near-real time by the automation system. In this article, we improve current approaches with the following contributions. We will exploit a higher fidelity representation of the physical system, and we will manipulate the resulting optimization problem accordingly, to allow for near-real-time solutions on conventional computing platforms onboard. We evaluate the quality of the proposal with a case study on a drilling unit equipped with six thrusters. The results will show that it is possible to achieve up to 5% of fuel savings with respect to conventional approaches.

Note to Practitioners—This article was motivated by the problem of minimizing fuel consumption in thrust allocation of DP systems. The current approaches simplify this issue by adopting simpler, yet related, optimization problems as surrogates, keeping the problem tractable for near-real-time control. We propose, instead, to solve the original problem with state-of-the-art modellization of the physical system and exploit reasonable and theoretical proprieties to achieve optimal solutions in near-real-time. The results on a drilling unit will show additional fuel savings of up to 5% with respect to alternative state-of-the-art approaches.

Index Terms—Fuel consumption optimization, marine dynamic positioning (DP) systems, near-real-time optimization, thrust allocation problem.

I. INTRODUCTION

W ITH offshore operations moving to deeper waters, more and more vessels are equipped with dynamic positioning (DP) systems. The DP system is responsible for maintaining a vessel’s fixed position and heading by using exclusively its thrusters and propellers [1]. It is an alternative to the conventional mooring system and is used in situations in which it is neither possible nor economically feasible to moor the vessel. This capability is critical in several operations that experienced an increase in demand and complexity during the last decades [2]: offshore drilling, coring, undersea pipe-laying operations, dredging [3], the offloading process of floating production storage and offloading (FPSO) units [4], and offshore support vessels that need to maintain close, but safe, distance from offshore platforms [5].

Consequently, there is the necessity to design and operate increasingly complex DP systems, capable of providing the required maneuvering precision, operational and environmental efficiency, and durability [6]. Over the past 70 years, these requirements have motivated the research community to study and develop increasingly sophisticated DP controllers. Starting from the simple proportional-integral-derivative (PID) controllers aiming to compensate horizontal modes of motion (surge, sway, and yaw), DP systems evolved into highly sophisticated tools leveraging on multivariate optimal controls and Kalman filtering [7]–[14], nonlinear or model predictive controls [3], [15]–[20], hybrid [21]–[24] or fault-tolerant controls [25]–[27]. A detailed review of the early history of DP systems is reported in [28], whereas the latest comprehensive surveys on technology advancements can be found in [29]–[31].

Currently, the design of an effective thrust allocation algorithm needs to address a multiobjective optimization problem [30]. On the one hand, we have to ensure the vessel position and heading in the most possible severe conditions taking into account the physical limitations of all related components, on the other hand, we need to minimize the environmental footprint by minimizing the fuel consumption. Contemporarily, the DP control system needs to run smoothly on the on-board automation platform in near-real-time, being able to promptly react to the evolution of the dynamical system. In the literature, it is possible to find a large variety of multiobjective cost functions and constraints for thrust allocation:
1) minimization of electrical load fluctuations [32];
2) minimization of total required power [6], [33]–[36];
3) maximization of thrust production efficiency [37];
4) minimization of fuel consumption [38]–[40];
5) constraints on thrust production due to physical equipment’s limits [1], [6], [33], [41], [42];
6) constraints of rudder dynamics which limit the number of combinations of propeller thrust directions [1], [43];
7) constraints for the safe operations of thrusters and steering machines [1], [6], [33], [44];
8) constraints on forbidden thrust production zones to reduce thrust losses [1], [33], [37], [44];
9) constraints on thrust allocation schemes for moments in the roll and pitch motions, which are fundamental for vessels with large draft and beam with respect to overall length [1].

Based on this literature review, the environmental footprint, and then the fuel consumption minimization is one of the most challenging and unexplored areas of research. The authors in [45] discuss this problem for the Varg FPSO unit and the West Venture drilling rig. However, the authors just focused on power management strategy (PMS), which is responsible for the allocation of power among the diesel generator (DG) sets. In particular, the authors considered power production and power demand as two separate problems, without coupling them. Analogously, the authors of [46] developed a PMS able to minimize operational costs and improve the fuel consumption of the DP system, purely from the PMS point of view. This gap in the literature has been also pointed out in [38] and [39]. In particular, the authors of [38] exploited a platform supply vessel to investigate the possibility of using the thrust allocation algorithm to minimize, in near-real-time, the fuel consumption of the DG sets. The authors formulated the thrust optimization problem as a standard convex quadratic programming (QP) one, by simplifying the original formulation, roughly approximating the involved objectives and constraints with linear and quadratic functions around the vessel’s operating point. The authors demonstrated that their proposal can lead to up to 2% in fuel savings with respect to the thrust allocation, which minimizes simply the thrust power. The authors of [39] exploited a turret-moored assisted FPSO as a case study, and a thrust allocation algorithm was developed to minimize fuel consumption based on penalty programming. Furthermore, in this case, the fuel consumption of the online DG sets was approximated as a second-degree polynomial with respect to power output. By using penalty programming, the authors reformulated the constrained optimization problem by means of a combination of unconstrained ones. The authors compared the proposed approach with other existing thrust allocation algorithms and showed additional fuel savings up to 7%.

The weakness of the approaches focusing on fuel consumption minimization is that they only consider the fuel efficiency of the prime movers, disregarding any losses that occur during conversion and transmission of the power to the propellers. Furthermore, most traditional control approaches consider either fixed-speed controllable pitch propellers (CPPs) or variable-speed fixed pitch propellers (FPPs) [3].

In this work, to the best knowledge of the authors, we will focus for the first time on fuel consumption minimization during DP operations considering a holistic view of the problem. In particular, state-of-the-art CPP thruster model and precise physical modeling of the DP system (including thrusters—THs, gearboxes—GBs, electric motors—EMs and DGs). The complexity of this research, and then its novelty, lies in finding the right tradeoff between computational complexity and accuracy in representing the fuel consumption and power losses, to make the problem tractable in near real-time by standard solvers that are readily available in most numerical platforms. As the authors will show in Sections II and III, this balance is accomplished by a series of theoretically founded simplifications, both in terms of the representation of the physical system itself and during the formulation of the optimization problem. Furthermore, we will quantify the effects of exploiting CPPs as an extra degree of freedom (DOF), in the optimization problem. Finally, to demonstrate the quality of the proposed approach, we will compare against state-of-the-art methods [5], [32], [33], [36], [38], [42], [43], [47]–[54] where the minimization of total thrust and total propeller power, with and without propeller pitch control, are proposed instead of directly optimizing the fuel consumption.

The rest of this article is organized as follows. In Section II, we present the modeling approach employed for the environmental conditions and all the components used in a DP system. Section III presents the formulation of the thrust allocation problem and the associated proposed solution method. Section IV presents the case study employed in this work. Section V presents the results of applying the methodology proposed in Section III using the case study presented in Section IV. Section VI concludes this article. Note that, to improve the readability of this article, the authors reported in Tables II and I, respectively, the summary of the acronyms and nomenclature exploited during the presentation.

II. PHYSICAL MODEL DESCRIPTION

In this section, we will describe in detail the mathematical framework required to accurately represent the environmental disturbances acting on a vessel, and all the components of a DP system that affect the total fuel consumption.

A. Vessel Kinematics

Any vessel in a seaway is mainly affected by different environmental disturbances, namely, waves, currents, and wind, which contain both slowly varying and high-frequency components [55]. For DP applications, by definition, the vessel is operating at close to zero speed, and only forces and moments in the horizontal plane are of interest [56]. We model the overall forces moments using the principle of superposition, which is the most commonly adopted model for thrust allocation optimization problem in DP [57]:

1) first-order waves-induced motion;
2) slowly varying disturbance motion produced by second-order wave effects, current, and wind;
3) control-induced motion produced by the thrusters.

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TABLE I
LIST OF SYMBOLS EXPLOITED IN THIS ARTICLE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
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<tbody>
<tr>
<td>$A_{FW}$</td>
<td>Frontal area projection</td>
<td>m$^2$</td>
<td>$S_{q,c,est}$</td>
<td>Stator quadrature current setpoint</td>
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</tr>
<tr>
<td>$A_{LW}$</td>
<td>Lateral area projection</td>
<td>m$^2$</td>
<td>$Z_{e}$</td>
<td>Reference frame transformation matrix</td>
<td></td>
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<tr>
<td>$A_{DF}$</td>
<td>Diagonal force coefficient matrix</td>
<td></td>
<td>$K_{e}$</td>
<td>Inductive coefficient vector</td>
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<td>$d_{max}$</td>
<td>Maximum azimuth angle</td>
<td>rad</td>
<td>$\theta_{a}$</td>
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<td>$\delta_{min}$</td>
<td>Minimum azimuth angle</td>
<td>rad</td>
<td>$L_{op}$</td>
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<td>$\beta$</td>
<td>Breadth</td>
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<td>$\beta_{op}$</td>
<td>Direct voltage setpoint</td>
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<td>$\gamma$</td>
<td>Current drain coefficient - yaw</td>
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<td>Stator direct voltage</td>
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<td>$\phi$</td>
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<td>$\nu_{T}$</td>
<td>Propeller torque coefficient</td>
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<td>$M_{e}$</td>
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However, the commercial autopilot and DP systems employed in real-world applications have various forms of wave filtering, such as cascaded notch and low-pass filters, to reduce wear and tear on both the steering machines and thrusters’ modulation [1], [58]. Note that only the slowly varying disturbances are counteracted by the steering and propulsion systems since the oscillatory motion due to waves should not be considered in the control feedback loop, to avoid unnecessary usage of the actuators [1], [55]. Hence, to be able to model the system in the most realistic possible way, we assume that these filtering techniques have been successfully implemented in the control system by neglecting the oscillatory motion. For low speed DP applications, it is common to consider the low frequency mathematical model in surge, sway, and yaw. The latter is dynamically linear and kinematically nonlinear [1]. In this mathematical model, the pitch and roll angles are assumed to be small, the ship has port-starboard symmetry, the Coriolis and centripetal terms are negligible, and the linear part of the damping matrix caused by wave drift damping and laminar skin friction has a dominating contribution [56]

$$\tau = \begin{bmatrix} x, y, \psi \end{bmatrix} \in \mathbb{R}^{3 \times 1}$$
is the output vector containing the vessel’s position \((x, y)\) and heading \((\psi)\) in the earth-fixed frame. \(v = [u, w, z] \in \mathbb{R}^{3 \times 1}\) contains the vessel's forward \((u)\), lateral \((w)\), and angular \((z)\) velocities in the body-fixed frame. \(v_c = [u_c, w_c, 0] \in \mathbb{R}^{3 \times 1}\) represents the forces and moments from the thrusters in the body-fixed frame [we will describe this later in Section II-C, specifically (12)]. \(f_{env} \in \mathbb{R}^{3 \times 1}\) represents the environmental disturbances coming from the wind, current, and waves (we will describe this later in Section II-B).

$$\mathbf{J} = \begin{bmatrix} 
\cos(\psi) - \sin(\psi) & 0 & 0 \\
\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1 
\end{bmatrix}$$

\(M = \begin{bmatrix} 
\Delta - X_g & 0 & 0 \\
0 & \Delta - Y_g & \Delta x - Y_g \\
0 & 0 & I_{N_g} 
\end{bmatrix}\)
the relative wind speed is defined as the square of the wind speed relative to the vessel. The norm of the wind speed components, \( V_{xw}, V_{yw}, V_{zw} \) of the vessel projected on the frontal and lateral projected areas of the nonsubmerged part is

\[
\frac{1}{2} \rho w \left[ C_{cx}(\gamma_{w})A_{Fw} + C_{cy}(\gamma_{w})A_{Lw} + C_{cw}(\gamma_{w})L_{nw} \right]
\]

where \( \rho w \) is the air density and \( A_{Fw}, A_{Lw}, \) and \( L_{nw} \) are the added mass forces for surge, sway, and yaw directions computed according to [60].

2) Current Forces: We have assumed irrotational and constant currents, whose effects in surge and sway motions were modeled as follows [1]:

\[
V_{ic} = \sqrt{u_{ic}^2 + w_{ic}^2}
\]

where \( u_{ic} = u - u_c \) and \( w_{ic} = w - w_c \) are the relative current velocities in the surge and sway motions, respectively.

Finally, by defining the current direction relative to the vessel heading as \( \gamma_{ic} = \gamma_{c} - \psi \), we can estimate the static current loads according to [60]:

\[
f_c = \frac{1}{2} \rho w V_{ic}^2 \left[ \begin{array}{c} C_{cx}(\gamma_{ic})L_{pp}^T \\ C_{cy}(\gamma_{ic})L_{pp}^T \\ C_{cw}(\gamma_{ic})L_{pp}^T \end{array} \right]
\]

where \( \rho w \) is the density of the water, \( C_{cx}, C_{cy}, C_{cw} \) are the current drag coefficients in surge, sway, and yaw, and \( L_{pp}, T \) are the length between perpendiculars and draft of the vessel.

3) Wave Forces: As stated at the beginning of this section, we assume that wave filtering techniques have been successfully employed. This allows us to neglect the first-order wave-induced forces and focus only on the wave drift forces, namely nonzero slowly varying components. Common practice is to solve the first-order problem using potential flow theory [61]. The mean drift forces can be obtained by applying the theory of conservation of momentum, namely, the far-field theory. For what concerns the case study considered in this work (Section IV), we utilized the ShipX software [62] to obtain the mean drift coefficients \( C_{wav,x}, C_{wav,y}, C_{wav,n} \). Hence, the mean wave drift forces follow from [61]:

\[
f_{wav} = \begin{bmatrix} C_{wav,x}H_s^2 \sin(\gamma_{wav} - \psi) \\ C_{wav,y}H_s^2 \sin(2(\gamma_{wav} - \psi)) \\ C_{wav,n}H_s^2 \cos(\gamma_{wav} - \psi) \end{bmatrix}
\]

where \( H_s \) is the significant wave height and \( \gamma_{wav} \) is the wave angle on the fixed body reference frame.

C. Azimuth Thrusters

As shown in Fig. 3(a), the vessel is equipped with \( k_p = 6 \) azimuth thrusters. We take into account the actuators’ force vector as follows:

\[
f = T_{tie}K_{tie}u_{tie}.
\]

\( T_{tie} \in \mathbb{R}^{3 \times 2k_p} \) [see (13)] is the extended thrust configuration vector, \( K_{tie} \in \mathbb{R}^{2k_p \times 2k_p} \) [see (14)] is the diagonal force coefficient matrix, and \( u_{tie} \in \mathbb{R}^{2k_p \times 1} \) [see (15)] is the vector of extended control inputs of the thrusters:

\[
T_{tie} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & \ldots & 0 & 1 \end{bmatrix}
\]

\[
K_{tie} = \text{diag}(k_1, k_1, \ldots, k_6, k_6)
\]

\[
u_{tie} = [u_{1,x}, u_{1,y}, \ldots, u_{6,x}, u_{6,y}]^T
\]
where \( h_{x,i}, h_{y,i} \) are the moment arms and \( u_{x,i} = u_i \cos(\alpha_i) \) and \( u_{y,i} = u_i \sin(\alpha_i) \) refer to the normalized thrust of each thruster in the longitudinal and lateral directions. Consequently, we can define the norm of the extended thrust vector \( \mathbf{u}_t \in \mathbb{R}^{k_p \times 1} \) and the azimuth angle vector \( \mathbf{a} \in \mathbb{R}^{k_p \times 1} \) as
\[
\mathbf{u}_t = \frac{u_{t,i}}{\sqrt{u_{x,i}^2 + u_{y,i}^2}} \\
\mathbf{a} = \arctan(u_y \otimes u_x)
\]
where \( \otimes \) indicates element-wise division.

Since the thrusters are identical, the diagonal elements of the matrix \( \mathbf{K}_e \) are equal to
\[
k_i = \frac{T_{p,\text{max}}}{0.5 \rho \omega \left(V_S^2 + (0.7 \pi n_p D_p)^2\right)}
\]
where \( T_{p,\text{max}} \) and \( n_p, D_p \) are the maximum thrust force and rotational speed of each thruster, \( D_p \) is the propeller diameter, and \( V_S \) is the norm of the vessel speed.

In this work, as a novel contribution, we generalize the traditional control approaches that consider either fixed-speed CPPs or variable-speed FPPs [3]. For this reason, we need to be able to predict the propellers’ thrust and torque characteristics based on the propeller pitch vector \( \mathbf{u}_p \in \mathbb{R}^{k_p \times 1} \) and the rotational speed vector \( \mathbf{n}_p \in \mathbb{R}^{k_p \times 1} \). To this end, we can make use of the four-quadrant open water data available in the literature [63], which describe the relationship between the nondimensional thrust force and rotational speed vectors and the set of nondimensional thrust and torque \( C_t, C_q \in \mathbb{R}^{k_p \times 1} \) coefficients.

Subsequently, we can establish a relationship between the nondimensional thrust and rotational speed vectors to derive the norm of the torque vector \( \mathbf{M}_p \in \mathbb{R}^{k_p \times 1} \) as follows:
\[
\beta = \arctan\left(\frac{v_a}{0.7 \pi n_p D_p}\right)
\]
\[
\nu_a = (1 - f_w) V_S
\]
\[
C_t = \mathbf{u}_t \otimes \left(u_a^2 + (0.7 \pi D_p n_p)^2\right)^{\frac{1}{2}}
\]
\[
C_q = \mathbf{M}_p \otimes \left(u_a^2 + (0.7 \pi D_p n_p)^2\right)^{\frac{1}{2}}
\]
where \( f_w, f_i, \eta_p \) are the wake factor, the thrust deduction factor, and the relative rotative efficiency of each propeller. It is worth noting that we assumed those quantities as constant for all the operating conditions and across all the propellers. We can also define the delivered power vector of the thrusters as follows:
\[
P_p = 2 \pi n_p \circ \mathbf{M}_p
\]
where \( \circ \) indicates element-wise multiplication.

When the vessel is in stationary position during DP operations, it is possible to assume that \( n_p \approx 0 \). Under this assumption, the torque coefficient of (23) can be expressed as a third degree polynomial taking into account propeller pitch angle
\[
\tilde{C}_q \approx \sum_{i=0}^{3} c_{q,0,i} \frac{u_p^{q,i}}{D_p^{q,i}}.
\]

Analogously, we can approximate also the thrust coefficient as follows:
\[
\tilde{C}_t \approx \sum_{i=0}^{1} c_{t,0,i} \frac{u_p^{t,i}}{D_p^{t,i}}.
\]

By means of these approximations, we can easily express the propellers’ rotational speed and torque on the basis of the thrust and pitch vectors
\[
C_q = \tilde{C}_q \Rightarrow \mathbf{M}_p = \frac{\pi p_w D_p^2}{8} \sum_{i=0}^{3} \left(c_{q,0,i} \frac{u_p^{q,i}}{D_p^{q,i}} \right) \circ (0.7 \pi D_p n_p)^{q,i}
\]
\[
C_t = \tilde{C}_t \Rightarrow n_p = \frac{(1 - f_i) T_{p,\text{max}}^{\frac{1}{2}}}{(0.7 \pi D_p)^{\frac{1}{2}}} \left(\mathbf{u}_t \otimes \sum_{i=0}^{1} c_{t,0,i} \frac{u_p^{t,i}}{D_p^{t,i}} \right)^{\frac{1}{2}}
\]

D. Gearbox and Shaft-Line

The literature related to the modeling and simulation of maritime GBs is rather limited, even though, lately, this subject has received increasing attention due to novel numerical modeling methods [64]. Overall, approaches consist of either complex thermal network models [65] or simple GB loss functions, as those presented in [66] and [67]. While the thermal network models are usually based on adimensional heuristic estimation models for the various loss sources, they, unfortunately, require very detailed design information of the GB, which is usually not readily available in real-world applications. However, the authors of [65] demonstrated that a properly calibrated empirical model can accurately match the predictions of a much more detailed thermal network model. In this work, we have exploited the quadratic model for the GB losses proposed in [68] and a linear model for the losses on the shaft bearings as functions of rotational speed and torque
\[
M_{sh} = M_p + M_{sh,\text{loss}}
\]
\[
M_{gb} = M_{sh} + M_{gb,\text{loss}}
\]
where \( M_{sh}, M_{sh,\text{loss}} \in \mathbb{R}^{k_p \times 1} \) are the normalized shaft torque and torque loss vectors, \( M_{gb} \) and \( M_{gb,\text{loss}} \in \mathbb{R}^{k_p \times 1} \) are the normalized GB torque and torque loss vectors, and the set of scalars \( c_{sh,i} \) and \( c_{gb,i} \) are fitted loss coefficients that can be obtained by the least squares error (LSE) estimation using the GB efficiency curves provided by the manufacturer.

E. Electric Motors

For the purpose of this work, it is necessary to accurately assess the losses of the asynchronous EMs at various operating conditions in terms of rotational speed and torque. Unfortunately, manufacturers report only the nominal efficiency. Consequently, to assess the losses of the asynchronous EMs, we employed the fifth-order state-space model defined in [69] and [70] from which we can obtain the efficiency map.
\[ \eta_{em} \in \mathbb{R}^{2x1} \]. We have assumed balanced supply voltage, thus neglecting the zero sequence current. For each EM, the model is defined as follows:

\[ \begin{bmatrix} v_{qs} & v_{ds} & v_{qr} & v_{dr}' \end{bmatrix}^T = \begin{bmatrix} r_s + \frac{1}{l_{ss}} & \frac{l_M}{2} & \frac{l_M}{2} & \frac{l_M}{2} \\ \frac{l_M}{2} & r_s - \frac{1}{l_{ss}} & \frac{l_M}{2} & \frac{l_M}{2} \\ \frac{l_M}{2} & \frac{l_M}{2} & r_s - \frac{1}{l_{ss}} & \frac{l_M}{2} \\ \frac{l_M}{2} & \frac{l_M}{2} & \frac{l_M}{2} & r_s + \frac{1}{l_{ss}} \end{bmatrix} \begin{bmatrix} i_{qs} \ i_{ds} \ i_{qr} \ i_{dr}' \end{bmatrix} \]

\[ i_{ss} = i_{ls} + i_M \]

\[ i_{rr}' = i_{lr}' + i_M \]

\[ M_{em} = \frac{3 q}{2} \left( \psi_{qdr}' \psi_{dqs}' - \psi_{qdr} \psi_{dqs} \right) \]

\[ \omega_{eq} = \omega_e - \frac{r_s}{2} \frac{i_{rr}'}{\psi_{dqs}} \]

\[ \omega_r = \omega_{eq} n = 2 \pi q \eta_{em} \].

\[ \psi_{qs}, \ \psi_{ds}, \ \psi_{dqs}, \ \psi_{qdr}, \ \psi_{dqs}, \ \psi_{qdr}' \] are the direct and quadrature currents, voltages, and flux linkages in the rotor and stator. \( r_s, r_r \) and \( l_s, l_r \) are the stator and rotor resistances and inductances. \( i_M \) is the mutual inductance. \( q \) is the number of poles of the induction motor. All these parameters can be obtained from EMs of similar size and plate data available in the literature [67] or by the manufacturer. Furthermore, \( \omega_{eq}, \ \omega_e, \ \omega_r, \ \omega_{em} \) are the base rotating reference frame, electric rotor, and EM shaft frequencies.

Furthermore, we assume that the EMs are controlled with direct field-oriented control, as proposed by Blaschke [71] and discussed in more detail in [69] and [72]. The quadrature and direct current references \( (i_{ds, set}, i_{qs, set}) \), in the synchronously rotating reference frame are determined from the torque and direct rotor flux references. The measured quadrature and direct currents are determined as follows:

\[ \begin{bmatrix} i_{qs, set} \\ i_{ds, set} \end{bmatrix} = \frac{M_{em}}{\omega_{eq} \left( r_s' + l_r' \right) l_M} \begin{bmatrix} \psi_{qdr}' \\ \psi_{dqs}' \end{bmatrix} + \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} \]

Subsequently, PID control is applied to obtain the voltage reference vector \( v_{ref} = [v_{ds, set}, v_{qs, ref}] \) with an extra feed-forward term to obtain the final quadrature and direct voltage references \( v_{set} = [v_{ds, set}, v_{qs, set}] \) as follows:

\[ v_{set} = K_u i_{set} + v_{ref} \]

\[ K_u = \begin{bmatrix} \frac{l_M^2}{T_r} & -\omega_{em} \frac{l_{ss}}{l_{in}} \\ \omega_{em} \frac{l_{ss}}{l_{rr}} & \frac{l_M^2}{T_r} \end{bmatrix} \]

\[ I_u = \begin{bmatrix} \frac{\partial i_{ds, set}}{\partial t} & i_{ds, set} \\ \frac{\partial i_{qs, set}}{\partial t} & i_{qs, set} \end{bmatrix} \]

\[ \omega_{mf} = \omega_e + \omega_{id} \]

\[ \omega_{sl} = \frac{R_{rr}}{l_{rr}} \psi_{dr, set} \]

where \( \omega_{sl} \) and \( \omega_{mf} \) are the slip and magnetic field angular speeds.

The efficiency map obtained from the state-space model can then be approximated by means of the Willans approach [73] as follows:

\[ \eta_{em} = (c_{em,0} n_{em} M_{em}) \wedge (n_{em} M_{em} + c_{em,1}) \]

\[ n_{em} = i_{gb} n_p \]

\[ M_{em} = M_p + M_{b, loss} + M_{gb, loss} \]

where \( n_{em} \) is the rotational speed of the EM, \( i_{gb} \) is the reduction ratio of the identical GBs, \( M_{em} \in \mathbb{R}^{2x1} \) refers to the normalized torque of the EM, and \( c_{em,i} \) are approximation constants obtained through LSE.

\subsection*{F. Frequency Converters}

Since we are investigating the fuel savings potential, we do not need to study the dynamics of the electrical network. In fact, the time constant of the electrical systems is at least one order of magnitude smaller than the mechanical one. Consequently, the transient operation does not have a significant effect on the overall fuel consumption. Moreover, we can safely assume that the induction motors are fed by an ideal voltage source [74]. Hence, frequency converters are modeled as ideal voltage sources providing the requested voltage and frequency to the EMs.

\subsection*{G. Diesel Generators}

We estimate the instantaneous fuel consumption of the DGs by means of a quadratic relation between torque and injected fuel per cycle, as proposed by Wood et al. [75] and Grimmelius et al. [76]. The auxiliary DGs, namely, DGs exploited for the on-board electrical power generation, are usually operated at the nominal fixed speed to optimize their efficiency. Hence, we will omit the dependence on engine speed, since it is constant, and we can directly link the specific fuel consumption vector of the DGs \( s_{fe} \in \mathbb{R}^{k_{ds} \times 1} \) to the control vector \( u_{dg} \in \mathbb{R}^{k_{ds} \times 1} \) as follows:

\[ s_{fe} = z_{dg} \sum_{i=0}^{z_{dg}} \hat{c}_{dg,i} u_{dg}^T \]

\( z_{dg} \in \mathbb{R}^{k_{ds} \times 1} \) is the vector of binary variables indicating if each DG is connected to the grid. \( k_{dg} \) is the number of DGs on the system. \( c_{dg,i} \) are scalar values that can be derived by means of LSE from the manufacturer fuel consumption curves.

Finally, the total fuel consumption of the DP system \( \hat{m}_{fa} \) can be expressed as the sum of the fuel consumption among all DGs:

\[ \hat{m}_f = u_{dg}^T s_{fe} \]
III. FORMULATING AND SOLVING THE THRUST ALLOCATION PROBLEM

The control system needs to solve the so-called thrust allocation problem, to ensure the vessels’ position and heading using its own thrusters to compensate the exogenous disturbances. This requires the development of a thrust allocation algorithm able to contemporary maintain the vessel position and heading while minimizing the fuel consumption. The resulting thrust allocation optimization problem can be sketched as follows:

\[
\begin{align*}
\min & \quad f(u, w) \\
\text{s.t.} & \quad h(u, w) = 0, \quad g(u, w) \leq 0
\end{align*}
\]

where \( u = [u_{t,e}, u_p, u_d, z_{dg}]^\top \) is the combined control vector, \( w = [V_u, V_v, V_{swv}, \gamma_u, \gamma_v, \gamma_{swv}]^\top \) is the exogenous inputs vector, \( f(u, w) \) is the objective function to be minimized (the fuel consumption), and \( h(u, w) \) and \( g(u, w) \) are the equality (e.g., keep the vessel position and heading) and inequality constraint (e.g., operational limitation) vectors. Furthermore, in this work, we will study and manipulate this optimization problem in such a way that it will be possible to solve it in near-real-time on conventional on-board computing platforms.

In the next sections, we will present the complete, detailed, and complex thrust allocation optimization problem leveraging on the notions described in Section II. We will quantify the extra savings in fuel consumption with respect to the more conventional modelization approaches adopted in the literature (e.g., minimization of the propeller power or the total thrust). Furthermore, to reduce the computational requirement and be able to adopt the proposal in near-real-time control, we will exploit reasonable and theoretical proprietaries.

A. Constraints

The first set of constraints are the boundaries for the decision variables, namely the pitch, the thrust, and diesel-generator power set-points, which represent the predefined manufacturer limits

\[
\begin{align*}
\rho_{\text{min}} \leq u_p & \leq \rho_{\text{max}} \\
0 & \leq u_{dg} \leq P_{dg,\text{max}} \\
z_{dg} & \in \mathbb{Z}_2.
\end{align*}
\]

Then, we have to take into account other kinds of constraints. In particular, we have to counteract the environmental disturbances, balance the propulsive power supply and demand, and satisfy the operating limits of all the components of the DP system. For this purpose, we define the constraints that limit the maximum thrust that can be delivered by each thruster

\[
0 \leq (u_{x}^2 + u_{y}^2)^{\frac{1}{2}} \leq T_{p,\text{max}}.
\]

Since the 3-DOF kinematic model has been employed in this work, the DP system must counteract the environmental disturbances in the surge, sway, and yaw motions

\[
T_{t,e}K_{t,e}u_{t,e} = f_{\text{wav}} + f_c + f_w = f_{\text{env}}.
\]

Leveraging on (7), (10), (11), (13)–(16) we can express the equality constraints on the basis of the control vector \( u \), and the exogenous inputs \( w \)

\[
\begin{align*}
\sum_{i=1}^{k_p} u_{x,i} & = \frac{1}{\sigma_1} \begin{bmatrix} f_{\text{env},x} \\ f_{\text{env},y} \\ f_{\text{env},n} \end{bmatrix}
\end{align*}
\]

where

\[
\sigma_1 = \frac{(1 - f_i)T_{p,\text{max}}^2}{(0.7\pi D_p)^2 (\frac{z_{w}}{8} D_p^2)^2}.
\]

The balance between power supply and demand can be formulated as a power balance across the main switchboard (SB). Under the assumption of an ideal SB [74], the power required from the EMs to counteract the environmental disturbances must be equal to the power provided by the DGs

\[
\sum_{j=1}^{k_d} u_{d,j} = \sum_{i=1}^{k_m} 2\pi n_{em,i} M_{em,i} \frac{1}{\eta_{em,i}}.
\]

By means of a series of technical steps, exploiting (27) and (28) in (29)–(49) and (47) and (48), it is possible to express the power balance in terms of the control vectors \( u_p, u_t, u_d \)

\[
\sum_{j=1}^{k_d} u_{d,j} = \sum_{i=1}^{k_m} \sigma_2 + \sigma_3 \left( \sum_{j=0}^{1} c_{0,j} u_{\text{dg},j} \right)^{\frac{1}{2}} + \sigma_4 \left( \sum_{j=0}^{1} c_{0,j} u_{\text{dg},j} \right)^{\frac{1}{2}}
\]

\[
+ \left( \sum_{j=0}^{1} c_{0,j} u_{\text{dg},j} \right)^{\frac{1}{2}} \left( \sum_{j=0}^{1} c_{0,j} u_{\text{dg},j} \right)^{\frac{1}{2}} + \sigma_5 \left( \sum_{j=0}^{1} c_{0,j} u_{\text{dg},j} \right)^{\frac{1}{2}} \left( \sum_{j=0}^{1} c_{0,j} u_{\text{dg},j} \right)^{\frac{1}{2}}\right)
\]

where

\[
\sigma_2 = \frac{c_{em,1}}{c_{em,0}}
\]

\[
\sigma_3 = \frac{2\pi}{c_{em,0}} \left( \frac{(1 - f_i)T_{p,\text{max}}^2}{(0.7\pi D_p)^2 (\frac{z_{w}}{8} D_p^2)^2} \right)
\]

\[
\times \left( \frac{c_{\text{sh},1}}{1 - c_{\text{sh},3}} + \frac{c_{\text{sh},3}}{1 - c_{\text{sh},3}} + \frac{c_{\text{sh},2}}{1 - c_{\text{sh},3}} \right)
\]

\[
\sigma_4 = \frac{2\pi}{c_{em,0}} \left( \frac{(1 - f_i)T_{p,\text{max}}^2}{(0.7\pi D_p)^2 (\frac{z_{w}}{8} D_p^2)^2} \right)
\]

\[
\times \left( \frac{c_{\text{sh},2}}{1 - c_{\text{sh},3}} \right)
\]
Closely placed azimuth thrusters exhibit a decrease in efficiency when they operate in the wake stream of each other [5]. In order to avoid these adverse interaction effects, forbidden zones for some azimuth angles must be defined. Using the dynamic positioning system (DPS) guide of the American Bureau of Shipping [77], we were able to define the forbidden zones on the basis of the relative positions of the thrusters and the propeller diameter. To this end, we have defined the feasible regions for all thrusters to lie between predefined azimuth angle ranges $\alpha_{\text{min}}, \alpha_{\text{max}} \in \mathbb{R}^{k_{i} \times 1}$

$$\alpha_{\text{min}} \leq \arctan(u_y \odot u_x) \leq \alpha_{\text{max}}.$$  

Constraints on the torque output of the EMs are also necessary to prevent operations outside the predefined manufacturer limits

$$0 \leq M_{\text{em}} \leq M_{\text{em,max}}.$$  

We can express these constraints on the basis of the control vectors $u_i$, $u_p$ by means of (23), (27), and (29)–(49)

$$0 \leq \sigma_5 + \sum_{i=1}^{n} c_{gb,i} \frac{u_{p,i}}{D_p} + \sigma_8 \left( \frac{u_{x,i}^2 + u_{y,i}^2}{u_p} \right)^{0.8} \leq M_{\text{em,max}}.$$  

$M_{\text{em,max}}$ is the maximum admissible torque output of the identical EMs. $\sigma_i$ can be expressed as follows:

$$\sigma_5 = \frac{2\pi}{c_{em,0} \left( (1-f_i) \sigma_{P_{\text{max}}} \right)^{0.7}} \left( \frac{0.7\pi D_p}{\sigma_{P_{\text{max}}}} \frac{\sigma_{P_{\text{max}}}}{D_p} \right)^{0.7} \times \left( 1 + \frac{c_{gb,4}}{1-c_{sh,4}} \right) c_{gb,3}^{0.7}.$$  

We can combine the two constraints reported in (73) and (74) and express them in terms of the control vectors $u_i$, $u_p$ using (28) and (48)

$$0 \leq \left( u_{x,i}^2 + u_{y,i}^2 \right)^{0.8} \frac{1}{\sum_{i=0}^{n} c_{gb,i}} \frac{u_{p,i}}{D_p} \leq \sigma_{11}.$$  

Note that

$$\sigma_{11} = \min \left( n_{p,max}, \frac{n_{\text{em,max}}}{I_{gb}} \right) \left( \frac{1}{0.7\pi D_p} \left( \frac{1-f_i}{8D_p} \right)^{0.7} \right)^{-1}.$$  

Focusing on the DG sets, constraints on their power output are necessary to impose load sharing. For this reason, we need to define a matrix of dummy variables $d_{dg} \in \mathbb{R}^{k_{i} \times k_{\text{dg}}}$ indicating if a pair of DGs is connected to the SB

$$d_{dg} = 1 - z_{dg} z_{dg}^T.$$  

Therefore, load sharing can be imposed with the following constraints:

$$M d_{dg} \leq u_{dg} - u_{dg}^T \leq M d_{dg}$$  

where $M$ is a large enough constant.

Finally, an extra set of constraints is required to force to zero the power output of DGs not connected to the SB

$$0 \leq u_{dg} \leq z_{dg} P_{\text{dg, nom}}$$  

where $P_{\text{dg, nom}}$ is the nominal power output of each DG.

B. Objective Functions

In this article, we have considered three optimization criteria:

1) minimization of the DGs fuel consumption with pitch control (our proposal);
2) minimization of total propellers power
   a) with pitch control;
   b) without pitch control (proposed and discussed in [6] and [33]–[36]);
3) minimization of thrust delivered by the thrusters without pitch control (proposed in [11]).

Note that Problem 1 is our novel proposal, Problem 2(a) is an intermediate problem that we formulate for completeness, to assess the effect of considering the pitch as a control variable in Problem 2(b), while Problems 2(b) and 3 are state-of-the-art proposals available in the literature.

For what concerns the fuel consumption minimization problem (Problem 1), the objective function can be formulated as the sum of the specific fuel consumption of the DGs, as reported in (50) and (51)

$$m_f = \sum_{i=1}^{k_{i}} u_{d_{dg,i}} z_{d_{dg,i}} \frac{2}{\sum_{j=0}^{k_{i}} c_{d_{dg,j}} I_{d_{dg,j}}^2}.$$  

Regarding propeller power minimization [Problem 2], we can derive the objective function, with [Problem 2(a)]
or without [Problem 2(b)] pitch control, from (22)–(28) it is possible to obtain
\[
    P_{\text{tot},p} = \sigma_{12} \sum_{j=0}^{k_p} \left( u_{x,j}^2 + u_{y,j}^2 \right)^{\frac{1}{2}} \left( \sum_{i=0}^{3} \frac{1}{c_{y,i} \left( \frac{u_{y,i}^2}{\sigma_{12}} \right)} + \sum_{i=0}^{3} \frac{1}{c_{y,i} \left( \frac{u_{y,i}^2}{\sigma_{12}} \right)} \right) \right)
\]
(81)
\[
    \sigma_{12} = \frac{(1 - f) T_{p,\text{max}}}{\pi \rho_{\text{env}}} \frac{1}{\eta_i \pi^2 (0.7 D_p)^2}.
\]
(82)

For Problem 2(b), we have assumed that all the elements of \( \mathbf{u}_p \) are equal to the nominal pitch of the propeller (\( p_{\text{nom}} \))
\[
    P_{\text{tot}} = \sigma_{13} \sum_{j=0}^{k_p} (u_{x,j}^2 + u_{y,j}^2)^{\frac{1}{2}}
\]
(83)
\[
    \sigma_{13} = \frac{(8(1 - f)) T_{p,\text{max}}}{(\pi \rho_{\text{env}}) \eta_i \pi^2 (0.7 D_p)^2} \left( \sum_{i=0}^{3} \frac{1}{c_{y,i} \left( \frac{u_{y,i}^2}{\sigma_{12}} \right)} + \sum_{i=0}^{3} \frac{1}{c_{y,i} \left( \frac{u_{y,i}^2}{\sigma_{12}} \right)} \right)^{\frac{1}{2}}
\]
(84)

Finally, for the minimization of total thrust without pitch control (Problem 3), the objective function is the sum of all the elements of the vector \( \mathbf{u}_t \)
\[
    T_{\text{tot}} = \sum_{i=0}^{k_p} (u_{x,i}^2 + u_{y,i}^2)^{\frac{1}{2}}.
\]
(85)

**C. Simplifications of the Optimization Problems**

As a starting point, let us note that:

1. Problem 1 is a mixed-binary programming problem with nonconvex objective and nonlinear constraints;
2. Problem 2(a) is a continuous nonconvex, nonlinearly constrained optimization problem;
3. Problem 2(b) is a continuous convex, nonlinearly constrained optimization problem [1], [6];
4. Problem 3 is a continuous convex, nonlinearly constrained optimization problem [1], [6];

Note that for Problem 2(b) and Problem 3 a number of solution methods have been proposed [78]–[80].

Problem 1 is clearly a very difficult combinatorial problem which may be hard to solve in nearly real-time as required in real-life DP applications. For this reason, in order to meet these requirements, we have to apply a series of theoretically or physically grounded simplifications which, from one side, allow us to find a minimum thrust in nearly real-time and, from the other side, still take into account all the physical phenomena (e.g., fuel consumption, pitch, and thrust direction and intensity) [81]–[90].

The first and trivial step is to reduce the dimensionality of all problems, eliminating all the equality constraints of (57) by substituting them inside the objective functions and the inequality constraints [81], [86], [91], [92]. Hence, we expressed x- and y-thrust components of the first thruster (see in Fig. 3(a) \( u_{x,1} \) and \( u_{y,1} \)) and the x-component of the fourth thruster (see in Fig. 3(a) \( u_{x,4} \))
\[
    u_{x,1} = \frac{1}{\sigma_1} f_{\text{env},u} - \sum_{i=2}^{k_p} u_{x,i}
\]
(86)
\[
    u_{x,4} = \frac{1}{\sigma_1} f_{\text{env},u} + h_{x,4} u_{y,4} - \sum_{i=2}^{k_p} u_{x,i} - u_{x,4},
\]
(87)

As a second step, we eliminate the binary variable vector \( z_{dg} \), the control vector \( \mathbf{u}_{dg} \), and the load sharing constraints of (78) and (79) to reduce the number of decision variables from twelve to two, without really changing the original problem, namely we simply reformulate the original optimization problem with fewer variables. For this purpose, we have to introduce two new decision variables \( N_{dg} \) and \( u_{dg} \). \( N_{dg} \in \{1, 2, 3, 4\} \) is the number of DGs connected to the SB, and \( u_{dg} \) is their, equal for all, power output. Then, we can reformulate the load balance across the main SB of (60) and the objective function of (51) as follows:
\[
    N_{dg} u_{dg} = g
\]
(89)
\[
    \tilde{m}_f = N_{dg} u_{dg} \sum_{j=0}^{2} c_{dg,j} u_{dg}^j.
\]
(90)
\[
    g = (u_x, u_y, u_p)
\]
(91)

which is what we want to minimize as a function of the delivered power, for a particular DG, is very close to linearity. Consequently, this tells us that, to deliver a particular power, we can both 1) turn on all the DGs and split equally the load, or 2) turn on just the minimum number of DGs required to deliver the requested power. Both options 1) and 2) would not change significantly the total fuel consumption. For this reason, exploiting the observation on specific fuel consumption, we chose the 2) option since it guarantees to us that DGs will work closer to their design point. Based on this comments we can also eliminate \( N_{dg} \) and \( u_{dg} \) as we they can be directly derived from (92) and (93) as follows:
\[
    N_{dg} = \left[ \frac{g}{P_{dg,\text{max}}} \right]
\]
(92)
\[
    u_{dg} = \frac{g}{N_{dg}}.
\]
(93)

This practical and theoretically grounded approximation allows us to further simplify the problem and make it more manageable for near-real-time optimization.

For the convenience of the reader, we have summarized in Tables III and IV the different optimization problems reporting
their objective functions, constraints, parameters decision variables, and exogenous inputs.

D. Solution Methods

In order to solve the optimization problems depicted in Section III, we employed different optimization algorithms, namely Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and a Hybrid PSO with Interior Point Method (PSO-IPM). In fact, other methods (e.g., gradient-based or surrogates) were not able to handle the different optimization problems which require the minimization of highly nonlinear objective functions subject to highly nonlinear constraints. Moreover, practical applications, such as the thrust allocation problem for marine DP systems, require the use of standard libraries and methods stable enough to be employed in daily practice. Hence, GA, PSO, and PSO-IPM are natural choices since they are standard, robust, and reliable methods available in most numerical platforms.

1) Genetic Algorithm: GAs are a family of adaptive search procedures that have been described, extensively analyzed, and successfully employed in the literature [93]–[100]. They derive their name from the fact that they are based on models of genetic change in a population of individuals. These models consist of three basic elements: 1) a Darwinian notion of "fitness," which governs the extent to which an individual can influence future generations, 2) a "mating operator," which produces offsprings for the next generation, and 3) genetic operators that determine the "genetic makeup" of the offsprings based on the genetic material of their parents [101]. There is a large body of both theoretical and empirical evidence showing that, even for very large and complex search spaces, GAs can rapidly locate regions with high fitness ratings [70].

For all the problems reported in Table III, we employed 1000 individuals per generation, whose fitness scores were scaled according to their rank ($r$) as $1/\sqrt{r}$, to allow the evolution of more diverse populations. Five percent of the fittest individuals were copied directly to the next generation, and for the remaining, stochastic uniform selection was employed. Finally, we exploited scattered crossover with a probability of 80%, along with Gaussian mutation. For Problem 1-Original, we exploited the MI-LXPM GA algorithm of [102]. In brief, it consists of extended Laplace crossover and power mutation operators, along with a truncation procedure for the integer restrictions. The constraint handling method of [103] is employed, along with the tournament selection operator. In this case, the number of individuals in each population was increased to 1800, with a crossover probability rate of 90%, an Elite count of 10% of the population, and binary tournament selection. For Problems 2(a) and 3, the Augmented Lagrangian Genetic Algorithm (ALGA) was employed [104], with the constraint handling method proposed in [103].

2) Particle Swarm Optimization: PSO was originally developed by Eberhart and Kennedy [105] and Shi and Eberhart [106]. The method was inspired by the behavior of social organisms in groups, such as bird and fish schooling or ant colonies, and emulates the interaction between members to share information. The search of the optimal solution is performed through agents, referred to as particles, whose trajectories are adjusted by a stochastic and a deterministic component. Each particle has a position and a velocity in the search space, and, at every iteration, each particle trends toward the optimum based on its “best” achieved position and the group’s “best” position. The movement of the particles in the swarm is controlled by the cognitive and the social parameters, indicating the confidence of the particle in itself and the swarm, respectively, and the inertial weight which influences the convergence behavior by increasing the distance the particle will travel from its previous position. At the beginning of the optimization, the PSO algorithm generates a random population of particles over the search space, where the position of each particle represents a solution. These particles are evaluated by computing the values of the objective function to obtain a fitness score, based on which the new position of each particle can be evaluated. The advantages of PSO are the reduced number of parameters to tune, constraints acceptance, and speed in providing good solutions [107], [108]. Furthermore, the stochastic properties of the algorithm allow for solution variability and thorough exploration of the search space in the initial iterations, with a local search behavior during the final iterations [109].

For all the problems reported in Table III excluding Problem 1-Original (which cannot be addressed with PSO since it does not handle properly discrete variables), we set
the swarm size to 180 particles, and the initial particles were randomly and uniformly distributed on the search space. We applied linearly decreasing inertia with a starting value of 1.1, and set the velocity of each particle to be influenced by a local neighborhood of 90% of the entire swarm. Finally, for the velocity adjustment of each particle between iterations, the relative weighting of each particle’s best position and the local neighborhood’s best position were both set to 1.49.

However, when applied in highly nonlinear problems, PSO is known to converge to suboptimal solutions, and its behavior is rather sensitive with respect to its parameters [110]–[112]. Although several studies have been dedicated to its convergence analysis [113]–[117], how to properly adjust its control parameters to achieve good performance is still an open issue.

3) Hybrid PSO With Interior Point Method: To mitigate the PSO problems, we decided to test the PSO-IPM algorithms, in which the best value of all particles in PSO, being close to an optimum point, is given as the initial starting point for the IPM, to perform a more efficient local search. IPMs originate from [118], whereas the modern era of IPM started with the work of [119]. They are classified into three main categories: projective methods, affine-scaling methods, and primal–dual methods. Among the different IPMs, the primal–dual algorithms have gained a reputation for being the most efficient, which is the reason we employ them here. In brief, optimality with every IPM is achieved through the following steps: transforming an inequality constrained optimization problem to equality constrained one, formulate the Lagrange function using logarithmic barrier functions, set the first-order optimality conditions, and apply Newton’s method to the set of equations coming from these optimality conditions.

For PSO-IPM, the parameters have been set as in the PSO algorithm.

IV. CASE STUDY

We considered the 89 800 ton Magellan Class drilling unit depicted in Fig. 1. The main characteristics of the vessel are reported in Table V, while its diesel electric propulsion scheme is shown in Fig. 2. The vessel is equipped with $k_p = 6$ azimuth thrusters, whose layout is reported in Fig. 3(a), and the corresponding forbidden zones are presented in Fig. 3.

The open water diagram of the vessel’s CPPs is available from the Maritime Research Institute Netherlands [63] and is shown in Fig. 4. The quality of the approximations exploited for the representation of the open water diagram at near zero speeds, by means of (25) and (26) of Section II-C, is illustrated in Fig. 5.
Furthermore, the empirical model for the GB losses discussed in Section II-D is parameterized using the set of on-board measurements from the marine GB presented in [67], under several steady-state operating conditions. We present the quality of these approximations in Fig. 6. In Fig. 7(a), the approximations regarding the fuel consumption of the DGs can be appreciated, as discussed during the modelization of the physical system in Section II-G, and for the simplification of the optimization problem in Section III-C.

The approximation errors between the state-space model of the EMs and the Willans approach discussed in Section II-E are presented in Fig. 8. Considering a grid of 25 equidistant points along each dimension (n_em, M_em) of the efficiency map, the error of the model is around 4%. However, Fig. 8 indicates that the errors magnitude depends from the operating conditions. In particular, the error for most of the operating conditions is less than 2%; however, in the near-zero speed and torque regions, the error is around 20%. Apparently, this is a weakness of our approach, but, for the purpose of this work, it turns out that it generates a positive side effect. In fact, in the operating conditions where the efficiency of the EM is low (≤ 30% according to the state-space model) our approximation underestimates the efficiency even further. This provides an extra incentive to the optimization algorithm to avoid these operating conditions for the EMs.

V. Results

In this section, we will analyze the results of exploiting the different optimization problems, summarized in Table III, for DP operations with the optimization algorithms described in Section III-D.

In particular, we will first perform a number of checks on the physical plausibility of the thrust allocation found
by the optimization algorithm. Then we will study the fuel consumption savings when the proposed modelizations are exploited instead of the state-of-the-art ones.

All experiments were performed on a machine equipped with two Intel Xeon Silver 4216, 128 GB of RAM, and 512 GB SSD running Windows Server 2019 and equipped with MATLAB R2020a. MATLAB R2020a already implements the GA in the function `ga`, the PSO in the function `pso`, and the IPM algorithm in the function `fmincon`. The PSO-IPM hybrid approach can be easily performed by setting the `HybridFcn` option of `pso` to `fmincon`.

The parameters employed for the GA, the PSO, and the PSO-IPM optimization algorithms are summarized in Table VI. This parameters have been chosen to obtain the best results (minimum cost) through all the experiments. In Table VI, the optimization problems of Table III solved with the specific optimization algorithm are also reported.

Before presenting our results, we want to make clear that even with extensive parameter tuning, the MI-LXPM GA could not solve the majority of the instances of Problem 1-Original even with many hours of computation. This made clear the necessity of switching from Problem 1-Original to Problem 1-Equivalent as described in Section III-C. For this reason, from now on, we will report only the results of Problems 1-Equivalent, 2(a), 2(b), and 3.

The rest of this section is organized as follows. Section V-A reports the analysis, from a physical point of view, of the solutions of Problem 1-Equivalent in a particular environmental condition. Then Section V-B illustrates the performance of the different DP problems of Table III, using also the different optimizers (see Section III-D), in terms of fuel consumption, fuel savings, and computational requirements.

### A. Physical Plausibility

To ensure the alignment between the results of the thrust allocation and the DP physical problem, we carried out a number of physical plausibility checks. In particular, we chose Problem 1-Equivalent for these checks since, compared with the other problems, it is the most exhaustive and physically accurate model (see Section III). Moreover, it guarantees the minimum fuel consumption (see Section V-B) and it is effectively exploitable in operations with standard optimization algorithms in near real-time (see Section V-B4).

Fig. 9(a)–(e) reports the solutions to Problem 1-Equivalent for five different operating conditions, in terms of relative wind speed $V_{rw}$ and direction $\gamma_{rw}$ of the environmental forces. In particular, for $\gamma_{rw} = \{0^\circ, 45^\circ, 90^\circ, 135^\circ, 180^\circ\}$, namely Fig. 9(a)–(e), we considered $V_{rw}$ as the maximum value that the vessel can sustain while still being able to maintain its position. This operating point is referred to, in literature, as maximum capability [1]. Note that $\gamma_{rw} = \{-45^\circ, -90^\circ, -135^\circ\}$ are not reported since symmetric w.r.t. $\gamma_{rw} = \{45^\circ, 90^\circ, 135^\circ\}$. In each figure, we reported:

1) top left: the operating points of the DG sets [see also Section II-G and Fig. 7(a)];
2) middle left: the operating points of the propellers (see also Section II-C and Fig. 5);
3) bottom left: the operating points of the EMs (see also Section II-E and Fig. 8);
4) right: the delivered thrust configuration, namely, a blue vector representing the thrust’s direction and intensity of each thruster, and the direction and intensity of the environmental forces as red vector applied to the center of gravity of the vessel.

We chose these specific conditions as they can be easily checked for their physical plausibility. In fact, since

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameter</th>
<th>Value/Type</th>
<th>Problem (Table III)</th>
</tr>
</thead>
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<tr>
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<td>Rank</td>
<td>(ii) (a)</td>
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<td>(iii)</td>
</tr>
<tr>
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<td>Crossover rate</td>
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<tr>
<td></td>
<td>Selection function</td>
<td>Stochastic Uniform</td>
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<td>Gaussian</td>
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<tr>
<td>MI-LXPM GA</td>
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</tr>
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<td>Neighbour fraction</td>
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<td>(ii) (b)</td>
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<td>(iii)</td>
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<tr>
<td></td>
<td>Social adjustment weight</td>
<td>1.49</td>
<td></td>
</tr>
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</table>
in Fig. 9(a)–(e) we are considering the maximum $V_{rw}$ that the vessel can sustain, still being able to maintain its position, it is reasonable to assume that the DGs, the thrusters, and the EMs will work in similar operating condition, in terms of delivered total thrust and requested total power. In order to support this statement, in Table VII we reported, for each considered conditions of Fig. 9(a)–(e), the total trust, the total EMs power, and total DGs power. As expected, all the reported conditions are sharing a similar lever of total trust, total EMs power, and total DGs power.

From the results reported in Fig. 9(a)–(e), we can make a series of observations:

1) As expected the selected wind direction and speed puts the system in a state for which it should operate near its design point. In fact, the vessel exploits all the five DGs operating approximately at 74% of their maximum power to sustain the maxim wind speed at different angles. It is easy to verify that the operation of four or six DG sets for the same load would result in higher fuel consumption.

2) As expected (see Section II-C), all the propellers operate at their design pitch, maximizing their efficiency.

3) As expected (see Section II-E), the induction motors operate at their most efficient region, with an efficiency of 0.96, very close to their nominal point.

4) In Fig. 9(a) and (e), we can note that the thrusters produce thrust in exactly the opposite direction from the environmental forces. This is possible because for these operating conditions there is no interaction between any two thrusters. Note that, considering (90) and (60), any other thrust configurations with nonzero forces on the lateral ship axis $u_{y,i}$ would result in the production of unnecessary additional power from the DG sets, which subsequently would increase total fuel consumption.

5) In Fig. 9(c), the environmental conditions act only on the lateral axis of the vessel. Fig. 9(c) shows that the forbidden zone constraints (see Section III-A) prevent the thrusters from operating solely in the lateral direction due to thruster–thruster flow interaction. We can verify, with the support of Table VII, that this is the
optimal thrust configuration as it requires the same amount of total DGs power of the conditions reported in Fig. 9(a) and (e). Instead, the EMs operate at slightly different points of lower rotational speed and higher torque; however, their efficiency is still equal to the highest possible value of 0.96. Finally, we can see that the thrust configuration of the two thruster triplets on the fore and aft of the vessel is not symmetrical. This is due to the asymmetry of the hull on the fore and aft of the vessel, which produces a certain moment on the yaw direction that has to be accounted for, to keep the vessel in a stationary position on the earth-fixed reference frame.

6) We can also verify that the solutions of reported in Fig. 9(b) and (d) are physically reasonable, although the optimal solution is far from trivial to identify, primarily due to the forbidden zone constraints. An overview of the longitudinal and lateral components of the thruster and environmental forces and moments is presented in Table VIII. In order to present all these results, it is necessary to introduce the concept of capability plot (CP), which will be instrumental to present the results in an intelligible way.

1) Capability Plot: The DP CP is a crucial tool to assess a vessel’s ability to retain its heading and position under certain environmental conditions [1]. Based on a given vessel design, the CPs consider the total environmental forces (due to wind, waves, and ocean currents) both in terms of direction and intensity.

In particular, to build the CP, it is necessary:

1) To vary the wind direction $\gamma_{rw} \in [0, \ldots, 360]$ and speed $V_{rw}$. The rage of the speed will be clarified soon.

2) To fix the waves and the ocean currents intensity to their [e.g., considering the average significant wave high in Section II-B, (10) and (11)] while vary their direction in accordance with the wind direction. Namely, one has to consider the worst case scenario in which the wind, waves, and currents are acting in the same direction.

3) To compute the thrust allocation in accordance with the formulations reported in Table III.

4) To increase $V_{rw} \in [0, \ldots]$ until the vessel is able to maintain its stationary position, namely, the environmental forces are balanced by the thrust offered by the thruster configuration.

5) To report, for each $\gamma_{rw}$ and $V_{rw}$, the desired quantity (e.g., fuel consumption or fuel savings). The latter is presented on a polar plot, where $\gamma_{rw}$ is the angle coordinate and $V_{rw}$ is the distance coordinate. We assume that $0^\circ$ corresponds to forces acting on the bow, and $180^\circ$ corresponds to forces acting on the stern of the vessel.

2) Fuel Savings—All Conditions: In Fig. 10, we report different CPs when the sea state has been fixed to moderate as it is the most frequent sea state according to [121]. This particular scenario corresponds to degree 4 in the Douglas sea scale, for this reason, we utilized a significant wave height equal to 2 m as input to obtain the wave second-order assuming Pierson–Moskowitz wave spectrum [122].

Fig. 10(a)–(d) reports the fuel consumption of each DP optimization problem of Table III, apart from Problem 1-Original. In fact, even with extensive parameter tuning, the MILPXPM GA failed to solve the majority of the instances of Problem 1-Original, even at run-times an order of magnitude higher than the ALGA in Problem 1-Equivalent.

By examining the relative fuel savings achieved by Problems 1, 2(a), and 2(b) against Problem 3, as reported in Fig. 10(e)–(g), the advantage of using a more complex

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\gamma_{rw}$ $[\text{deg}]$ & $V_{rw} \text{ [m/s]}$ & $f_{s,1} \text{ [MN]}$ & $f_{s,2} \text{ [MN]}$ & $f_{s,3} \text{ [MN]}$ & $f_{s,4} \text{ [MN]}$ & $f_{s,5} \text{ [MN]}$ & $f_{s,6} \text{ [MN]}$ & $\sum f_s \text{ [MN]}$ & $\sum f_y \text{ [MN]}$ & $\sum f_x \text{ [MNm]}$ \\
\hline
0 & 100 & -0.92 & 0.00 & -0.92 & 0.00 & -0.92 & 0.00 & -0.92 & 0.00 & -0.92 \\
45 & 27 & -0.17 & 0.31 & -0.09 & 0.24 & -0.09 & 0.30 & -0.14 & 1.42 \\
90 & 37 & 0.03 & -1.09 & -0.24 & -1.03 & 0.01 & -1.09 & 0.02 & -1.03 \\
135 & 27 & 0.07 & 1.39 & 0.34 & 0.97 & 0.17 & 1.03 & 0.06 & 0.37 \\
180 & 100 & 0.92 & 0.00 & 0.92 & 0.00 & 0.92 & 0.00 & 0.92 & 0.00 & 0.92 \\
\hline
\end{tabular}
\caption{Thruster Forces and Environmental Disturbances for the Operating Conditions Presented in Figure 9}
\end{table}

\footnote{Note that, the conventional CP just reports the maximum $V_{rw}$ [1]}

In order to present all these results, it is necessary to introduce the concept of capability plot (CP), which will be instrumental to present the results in an intelligible way.

In order to present all these results, it is necessary to introduce the concept of capability plot (CP), which will be instrumental to present the results in an intelligible way.

$$1N$$
and detailed DP optimization problem becomes evident. From Fig. 10(e) it is possible to observe that if instead of minimizing for total thrust according to Problem 3, one minimizes the fuel consumption according to Problem 1-Equivalent results in fuel savings up to 6% for very low wind speeds and very low environmental forces. Problem 2(a) results in lower savings (up to 4%). Finally, Problem 2(b) provides essentially no additional savings. This result is expected, as the objective functions of Problem 2(b) and Problem 3 are equivalent, and just the constraints change (see Table III).

Finally, to better appreciate the fuel savings of Problems 1, 2(a), and 2(b) with respect to Problem 3, in Fig. 10(h) we report the fuel savings in the CP distribution and their averaged values. Clearly, Problem 1-Equivalent guarantees the best saving and Problem 1-Equivalent, 2(a), 2(b) provide an average fuel saving of 3.12%, 2.06%, and 0.01%, respectively.

3) Fuel Savings—In Operations: To assess the benefits of using the newly proposed DP optimization problems in actual operating conditions, we exploited the wind and wave (the sea state according to the Douglas sea scale) data of the North Atlantic and North Pacific area that were collected during an extensive measurement campaign in [121] together with their probability of occurrence. These data are reported in Table IX. In the same table, it is also reported the average fuel savings in that particular sea state, when using Problems 1, 2(a), and 2(b) against Problem 3. Note that, as expected and commented in Section V-B2, Problem 1 guarantees the largest savings in all sea states, and the largest savings are achieved in lower sea states.

Since the probability of occurrence of each sea state in the North Atlantic and North Pacific area, we can also estimate the lifetime average fuel savings using Problems 1, 2(a),
and 2(b) with respect to Problem 3. These quantities are reported in Table X. From this table it is possible to observe that, using Problem 1, guarantees and approximately extra 4% fuel savings in both regions.

4) Computational Requirements: In Fig. 11, we report different CPs in the same conditions described in Section V-B2 for Fig. 10.

In particular, Fig. 11(a)–(c) reports the fuel consumption for Problem 1-Equivalent (the best performing one according to the results of Sections V-B2 and V-B3) using different optimizers (GA, PSO-IPM, and PSO) described in Section III-D. By observing Fig. 11(a) and (b), the fuel consumption seems to be similar regardless of the exploited optimizer (GA or PSO-IPM), while for PSO the behavior is quite different. In order to better understand these results, in Fig. 11(d) and (e) we reported the percentage of fuel savings using, respectively, the PSO-IPM and PSO optimizers against GA. From these results, it is possible to observe that the PSO-IPM optimizer is slightly worse than the GA, while the PSO is largely worse than GA.

Moreover, Fig. 11(f)–(h) reports the time required to find the optimal solution, still for Problem 1-Equivalent using the different optimizers. From Fig. 11(a) and (b), it is possible to note that the time seems to be very high for GA and very low.
for PSO, and, as intuitively expected, the time grows as we get close to the CPs limits. Furthermore, in this case, to provide the reader with a better understanding of the reported results, in Fig. 11(i) and (j) we reported the percentage of time savings using, respectively, the PSO-IPM and PSO optimizers against the GA. For this reason, it is possible to observe that the PSO optimizer is the fastest method while the GA is the slowest. PSO-IPM is slightly slower than PSO.

From these results, it is possible to conclude that PSO-IPM is the method with the best tradeoff between accuracy and computational requirement, which allows to use Problem 1-Equivalent in near real-time DP applications.

VI. CONCLUSION

Marine DP systems play a central role in offshore maritime operations ensuring the mission of the vessels (e.g., drill line, pipe-laying, coring, and ocean observation) by maintaining the vessel’s position and heading using its own propellers and thrusters to compensate exogenous disturbances, like wind, waves, and currents. For these types of vessels, the DP system is the primary cause of fuel consumption, having a strong impact on its overall footprint. For this reason, this article faced the problem of optimizing the propellers-thruster allocation, namely, determining the thrust and direction of each propeller and thruster, to maintain its position and heading, while minimizing the fuel consumption.

State-of-the-art approaches commonly exploit a simplified approach where simpler (mostly convex), yet related, optimization problems are exploited as surrogates to keep the problem and the computational requirements at a level suited for a near-real-time control. This allows one to design a DP system able to operate in near-real-time, allowing its exploitation on-board during operation by simply integrating it in the automation system.

The novelty of this article lies in the following aspects. First, we propose to directly face the problem with a high fidelity modeling approach, second we exploit physical and theoretical funded proprieties for achieving optimal solutions in near-real-time with standard optimizers.

The authors showcased the proposed approach applying their methodology on a drilling unit equipped with six thrusters to evaluate the potential fuel consumption savings. The results showed that it is possible to achieve up to 5% of fuel savings with respect to conventional approaches.

Nevertheless, space for improvement still exists. For example, we could incorporate the thruster–thruster hydrodynamic interaction effects. In this work, the standard approach of avoiding these effects has been employed, by adding the forbidden zone constraints for some azimuth angles in which severe interaction occurs. However, the authors of [5] indicate that additional benefits arise by representing these effects with thrust efficiency curves, instead of using forbidden zone constraints. Another improvement could be related to the modelization of frequent DG starts and stops. Although these effects have not been investigated in this work, it is known that frequently starts and stops are harmful for the lifetime of any internal combustion engine [123]. Another interesting direction of research could be to examine the performance of our approach aiming at directly reducing the emissions [123]. Finally, custom optimizers for the proposed DP problems could be developed targeting higher efficiency and effectiveness with respect to standard general-purpose optimizers.

REFERENCES


TABLE IX

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<tr>
<th>Sea State</th>
<th>V_w [m/s]</th>
<th>Probability occurrence [%]</th>
<th>Average fuel savings w.r.t. Problem (a)</th>
<th>Problem 1</th>
<th>Problem 2(A)</th>
<th>Problem 2(B)</th>
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<td></td>
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<tr>
<td>1</td>
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TABLE X

<table>
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<th>Problem</th>
<th>Lifetime Average Fuel Savings [%]</th>
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<th>North Pacific</th>
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<td>Problem (b)</td>
<td>(4.04)</td>
<td>(3.98)</td>
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<td>(2.67)</td>
<td>(2.53)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>
A. J. Sørensen, S. T. Quek, and T. D. Nguyen, "Improved operability"


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